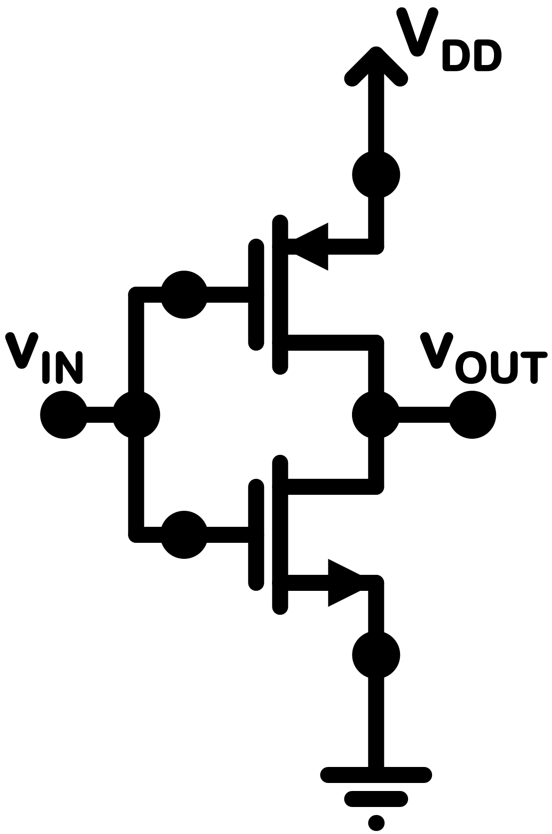
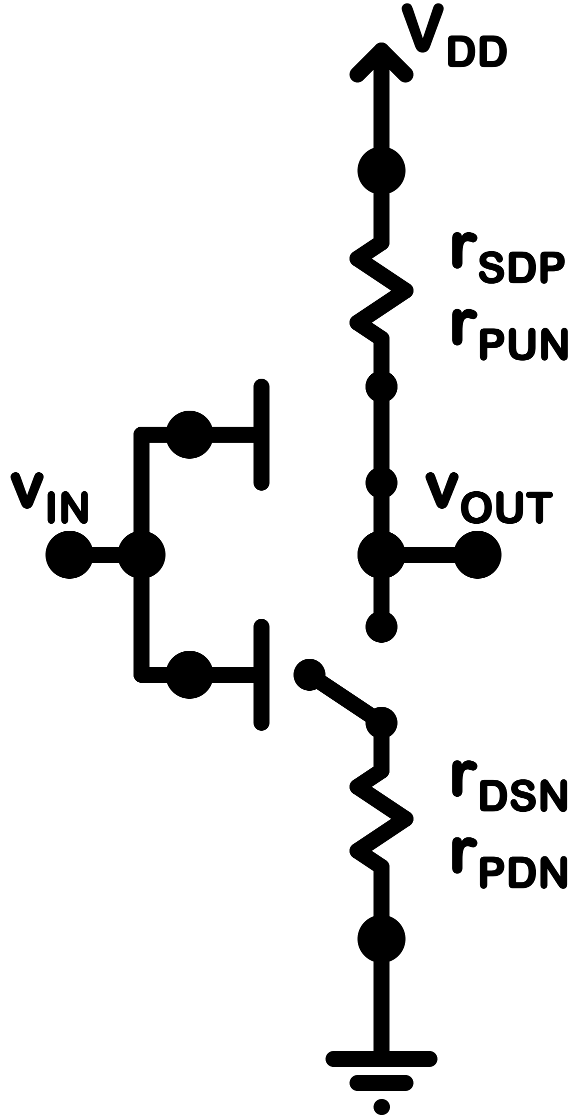
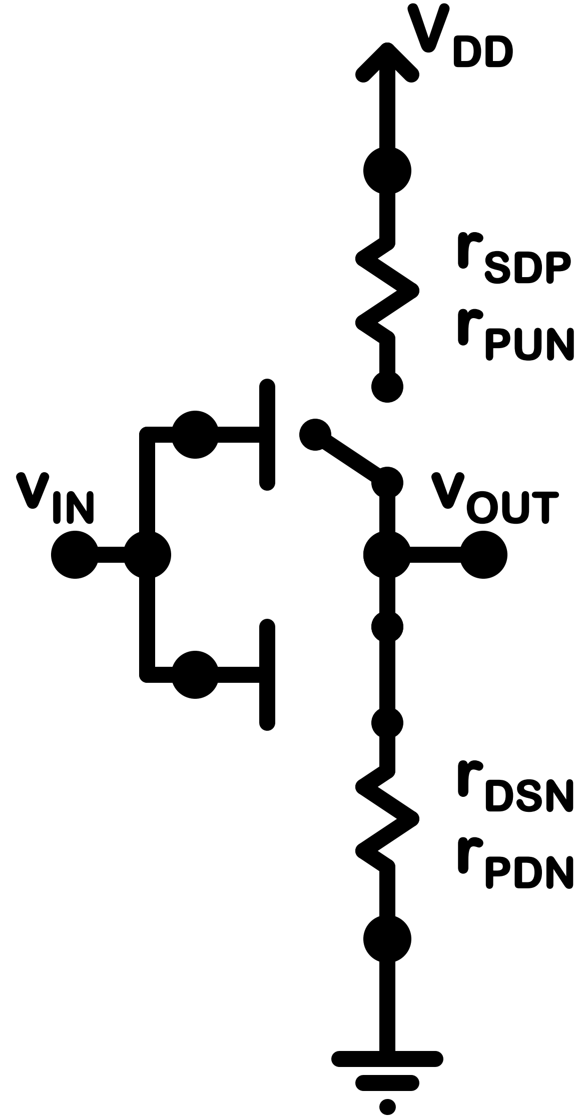
CMOS Inverter



$v_{IN} = 0$
$v_{OUT} = V_{DD}$



$v_{IN} = V_{DD}$
$v_{OUT} = 0$



CMOS Inverter

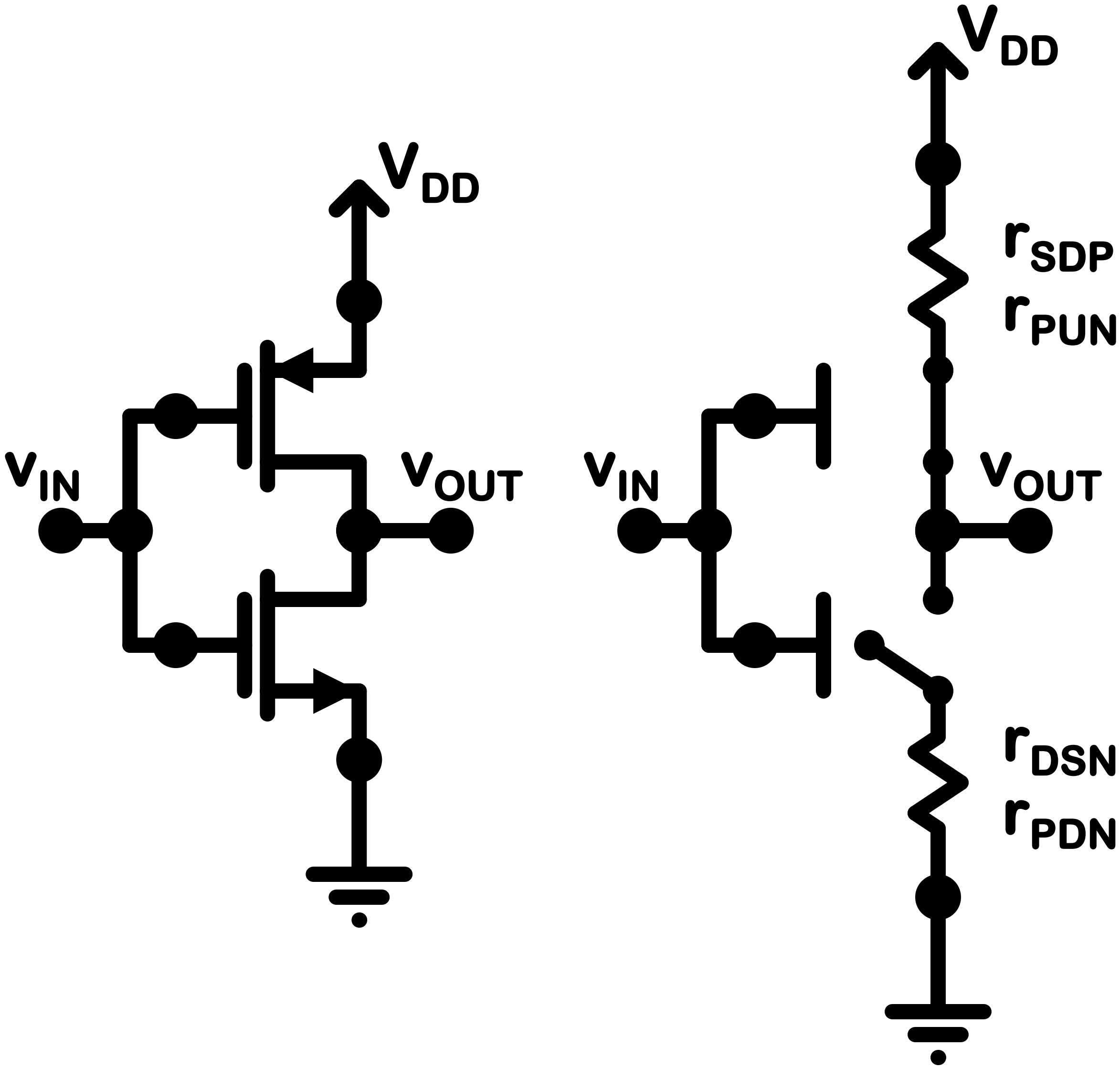
$v_{IN} = 0$

$v_{OUT} = V_{DD}$

Q_P: Triode small v_{SD}

Q_N: Cutoff

$r_{PUN} = r_{SDP}$



CMOS Inverter

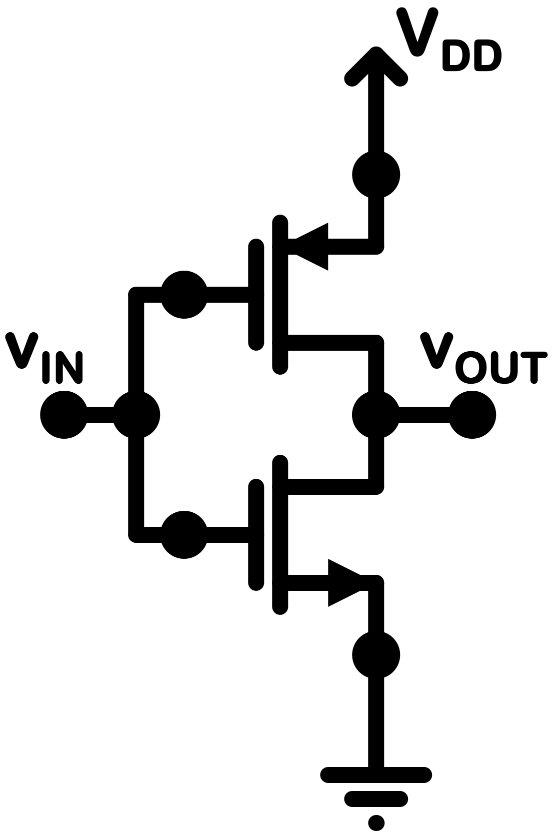
$v_{IN} = 0$

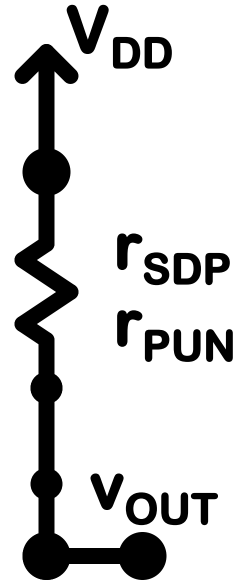
$v_{OUT} = V_{DD}$

Q_P: Triode small v_{SD}

Q_N: Cutoff

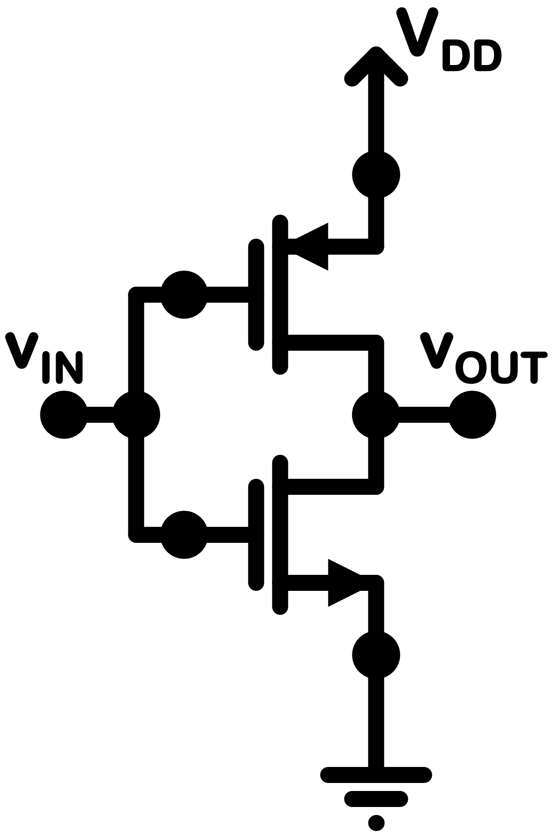
$r_{PUN} = r_{SDP}$

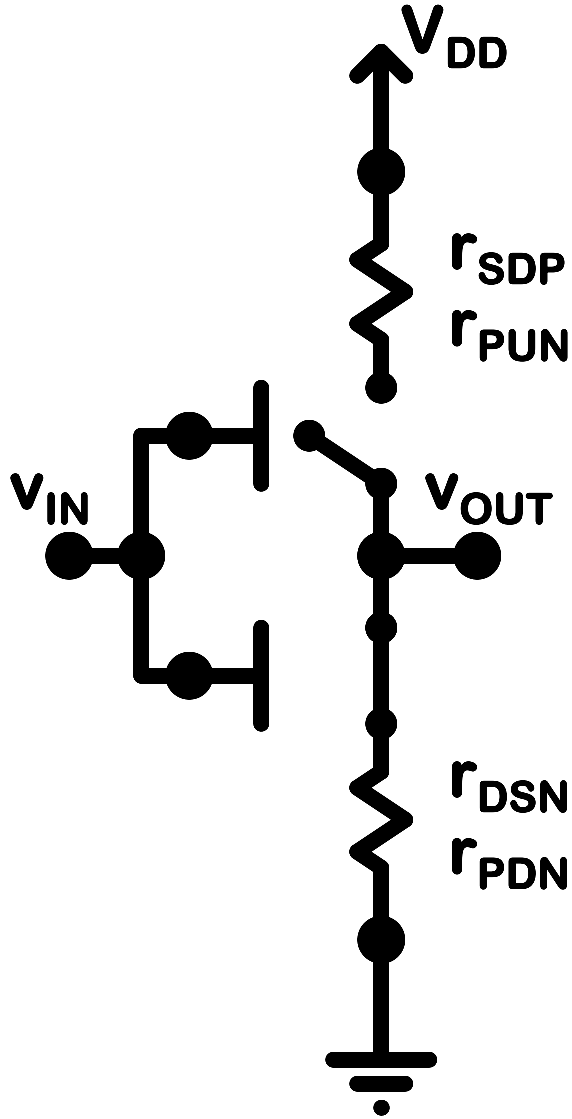




CMOS Inverter

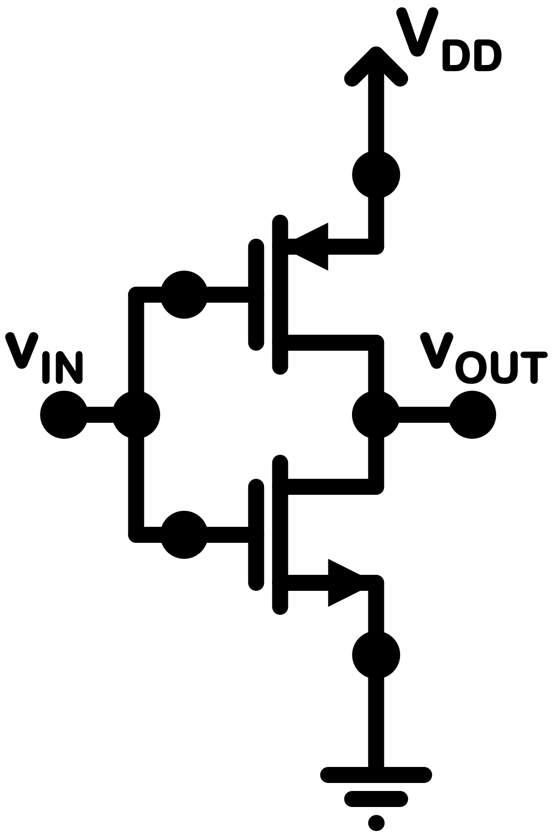
$v_{IN} = V_{DD}$
$v_{OUT} = 0$
Q_P: Cutoff
Q_N: Triode small v_{DS}
$r_{PDN} = r_{DSN}$

