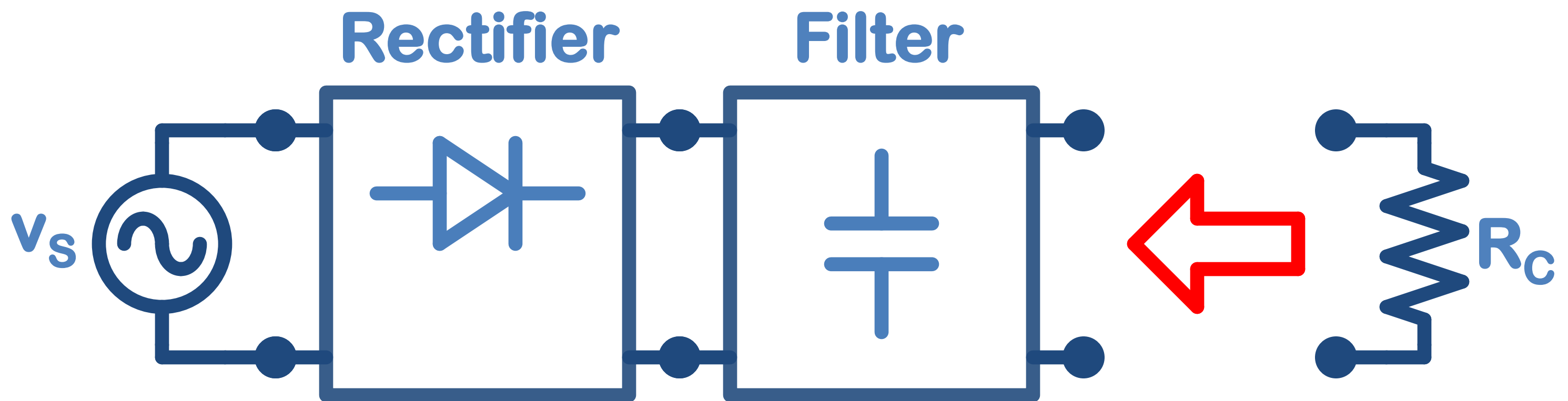
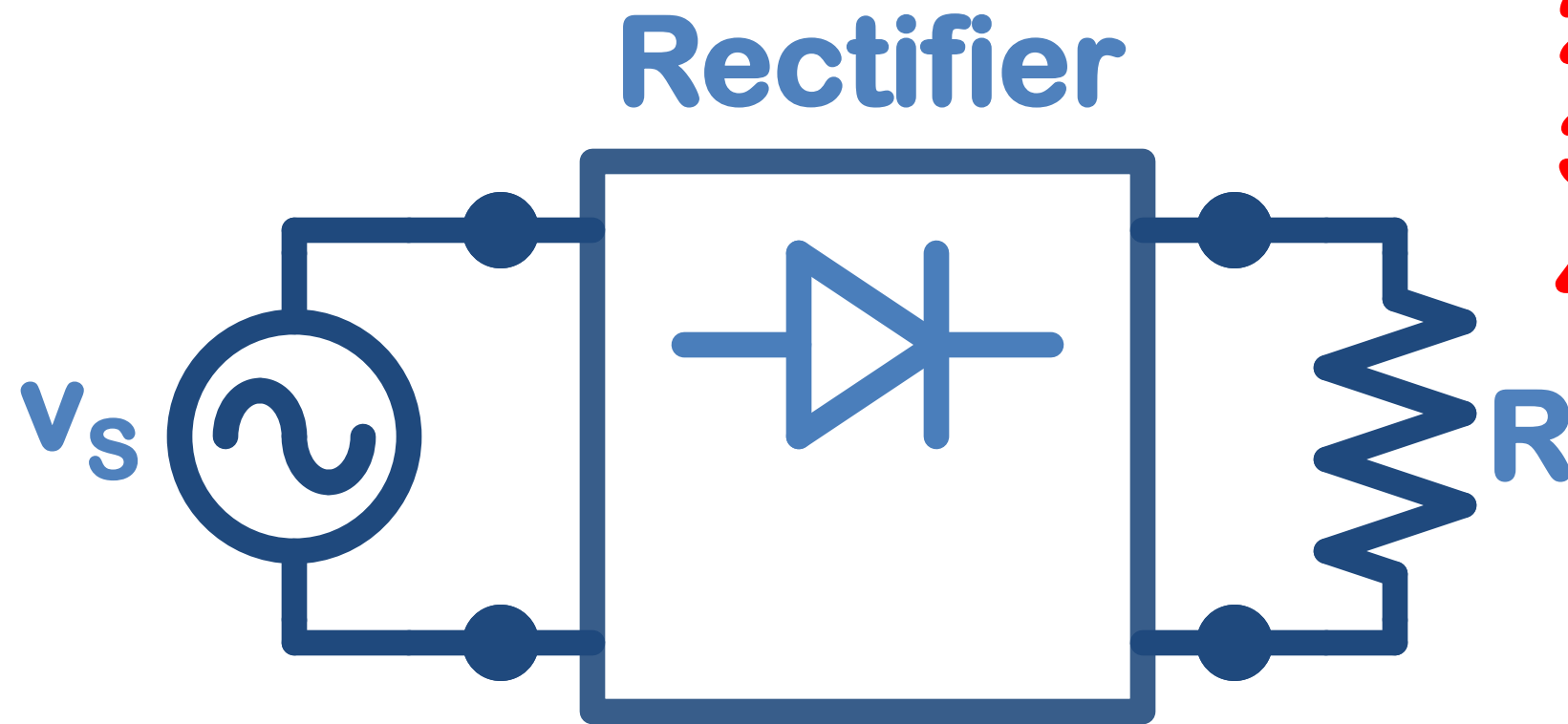
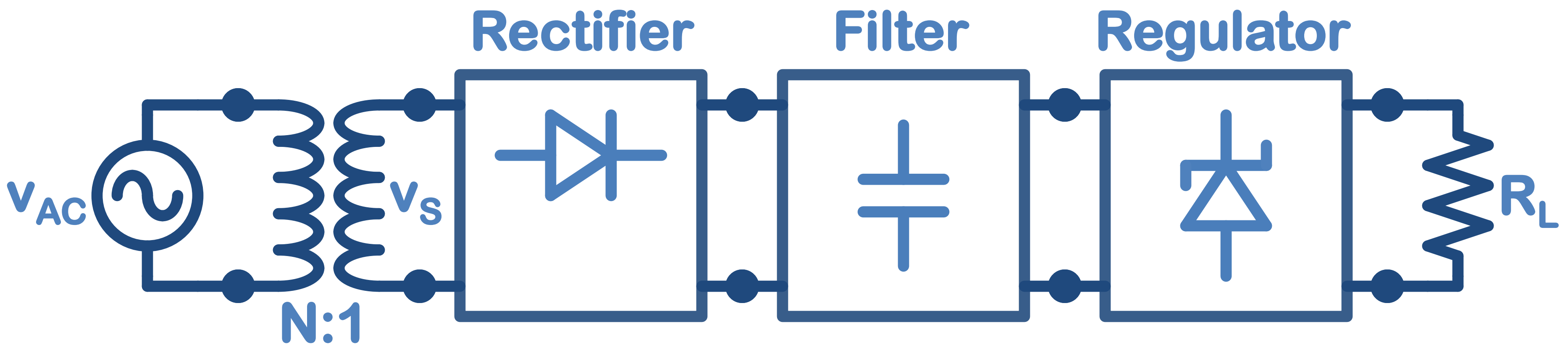


- 1) Rectifier Circuits w/Resistor.
- 2) Filter Capacitor.
- 3) Capacitor & Resistor.
- 4) Regulator.





# Positive Half Wave Rectifier