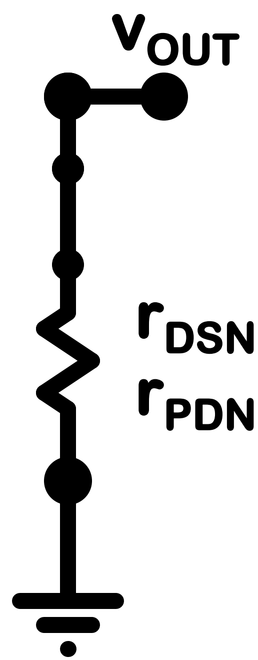




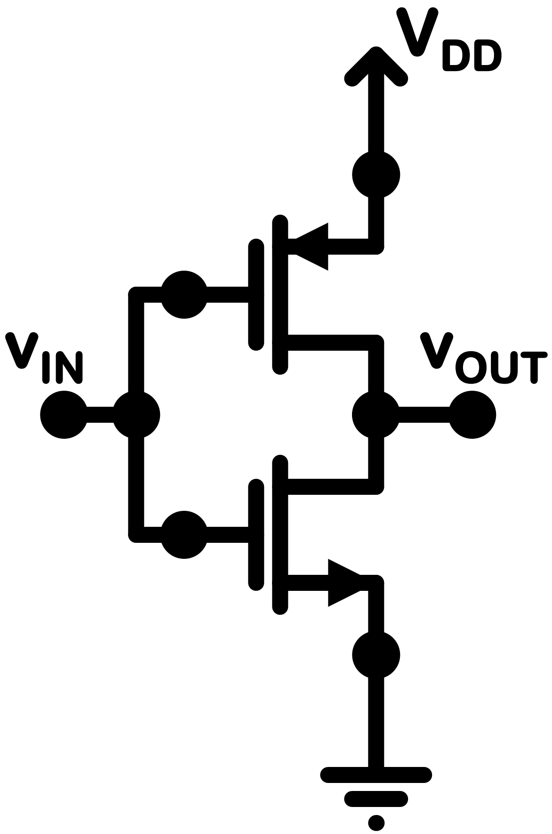
CMOS Inverter

$v_{IN} = V_{DD}$
$v_{OUT} = 0$
Q_P: Cutoff
Q_N: Triode small v_{DS}
$r_{PDN} = r_{DSN}$





CMOS Inverter



$v_{IN} = V_{DD}$

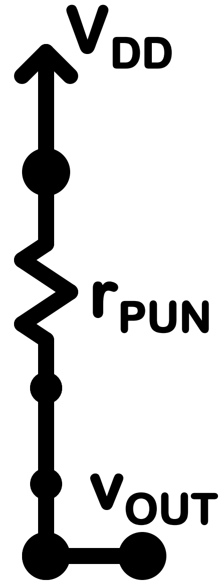
$v_{OUT} = 0$

Q_P: Cutoff

Q_N: Triode small v_{DS}

$r_{PDN} = r_{DSN}$

$r_{DSN} = 1/(k_n*(V_{DD}-V_{tn}))$



$v_{IN} = 0$

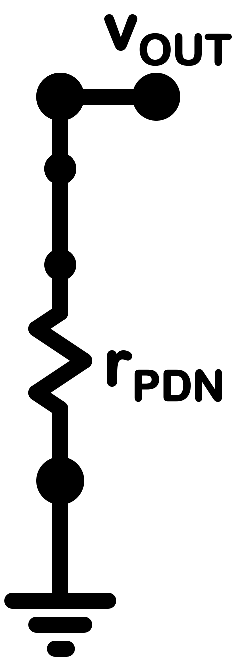
$v_{OUT} = V_{DD}$

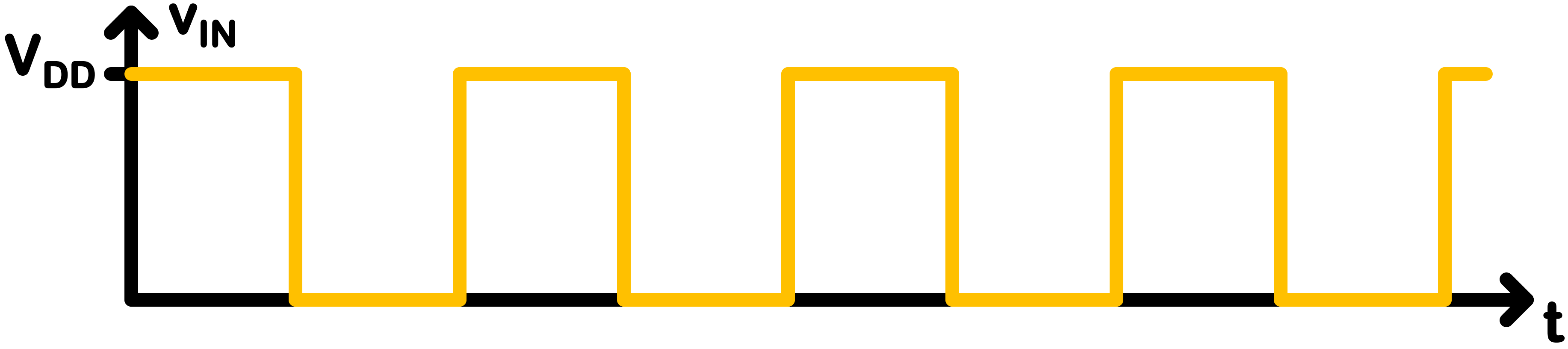
Q_P: Triode small v_{SD}

Q_N: Cutoff

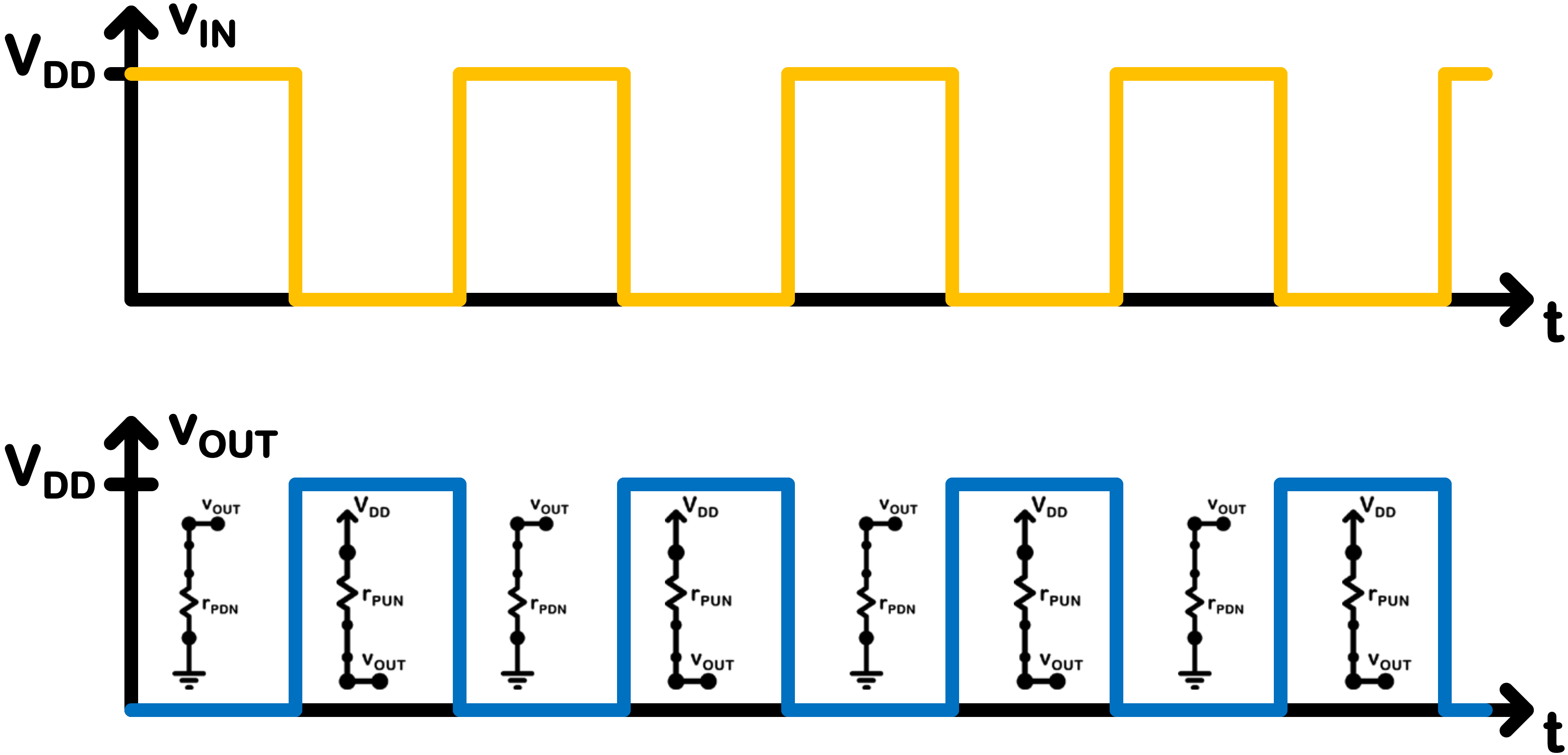
$r_{PUN} = r_{SDP}$

$r_{SDP} = 1/(k_p*(V_{DD}-|V_{tp}|))$



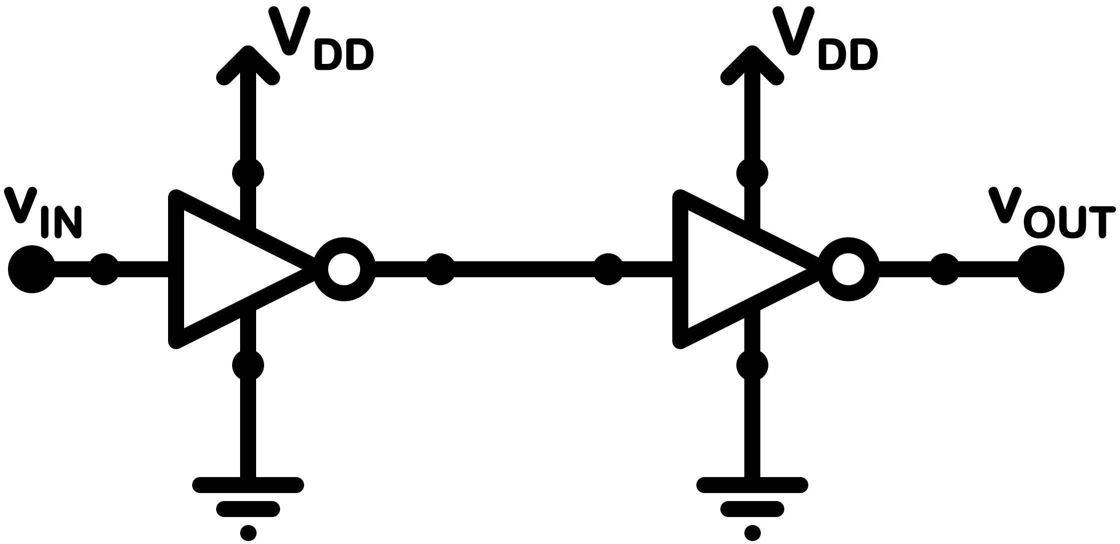


VDD
VIN
t

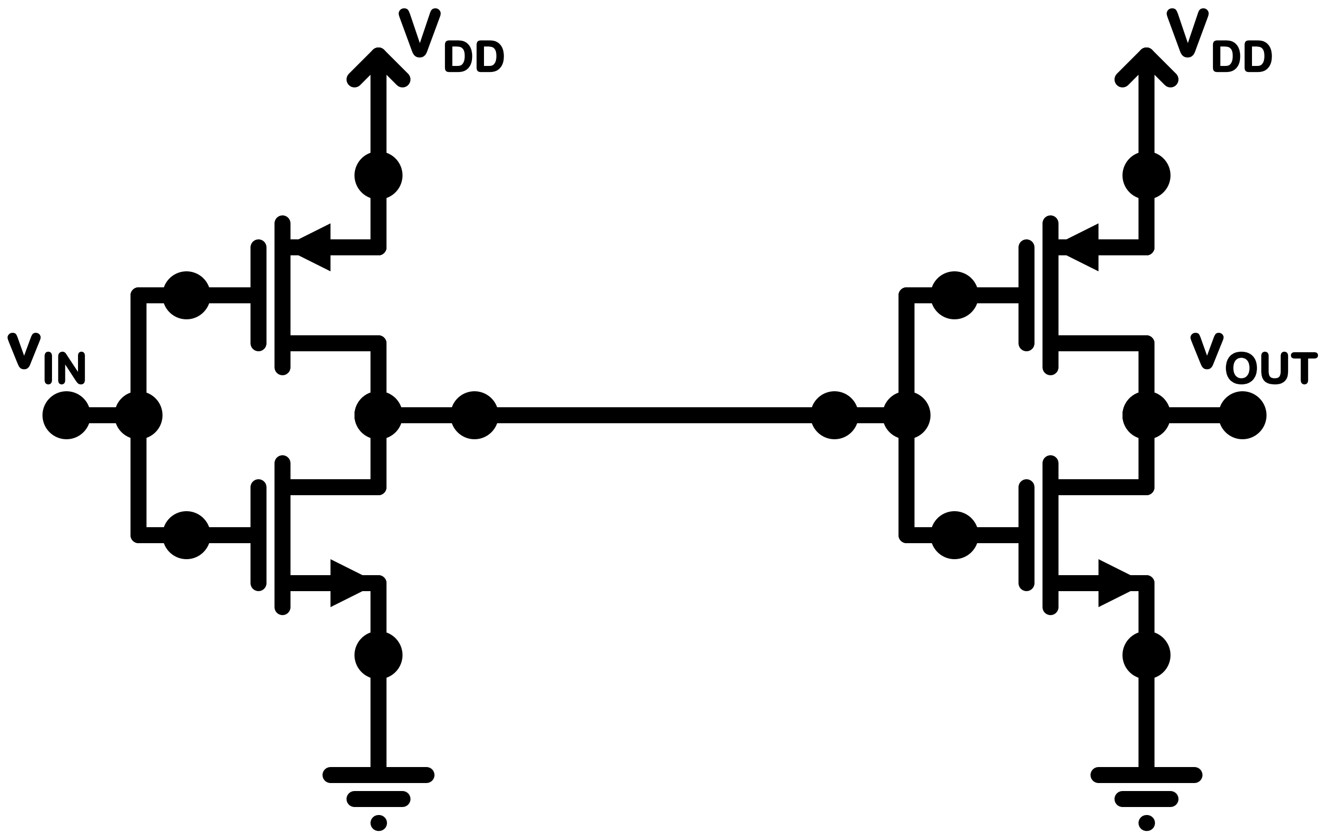


V_{DD}
v_{IN}
t
V_{DD}
v_{OUT}
v_{OUT}
r_{PDN}
V_{DD}
r_{PUN}
v_{OUT}
t

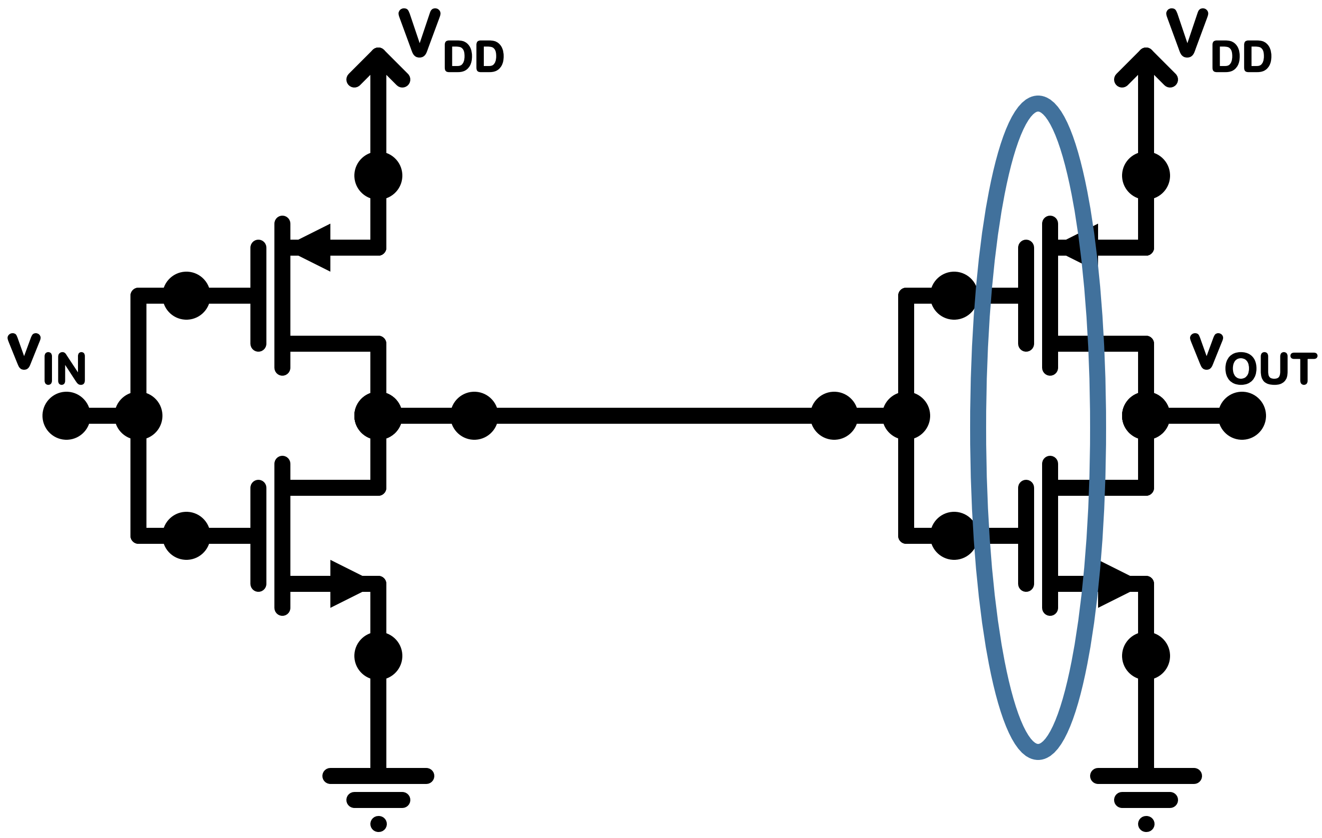
CMOS Inverter



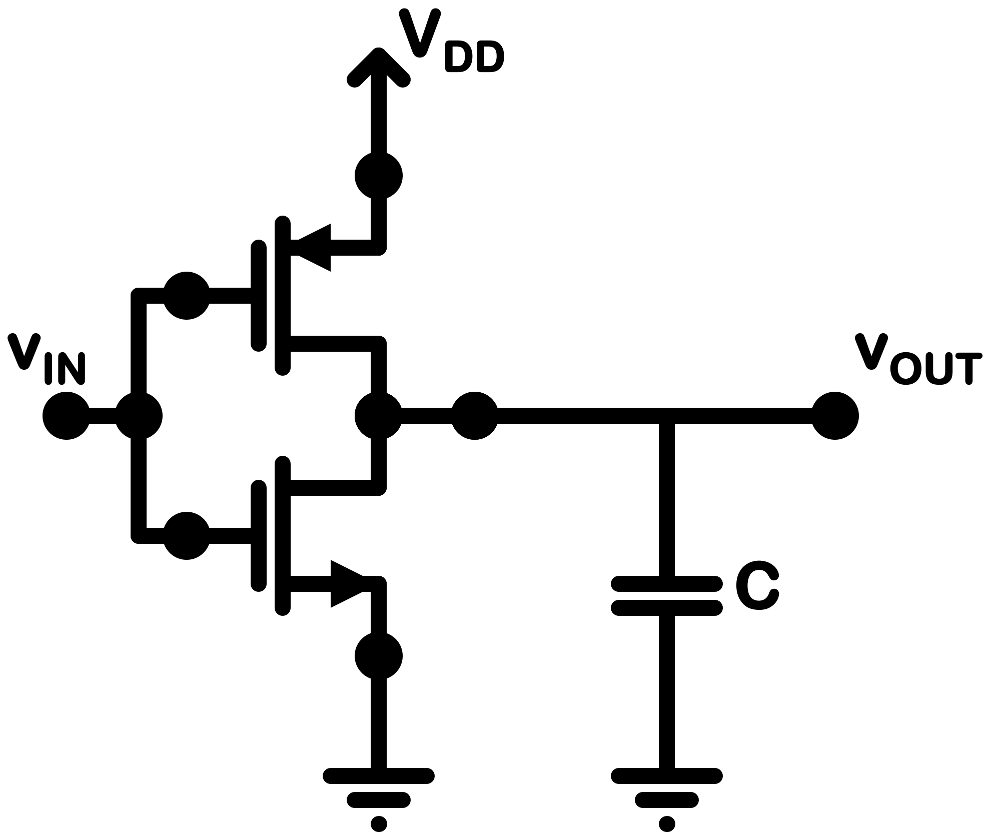
CMOS Inverter



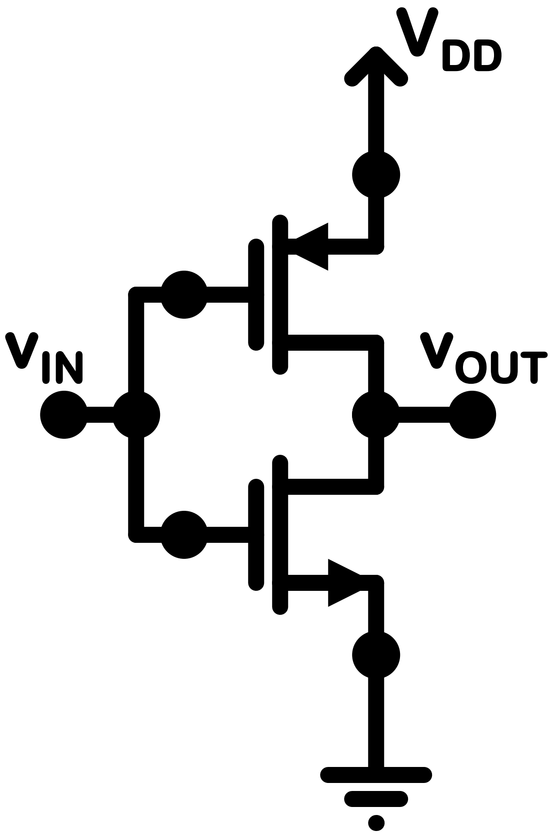
CMOS Inverter

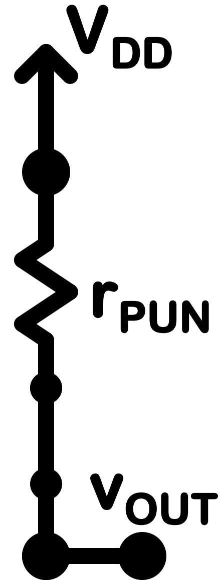


CMOS Inverter



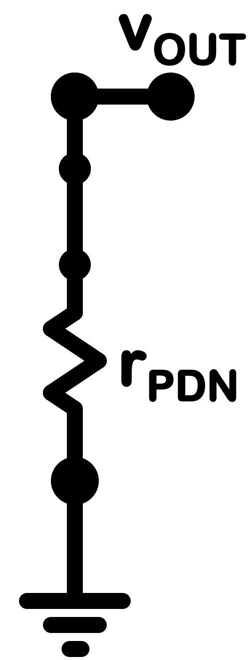
CMOS Inverter





$v_{IN} = V_{DD}$
$v_{OUT} = 0$
Q_P: Cutoff
Q_N: Triode small v_{DS}
$r_{PDN} = r_{DSN}$

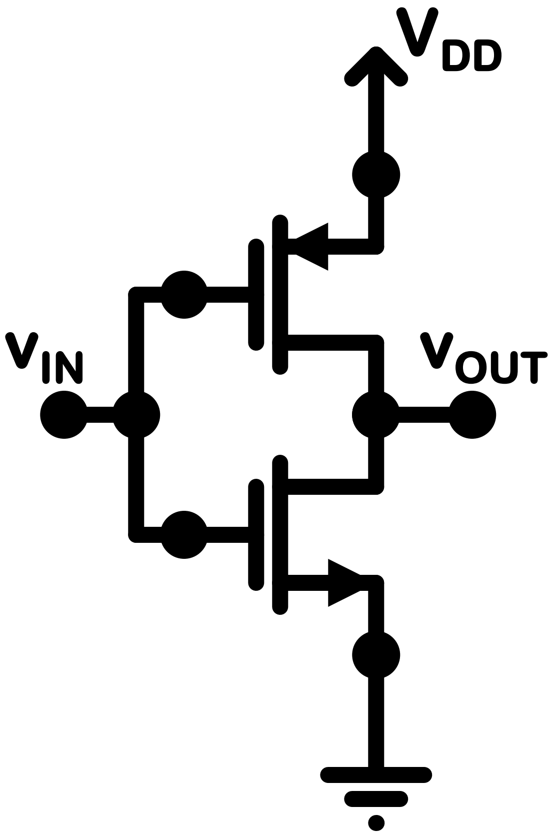
$v_{IN} = 0$
$v_{OUT} = V_{DD}$
Q_P: Triode small v_{SD}
Q_N: Cutoff
$r_{PUN} = r_{SDP}$

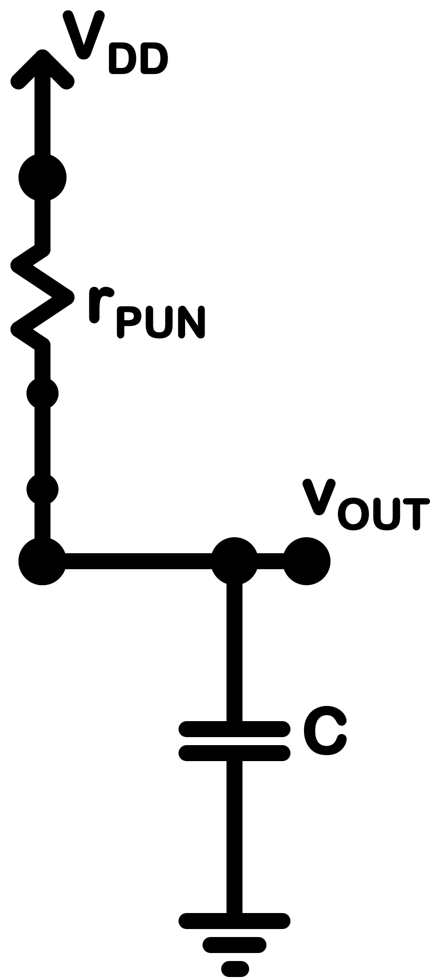


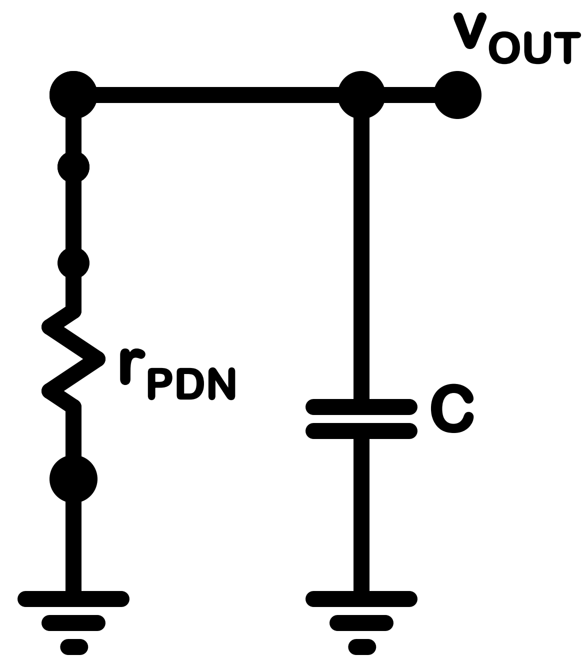
CMOS Inverter

$v_{IN} = 0$
$v_{OUT} = V_{DD}$
Q_P: Triode small v_{SD}
Q_N: Cutoff
$r_{PUN} = r_{SDP}$

$v_{IN} = V_{DD}$
$v_{OUT} = 0$
Q_P: Cutoff
Q_N: Triode small v_{DS}
$r_{PDN} = r_{DSN}$



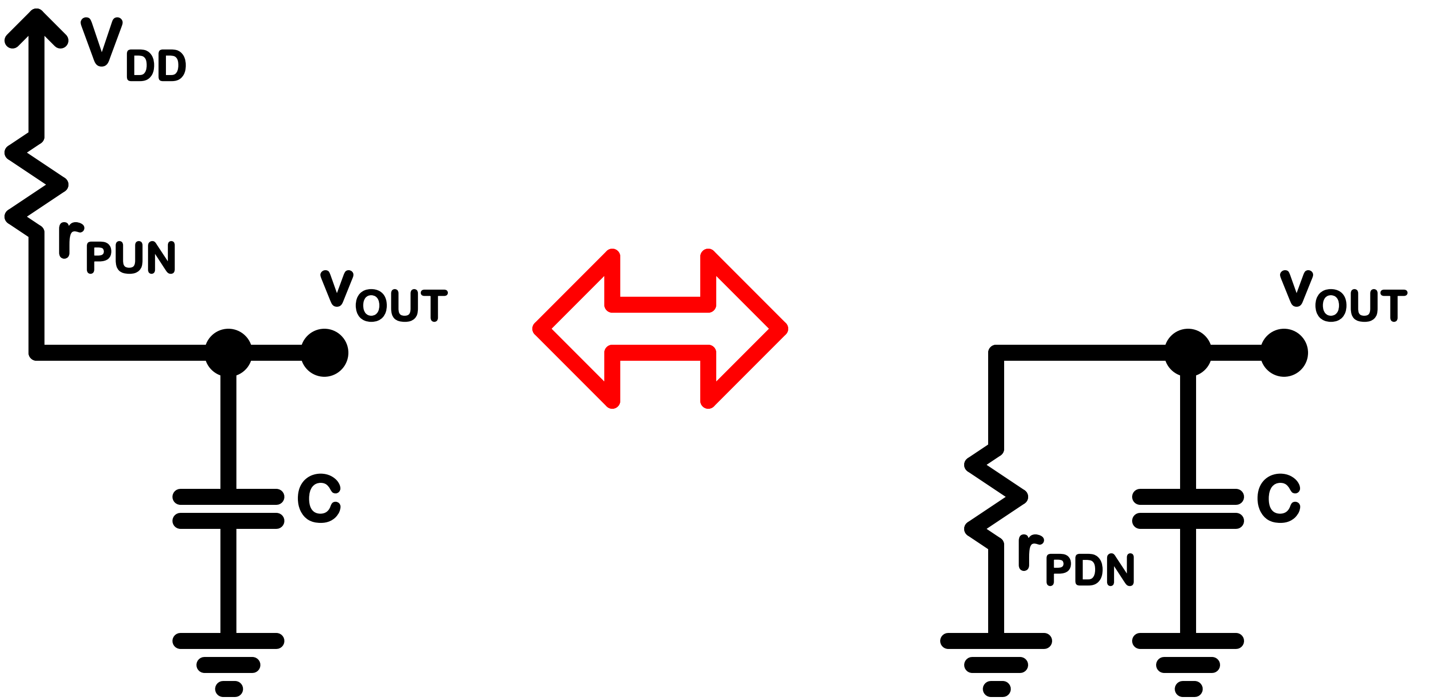


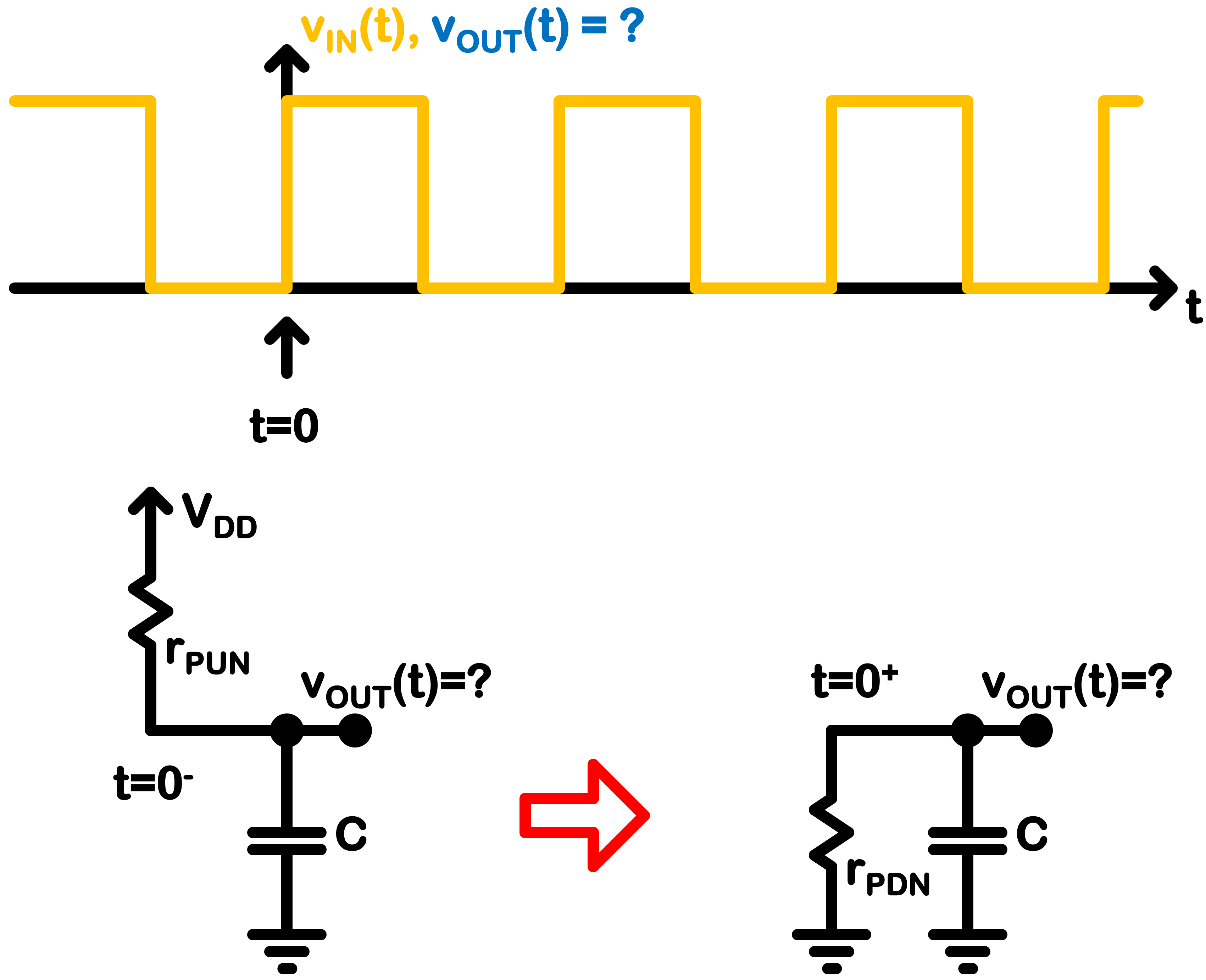


CMOS Inverter

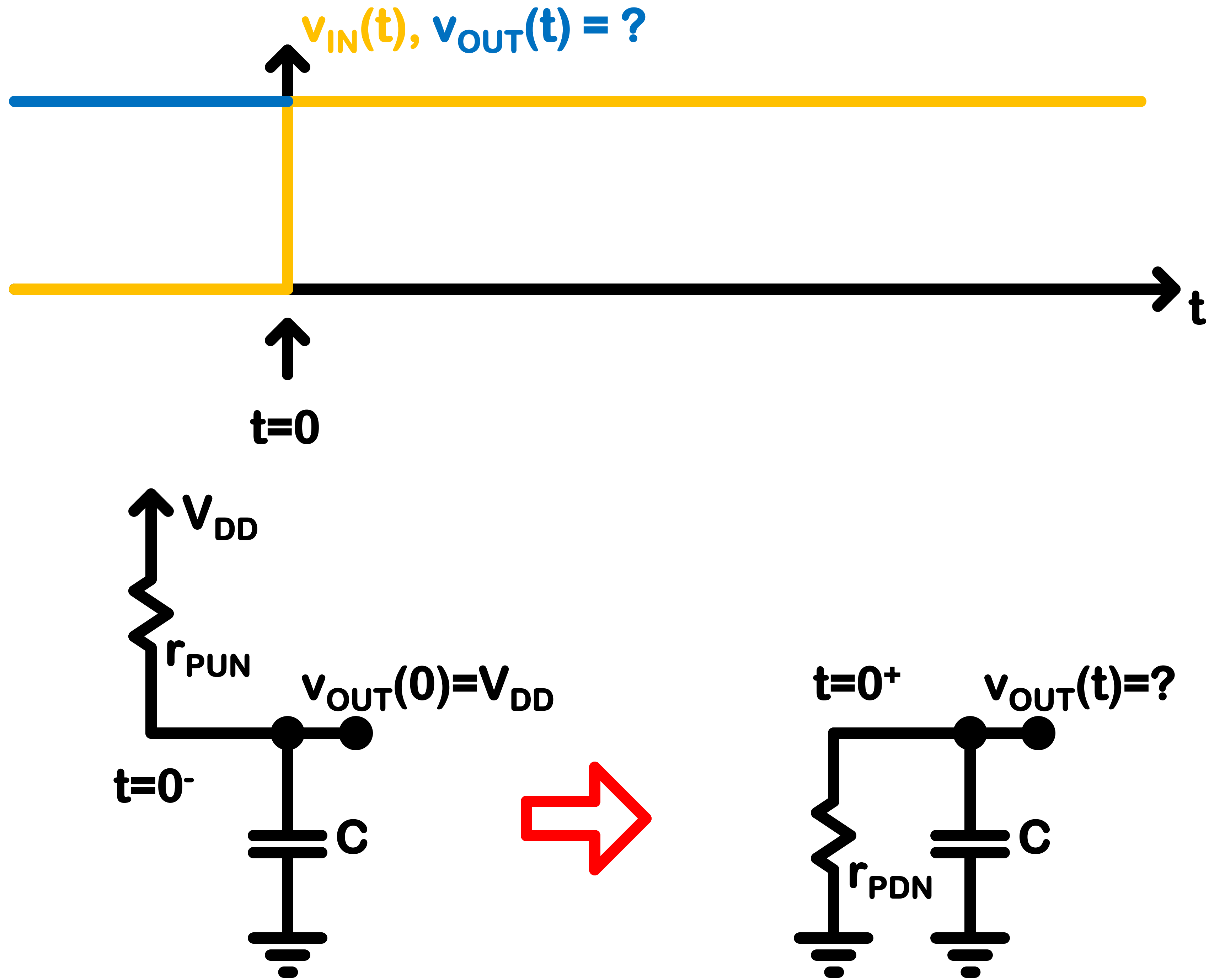
$v_{IN} = 0$
$v_{OUT} = V_{DD}$

$v_{IN} = V_{DD}$
$v_{OUT} = 0$

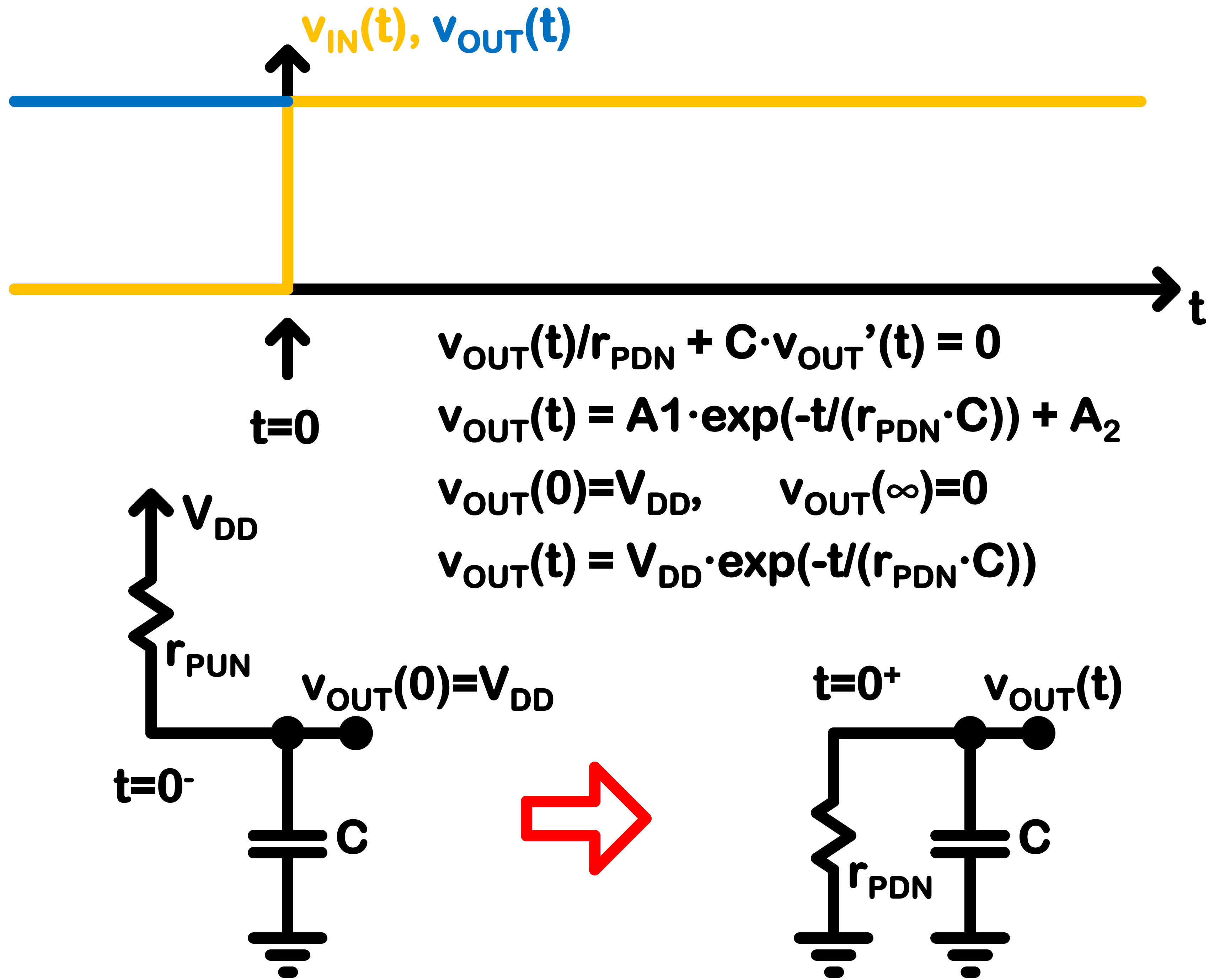




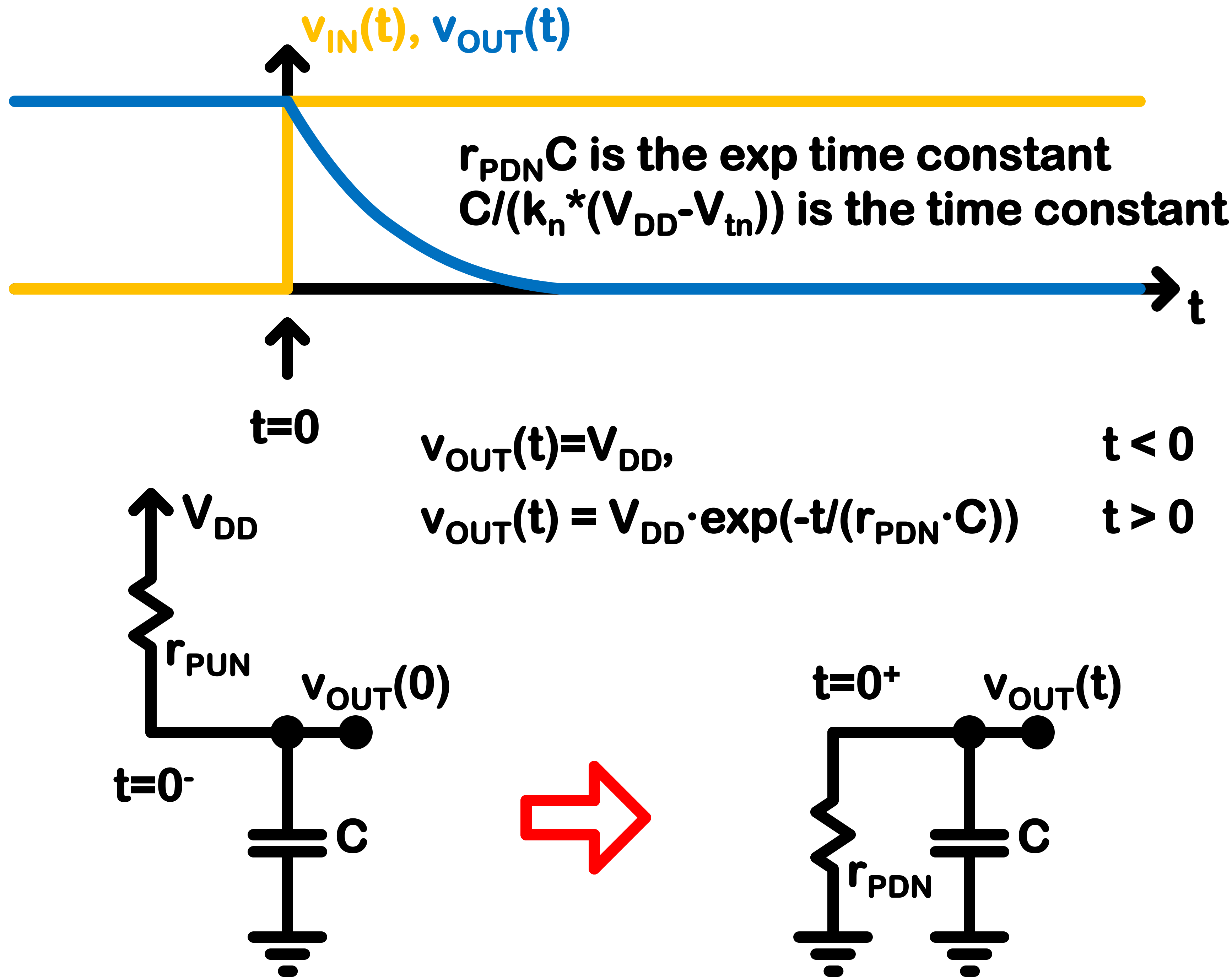
$v_{IN}(t)$, $v_{OUT}(t)$ = ?
t
t=0
V_{DD}
r_{PUN}
$v_{OUT}(t)$=?
t=0⁻
C
t=0⁺
$v_{OUT}(t)$=?
C
r_{PDN}



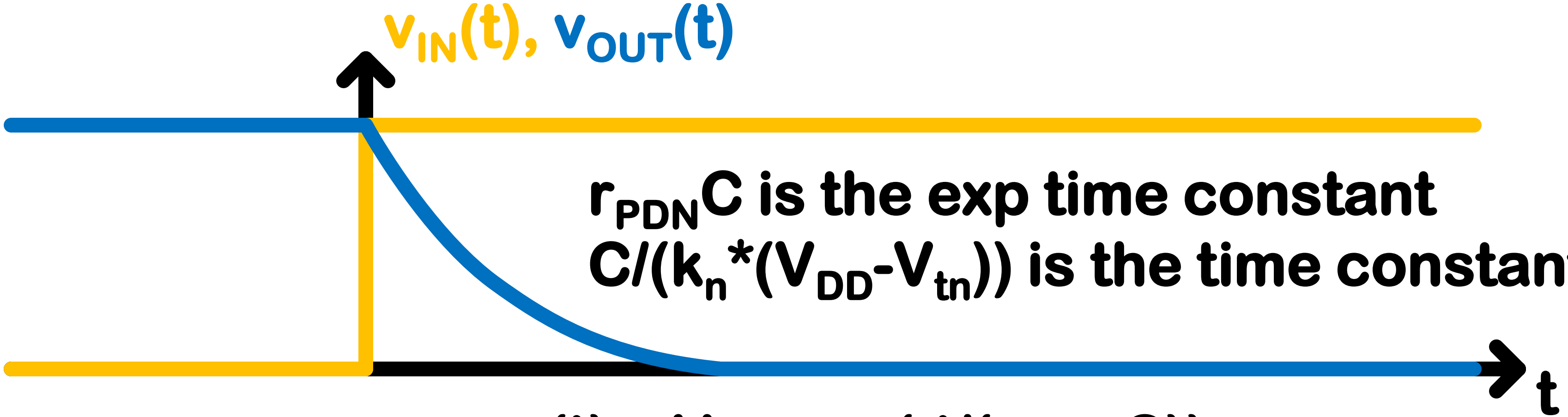
vIN(t), vOUT(t) = ?
t
t=0
VDD
rPUN
vOUT(0)=VDD
t=0-
C
t=0+
vOUT(t)=?
rPDN
C



$v_{IN}(t)$, $v_{OUT}(t)$
t
t=0
$v_{OUT}(t)/r_{PDN} + C\cdot v_{OUT}'(t) = 0$
$v_{OUT}(t) = A1\cdot \exp(-t/(r_{PDN}\cdot C)) + A_2$
$v_{OUT}(0)=V_{DD}$, $v_{OUT}(\infty)=0$
$v_{OUT}(t) = V_{DD}\cdot \exp(-t/(r_{PDN}\cdot C))$
V_{DD}
r_{PUN}
$v_{OUT}(0)=V_{DD}$
$t=0^-$
C
$t=0^+$
$v_{OUT}(t)$
r_{PDN}
C



$v_{IN}(t)$, $v_{OUT}(t)$
$r_{PDN}C$ is the exp time constant
$C/(k_n*(V_{DD}-V_{tn}))$ is the time constant
t
t=0
$v_{OUT}(t)=V_{DD}$, t < 0
$v_{OUT}(t) = V_{DD}\cdot exp(-t/(r_{PDN}\cdot C))$ t > 0
V_{DD}
r_{PUN}
$v_{OUT}(0)$
$t=0^-$
C
$t=0^+$
$v_{OUT}(t)$
C
r_{PDN}

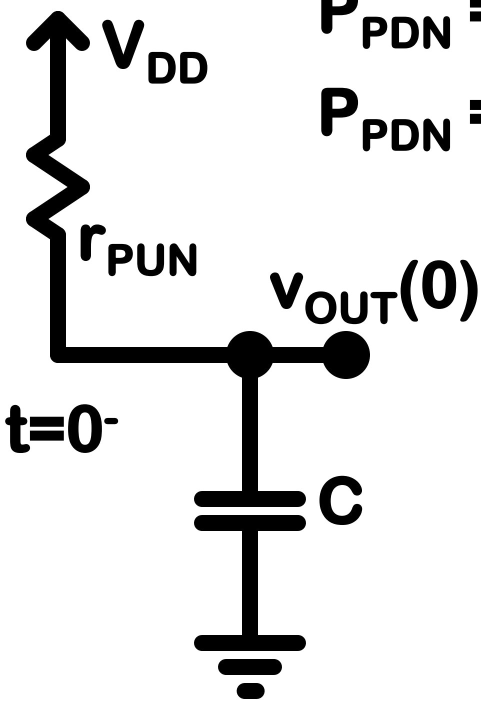


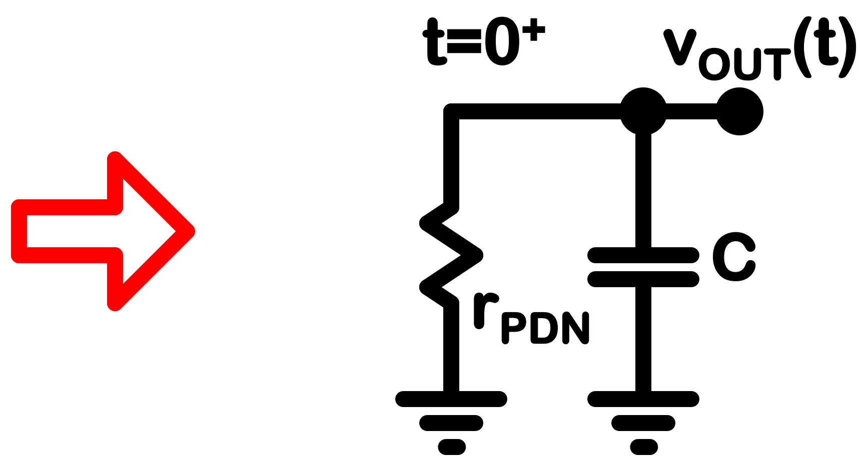
$$v_{OUT}(t) = V_{DD}\cdot\exp(-t/(r_{PDN}\cdot C))$$

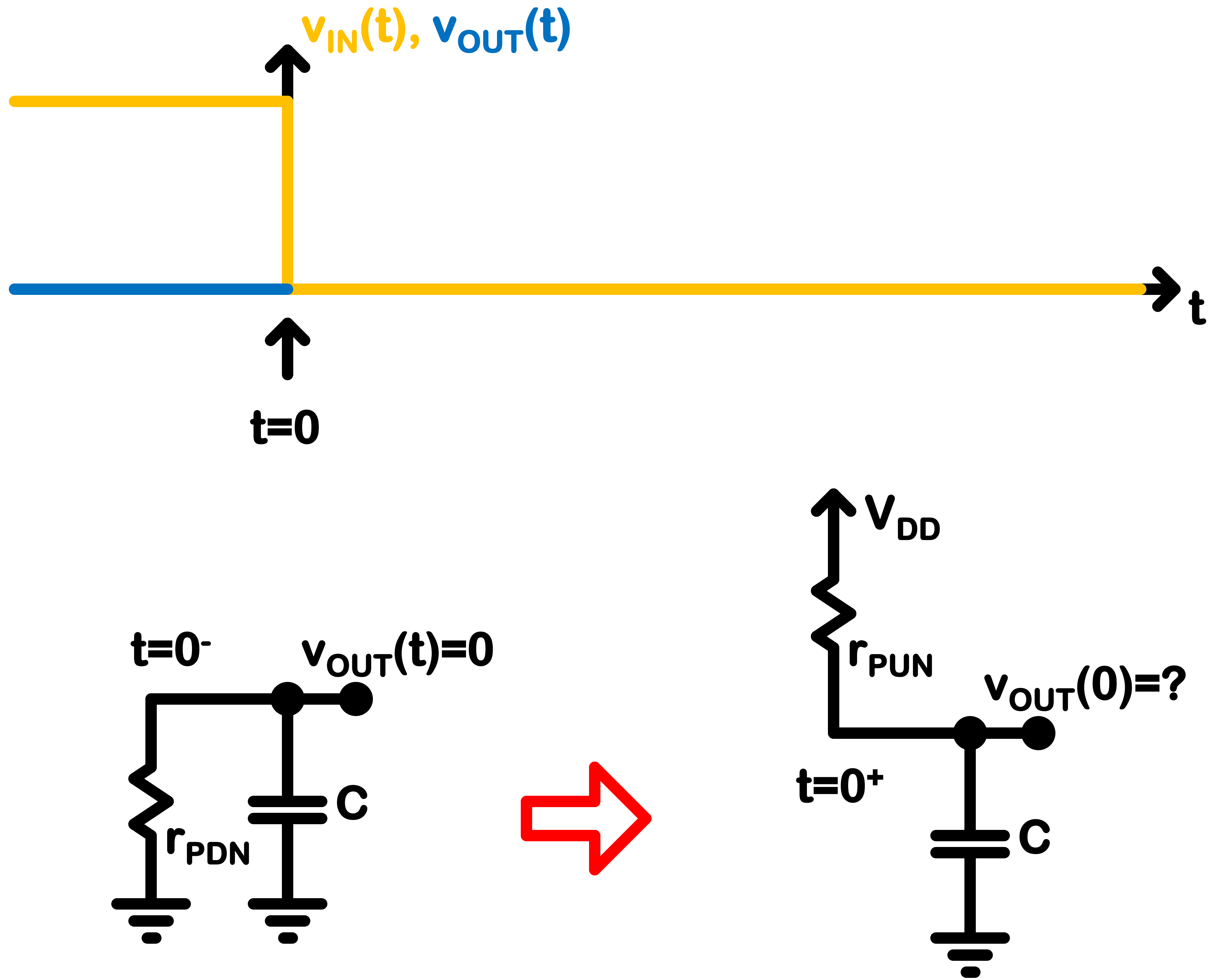
$$i_r = v_{OUT}/r_{PDN} = (V_{DD}/r_{PDN})\cdot\exp(-t/(r_{PDN}\cdot C))$$

$$P_{PDN} = v_{OUT}{}^2/r_{PDN}$$

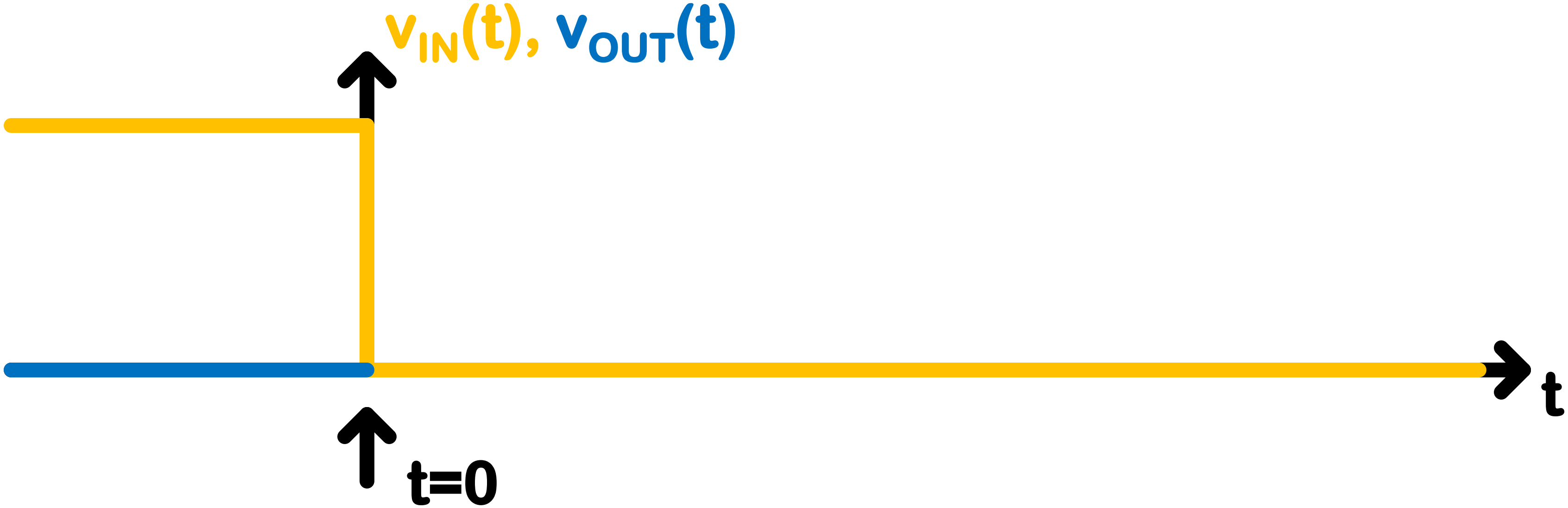
$$P_{PDN} = (V_{DD}{}^2/r_{PDN})\cdot\exp(-2t/(r_{PDN}\cdot C))$$





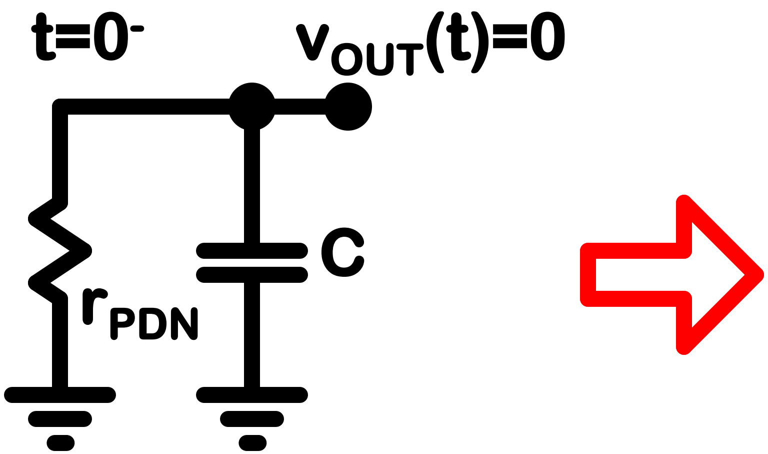


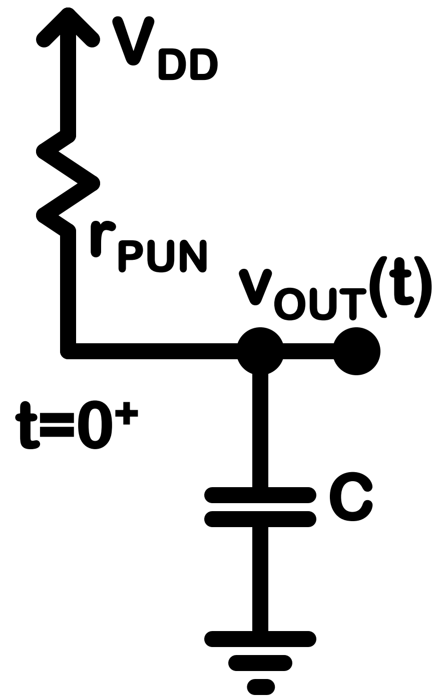
vIN(t), vOUT(t)
t
t=0
t=0-
vOUT(t)=0
C
rPDN
VDD
rPUN
vOUT(0)=?
t=0+
C

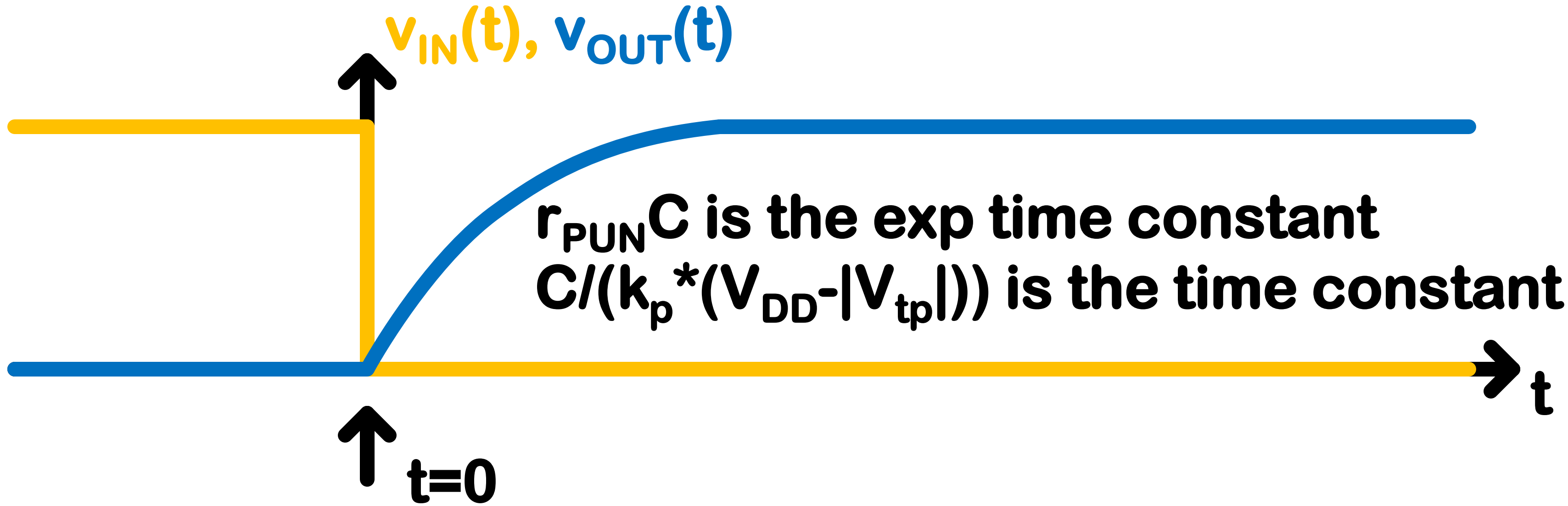


$$v_{OUT}(t) = 0, \qquad t < 0$$

$$v_{OUT}(t) = V_{DD} - V_{DD} \cdot \exp(-t/(r_{PUN} \cdot C)) \qquad t > 0$$

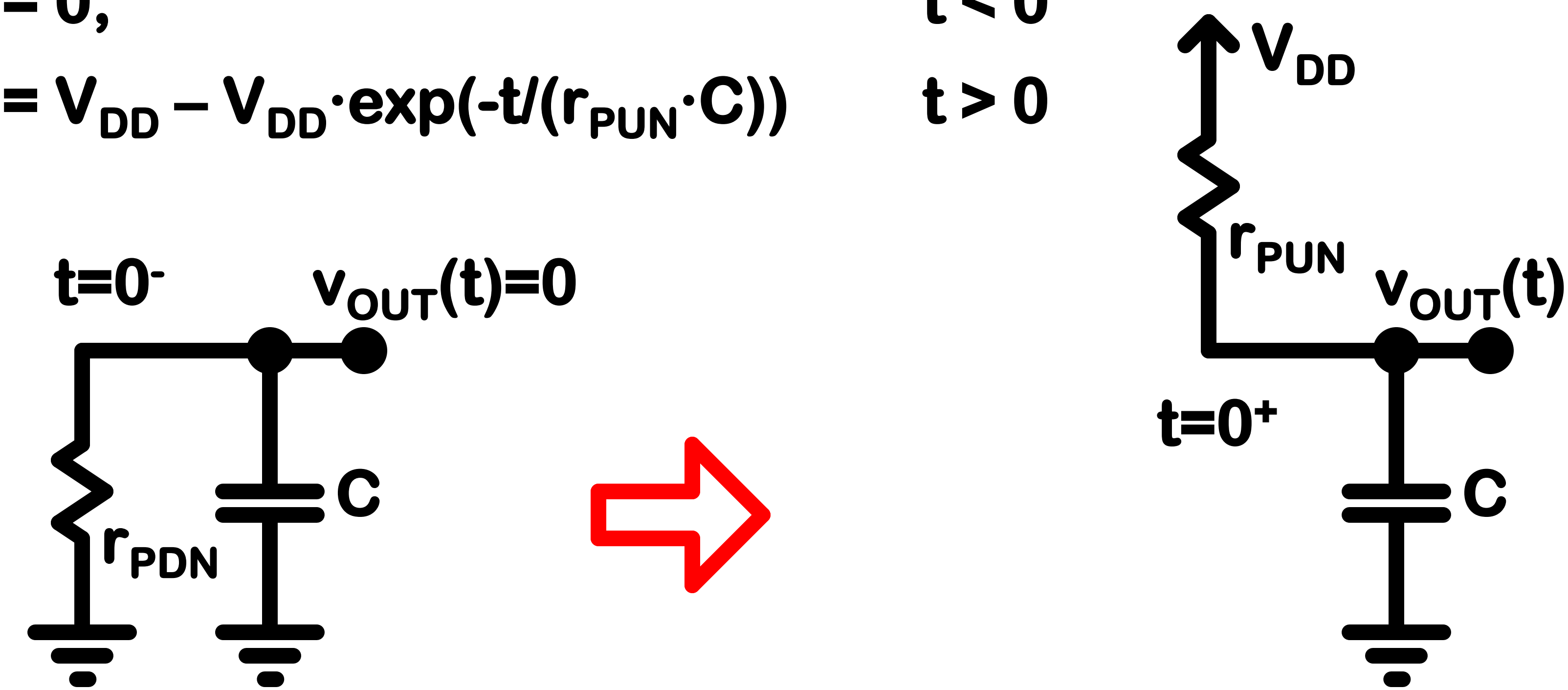


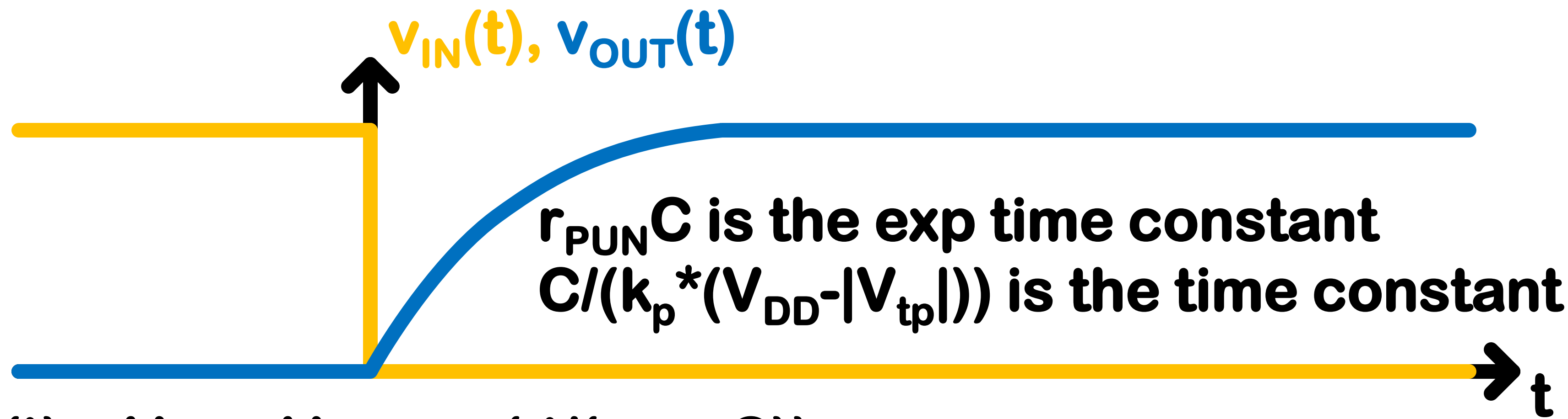




$v_{OUT}(t) = 0,$ $\quad t < 0$

$v_{OUT}(t) = V_{DD} - V_{DD} \cdot \exp(-t/(r_{PUN} \cdot C))$ $\quad t > 0$



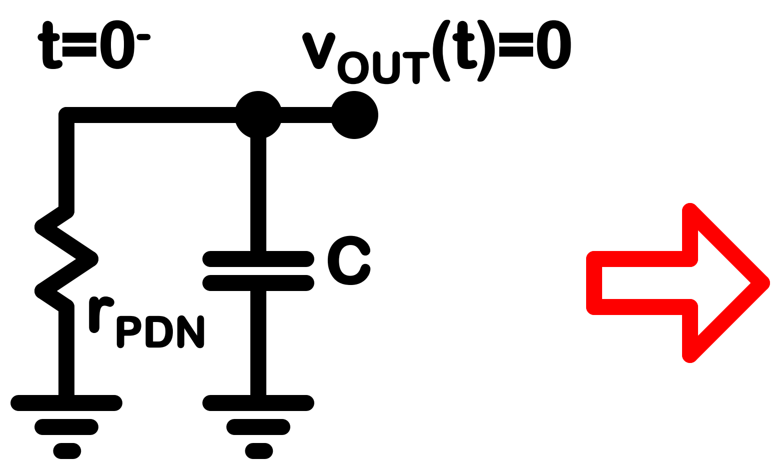


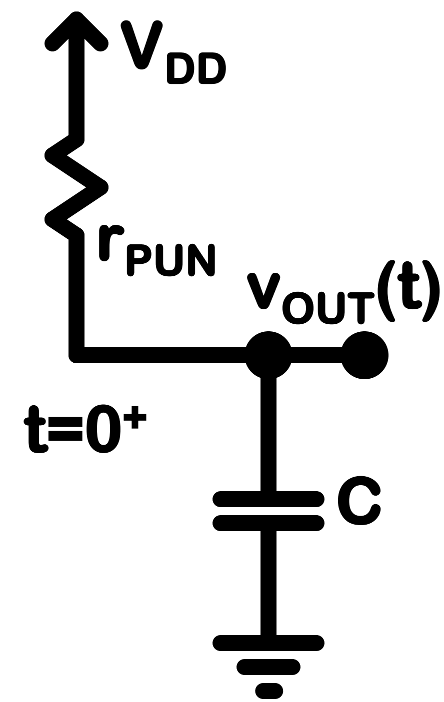
$$v_{OUT}(t) = V_{DD} - V_{DD}\cdot exp(-t/(r_{PUN}\cdot C))$$

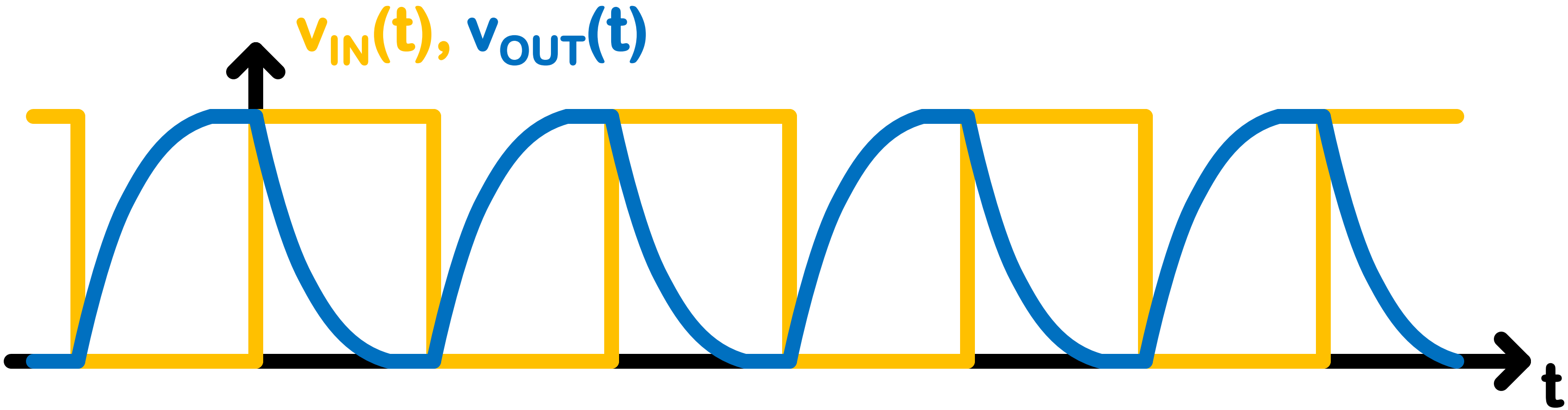
$$i_r = (V_{DD} - v_{OUT})/r_{PUN} = (V_{DD}/r_{PUN})\cdot exp(-t/(r_{PUN}\cdot C))$$

$$P_{PUN} = (V_{DD}^2/r_{PUN})\cdot exp(-2t/(r_{PUN}\cdot C))$$

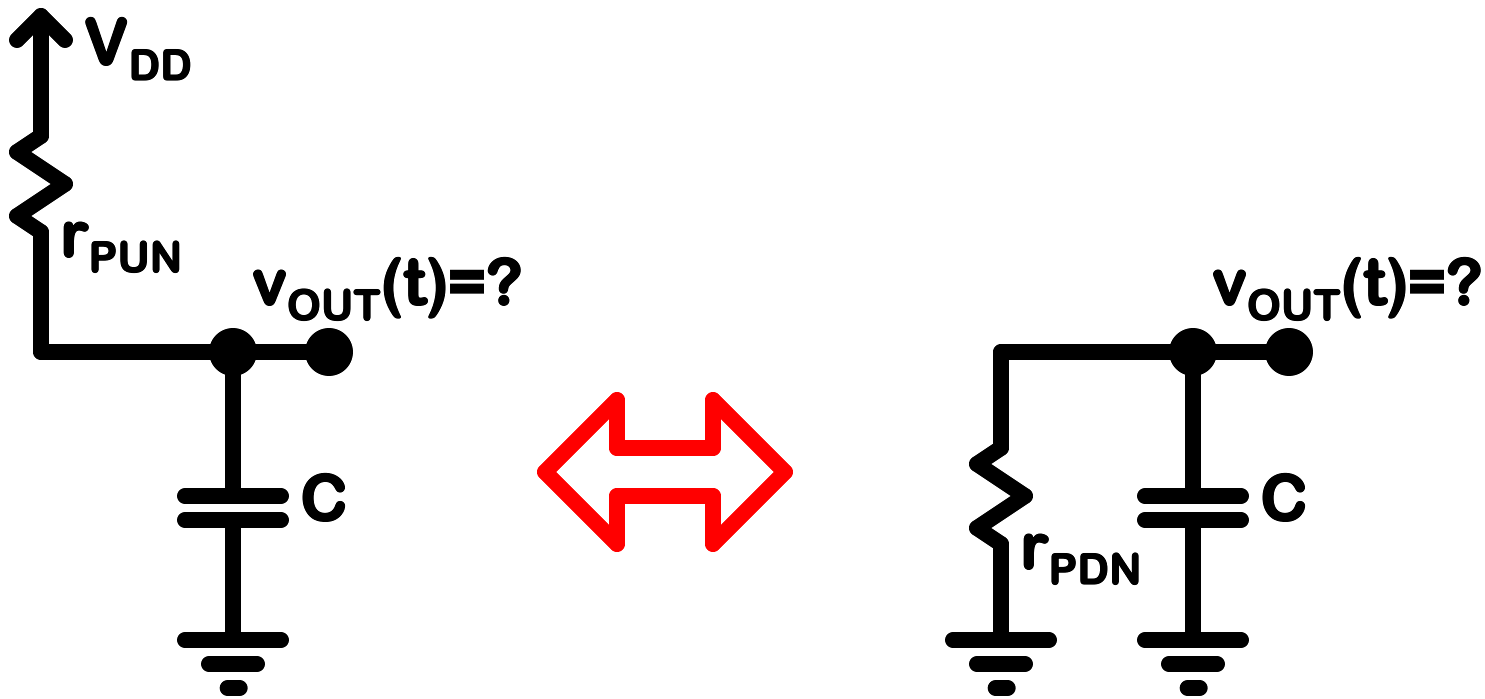
$$P_{SUPPLY} = (V_{DD}^2/r_{PUN})\cdot exp(-t/(r_{PUN}\cdot C))$$



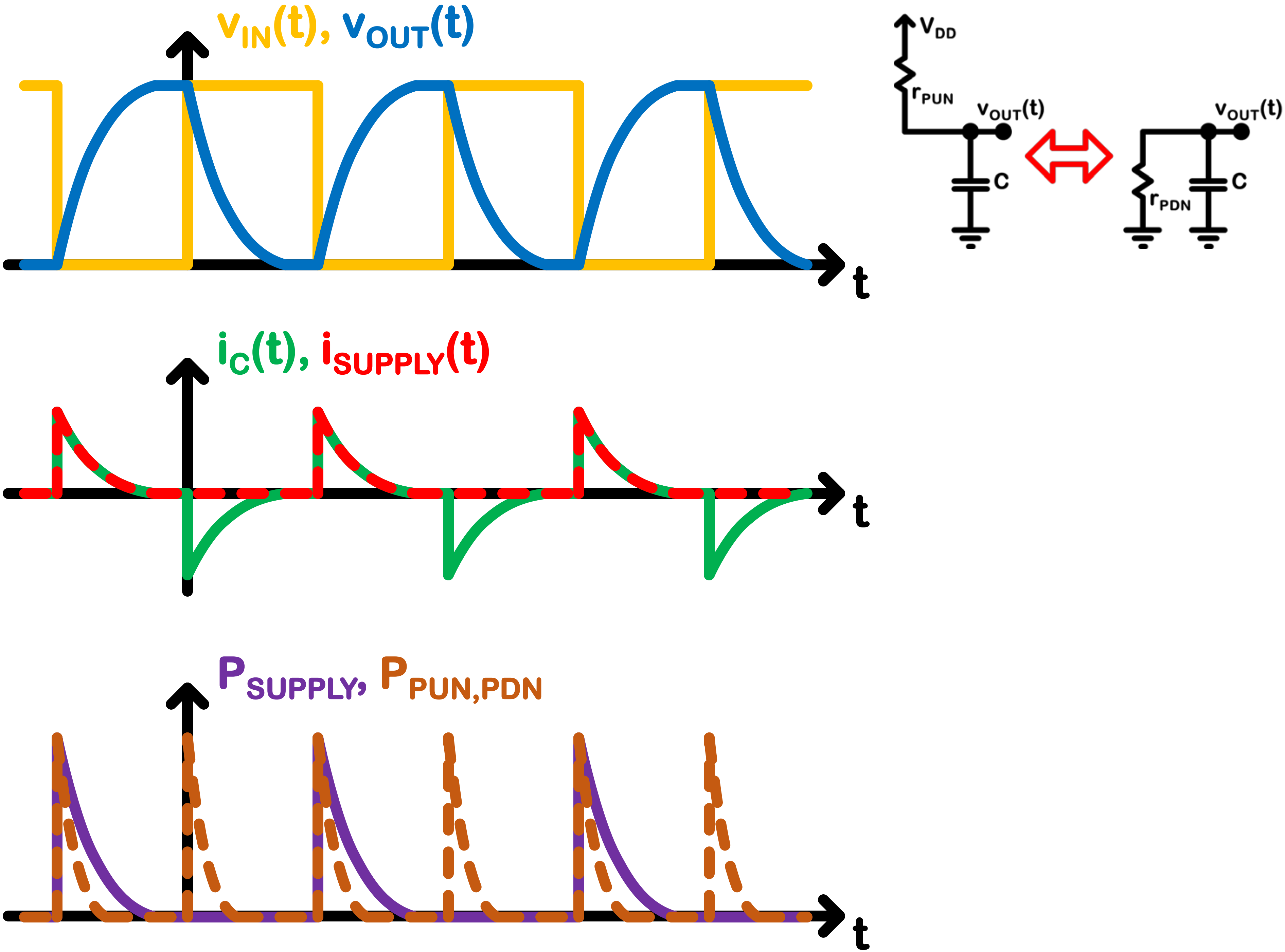




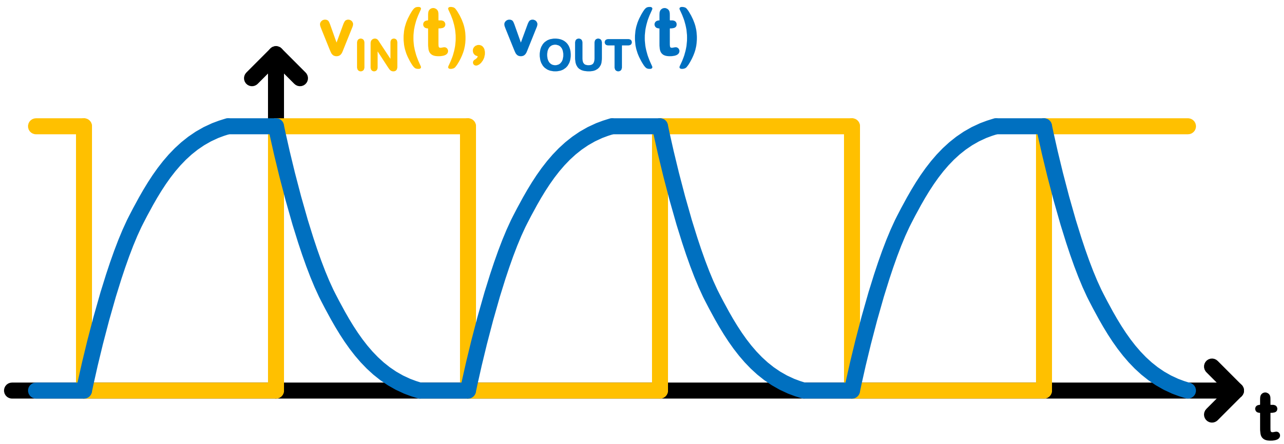
vIN(t), vOUT(t)
t

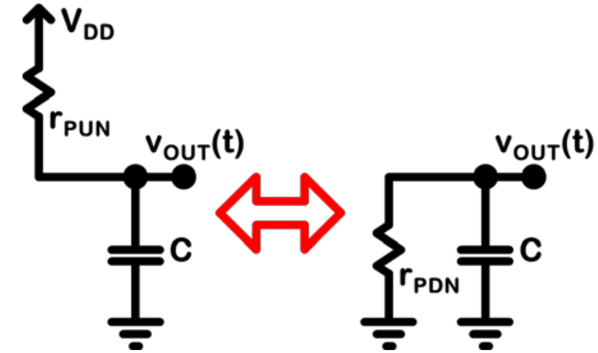


VDD
rPUN
vOUT(t)=?
C
vOUT(t)=?
rPDN
C



vIN(t), vOUT(t)
t
VDD
rPUN
vOUT(t)
C
rPDN
vOUT(t)
C
iC(t), iSUPPLY(t)
t
PSUPPLY, PPUN,PDN
t



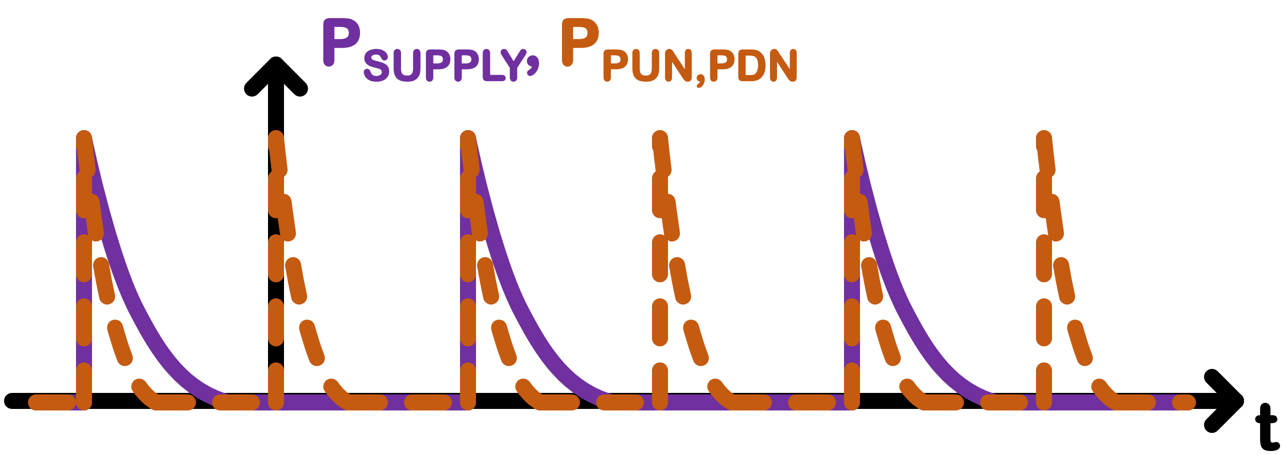


$$P_{SUPPLY} = (V_{DD}^2/r_{PUN})\cdot\exp(-t/(r_{PUN}\cdot C))$$

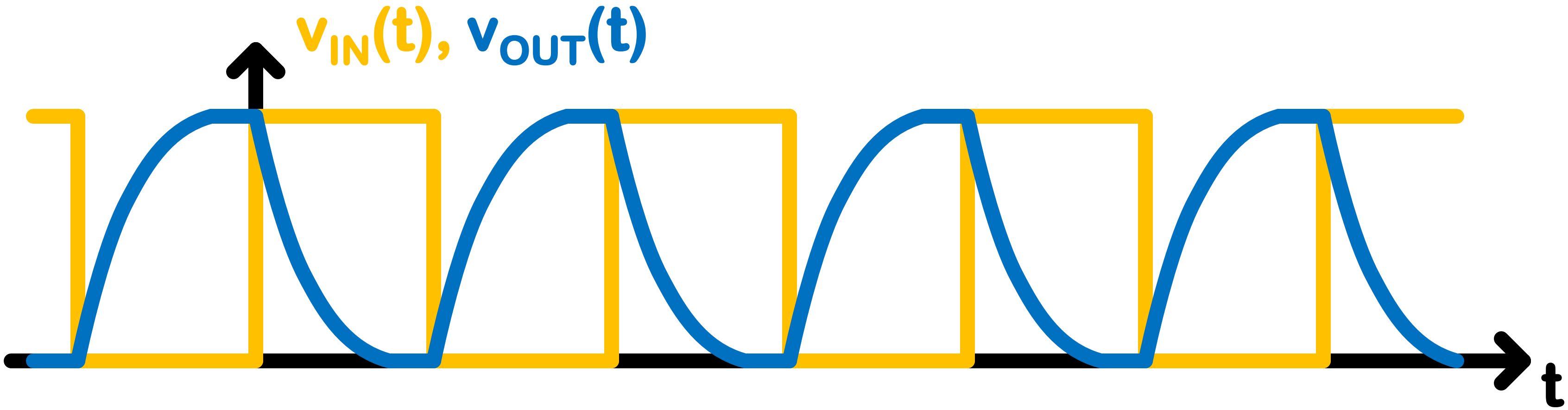
$$P_{AVG} = (1/T) \int_T (V_{DD}^2/r_{PUN})\cdot\exp(-t/(r_{PUN}\cdot C)\, dt$$

$$P_{AVG} = -(1/T)\, r_{PUN}\cdot C\, (V_{DD}^2/r_{PUN})\cdot\exp(-t/(r_{PUN}\cdot C) \,|_{0 \text{ to } T}$$

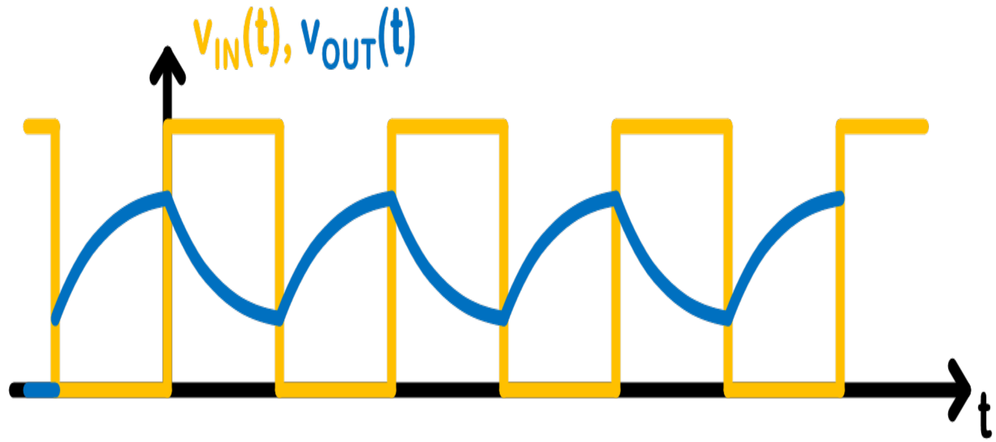
$$P_{AVG} = (1/T)\, C\, (V_{DD}^2) \quad \text{if} \quad r_{PUN}C << T$$



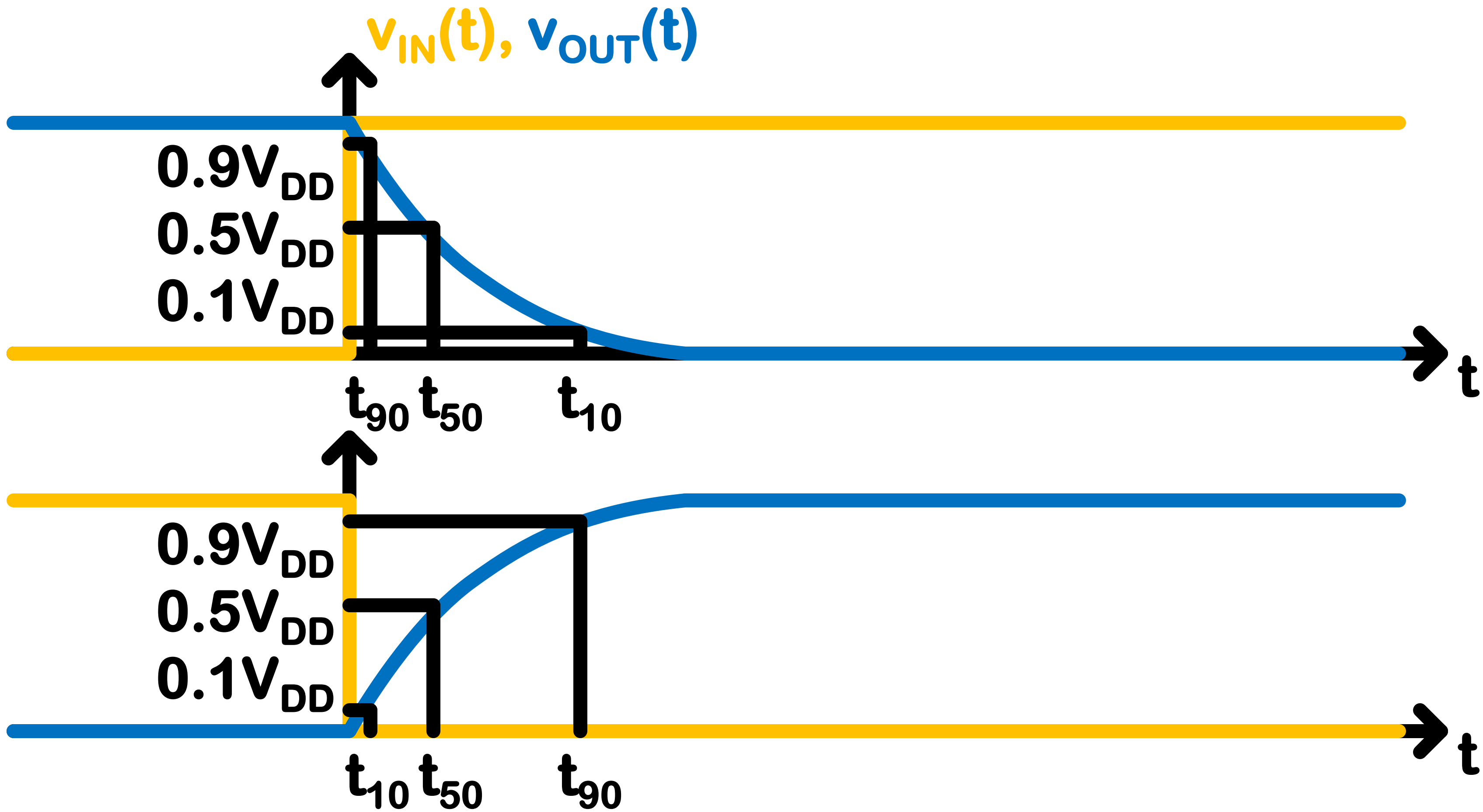
$$P_{AVG} = f\cdot C\cdot V_{DD}^2$$



vIN(t), vOUT(t)
t



vIN(t), vOUT(t)
t

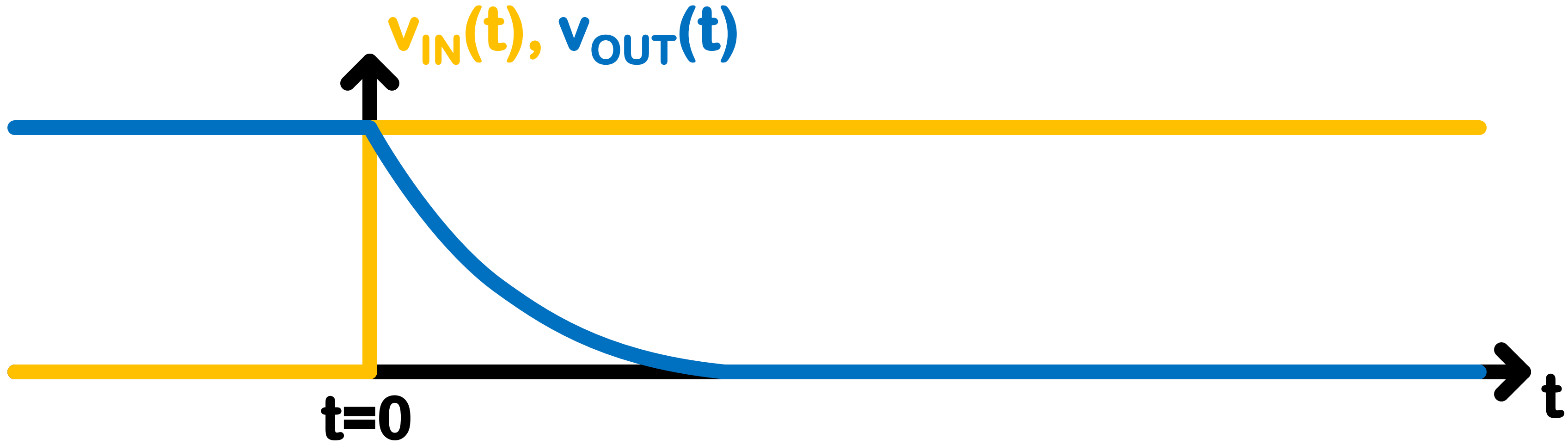


$\tau_{PHL} = t_{50}$ on top plot

$\tau_{PLH} = t_{50}$ on bottom plot

$\tau_{THL} = t_{10} - t_{90}$ on top plot

$\tau_{TLH} = t_{90} - t_{10}$ on bottom plot



$$v_{OUT}(t_{50}) = V_{DD}\cdot\exp(-t_{50}/(r_{PDN}\cdot C)) = V_{DD}/2$$

$$-t_{50}/(r_{PDN}\cdot C) = \ln(1/2)$$

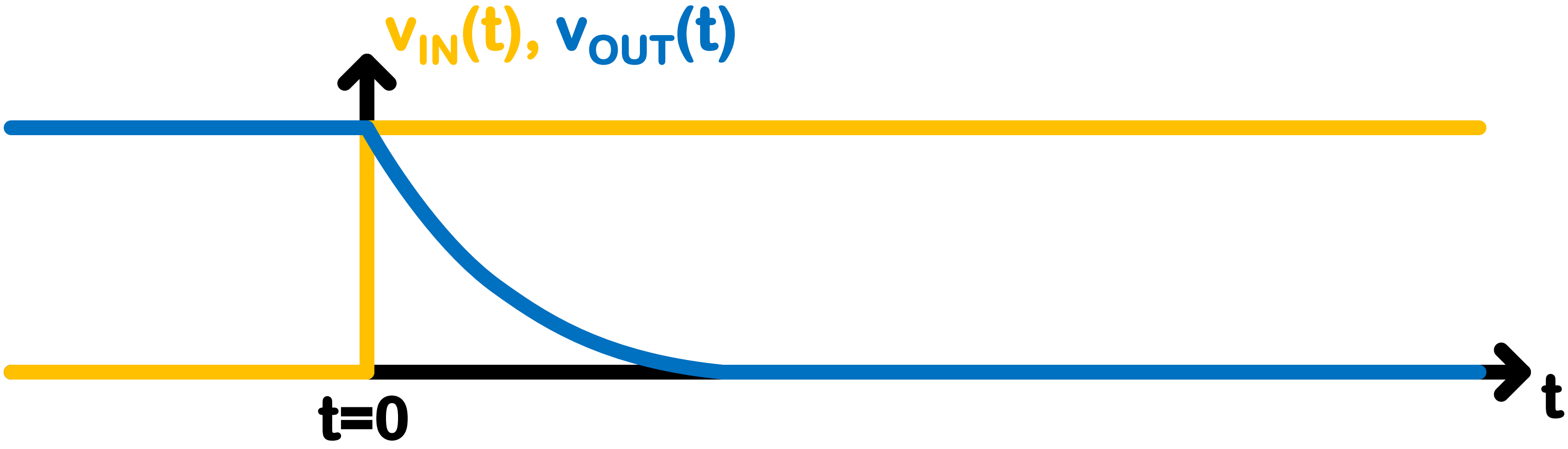
$$t_{50} = r_{PDN}\cdot C\cdot\ln(2)$$

$$-t_{10}/(r_{PDN}\cdot C) = \ln(1/10)$$

$$t_{10} = r_{PDN}\cdot C\cdot\ln(10)$$

$$-t_{90}/(r_{PDN}\cdot C) = \ln(9/10)$$

$$t_{90} = r_{PDN}\cdot C\cdot\ln(10/9)$$



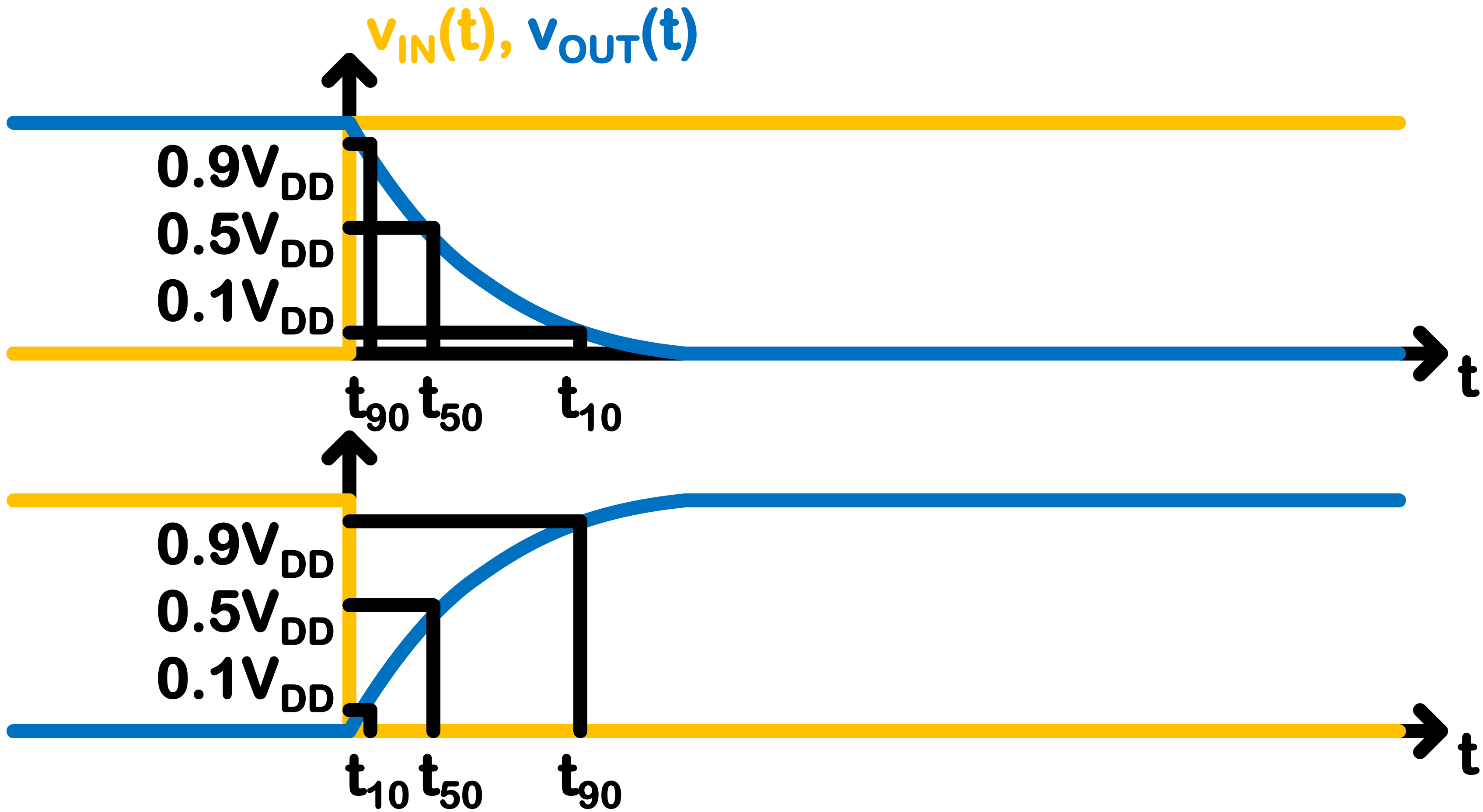
$$\tau_{PHL} = t_{50} = r_{PDN}\cdot C\cdot \ln(2)$$

$$\tau_{THL} = t_{10} - t_{90} = r_{PDN}\cdot C\cdot \ln(10) - r_{PDN}\cdot C\cdot \ln(10/9)$$

$$\tau_{THL} = r_{PDN}\cdot C\cdot (\ln(10) - \ln(10/9)) = r_{PDN}\cdot C\cdot \ln(10\cdot 9/10)$$

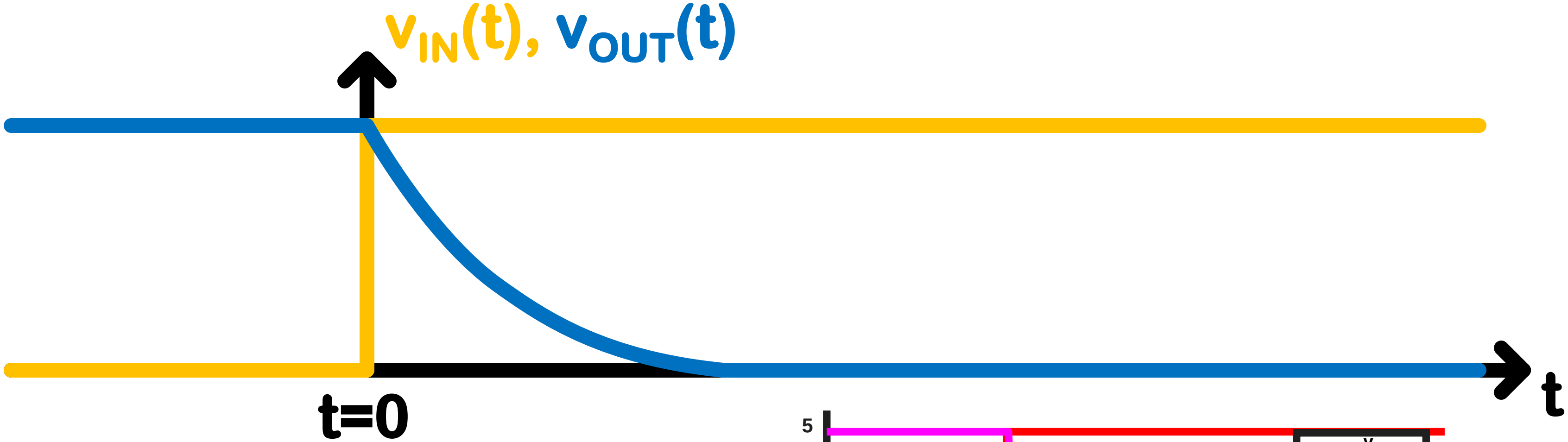
$$\tau_{PHL} = r_{PDN}\cdot C\cdot \ln(2)$$

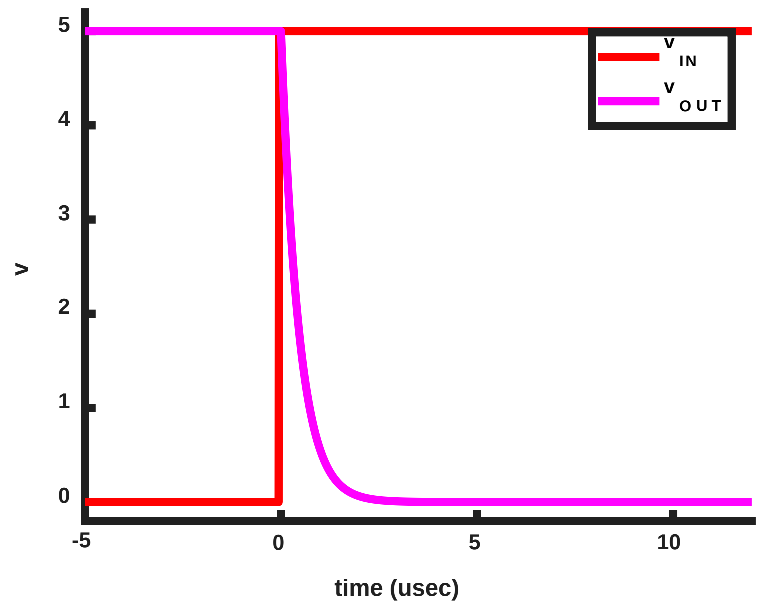
$$\tau_{THL} = r_{PDN}\cdot C\cdot \ln(9)$$



$$\tau_{PHL}, \tau_{PLH} = r_{PDN,PUN} \cdot C \cdot \ln(2)$$
$$\tau_{THL}, \tau_{TLH} = r_{PDN,PUN} \cdot C \cdot \ln(9)$$

This is for the simple model of a resisitor and capacitor.





$v_{OUT}(t) = V_{DD} \cdot \exp(-t/(r_{PDN} \cdot C))$

$k_n = 130$ mA/V

$V_{tn} = 1.62$ V

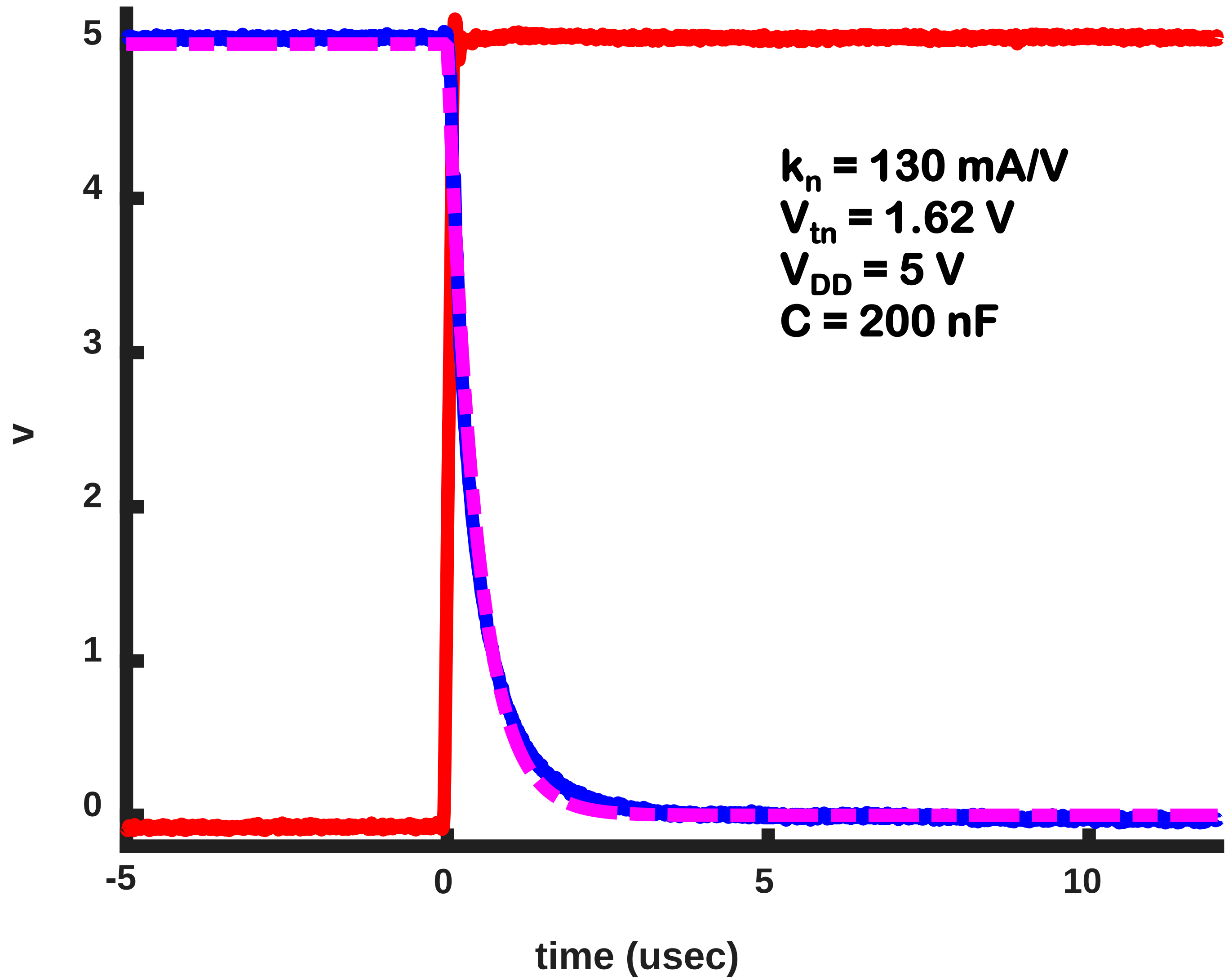
$V_{DD} = 5$ V

$C = 200$ nF

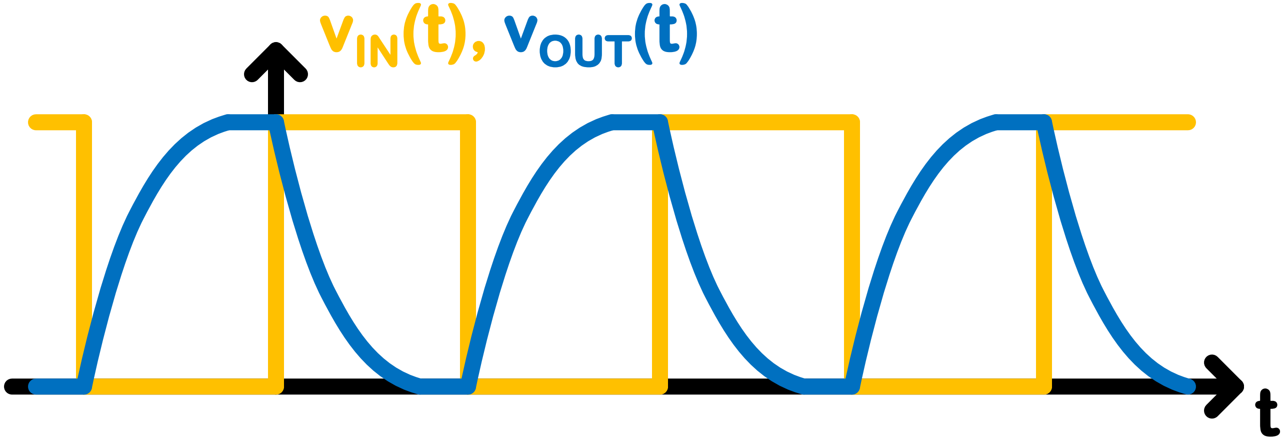
$r_{PDN} = r_{DS} = 1/(.130*(5-1.62)) = 2.27\ \Omega$

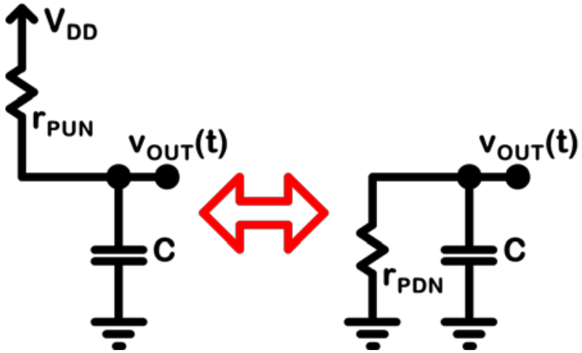
$r_{PDN} \cdot C = 454$ nsec

$v_{OUT}(t) = 5 \cdot \exp(-t/(454 \times 10^{-9}))$



5
4
3
2
1
0
v
-5
0
5
10
time (usec)
k_n = 130 mA/V
V_{tn} = 1.62 V
V_{DD} = 5 V
C = 200 nF





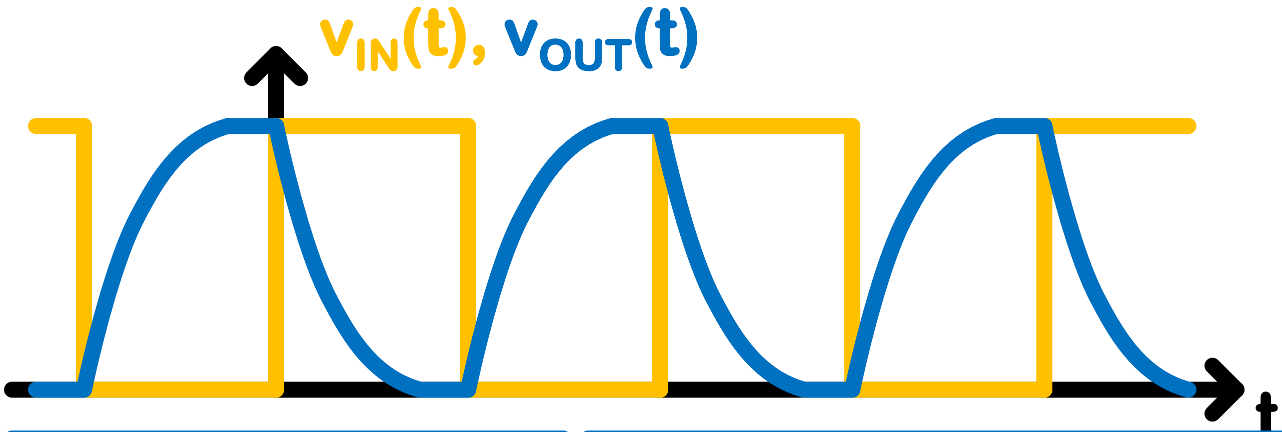
$$P_{AVG} = f \cdot C \cdot V_{DD}^2$$

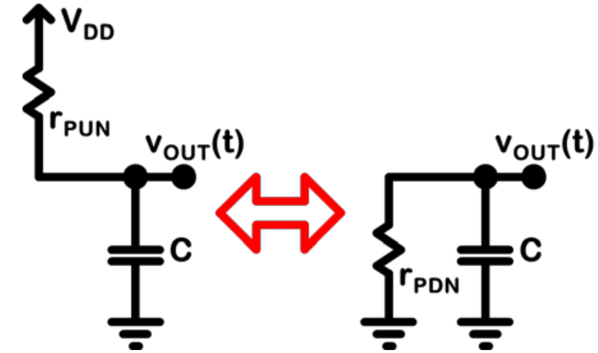
$$\tau_{PHL}, \tau_{PLH} = r_{PDN,PUN} \cdot C \cdot \ln(2)$$
$$\tau_{THL}, \tau_{TLH} = r_{PDN,PUN} \cdot C \cdot \ln(9)$$

For this model the minimum clock period is proportional to τ_{THL} or τ_{THL}.

$$f_{max} \sim 1/\tau_T \sim 1/(r_{PDN,PUN} \cdot C)$$

$$r_{SD,DS} = 1/(k'(W/L)(V_{DD}-V_t))$$





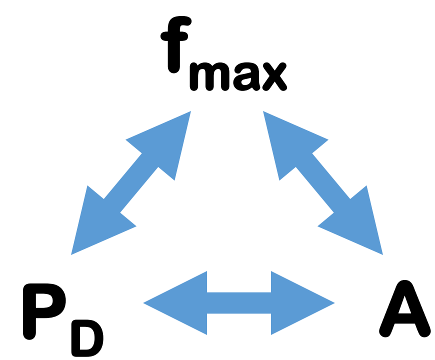
$P_{AVG} = N \cdot f \cdot C \cdot V_{DD}^2$

$f_{max} \sim (2/C) \cdot k' \cdot (W/L) \cdot (V_{DD} - V_t)$

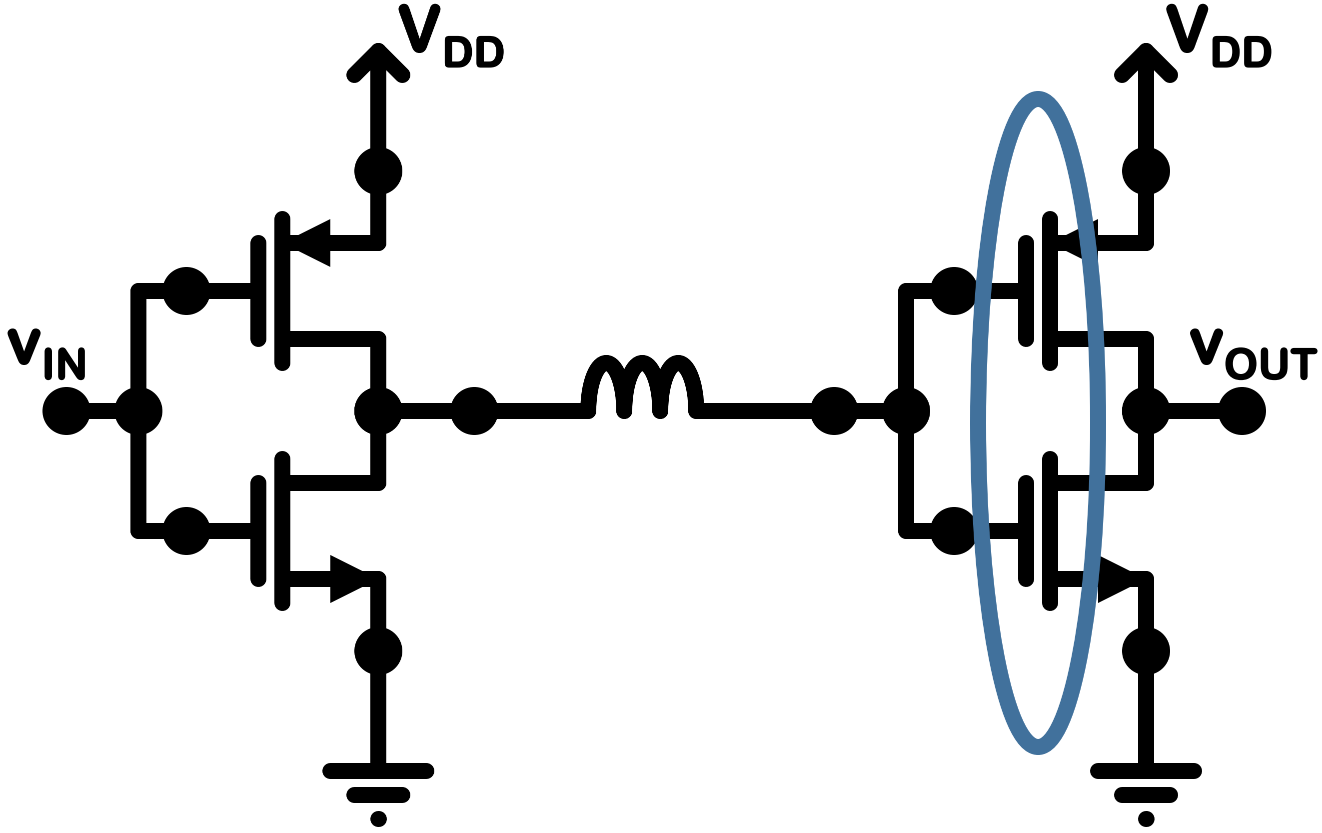
$A = N \cdot W \cdot L$

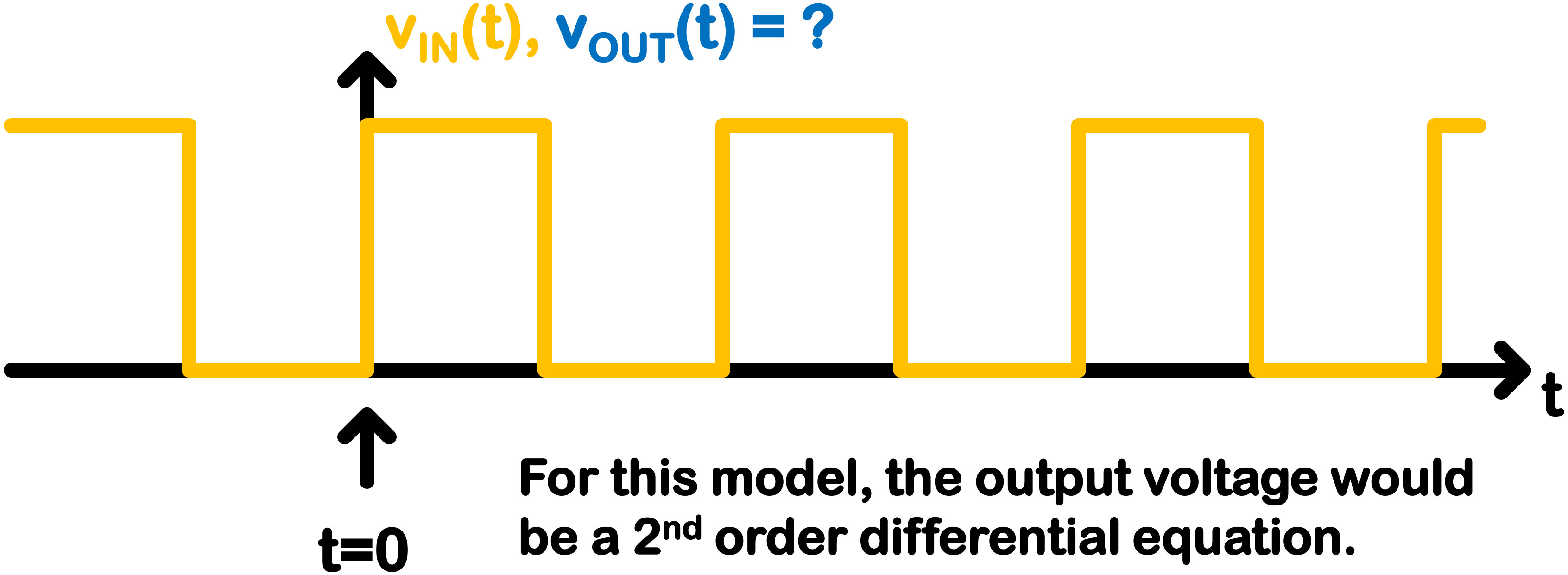
Performance (f_{max}) vs. Power Dissipation (P_D) vs. Area/Number (A,N) (assume always operating at f_{max})

1. Increase V_{DD} (Power vs f_{max})
   - f_{max} goes up by V_{DD}-V_t
   - Power goes up by $(V_{DD}-V_t) \cdot V_{DD}^2$
2. Increase W/L (f_{max} vs A)
   - f_{max} goes up by W
   - Power goes up by W (because of f_{max})
   - Uses more area on the IC
3. Find a better process that has
   - Larger k'
   - Smaller C, but $k' = \mu C_{ox}$, so maybe higher mobility.
   - Smaller V_t



CMOS Inverter





For this model, the output voltage would be a 2[nd] order differential equation.

a. overdamped.

b. underdamped (ringing)

c. critically…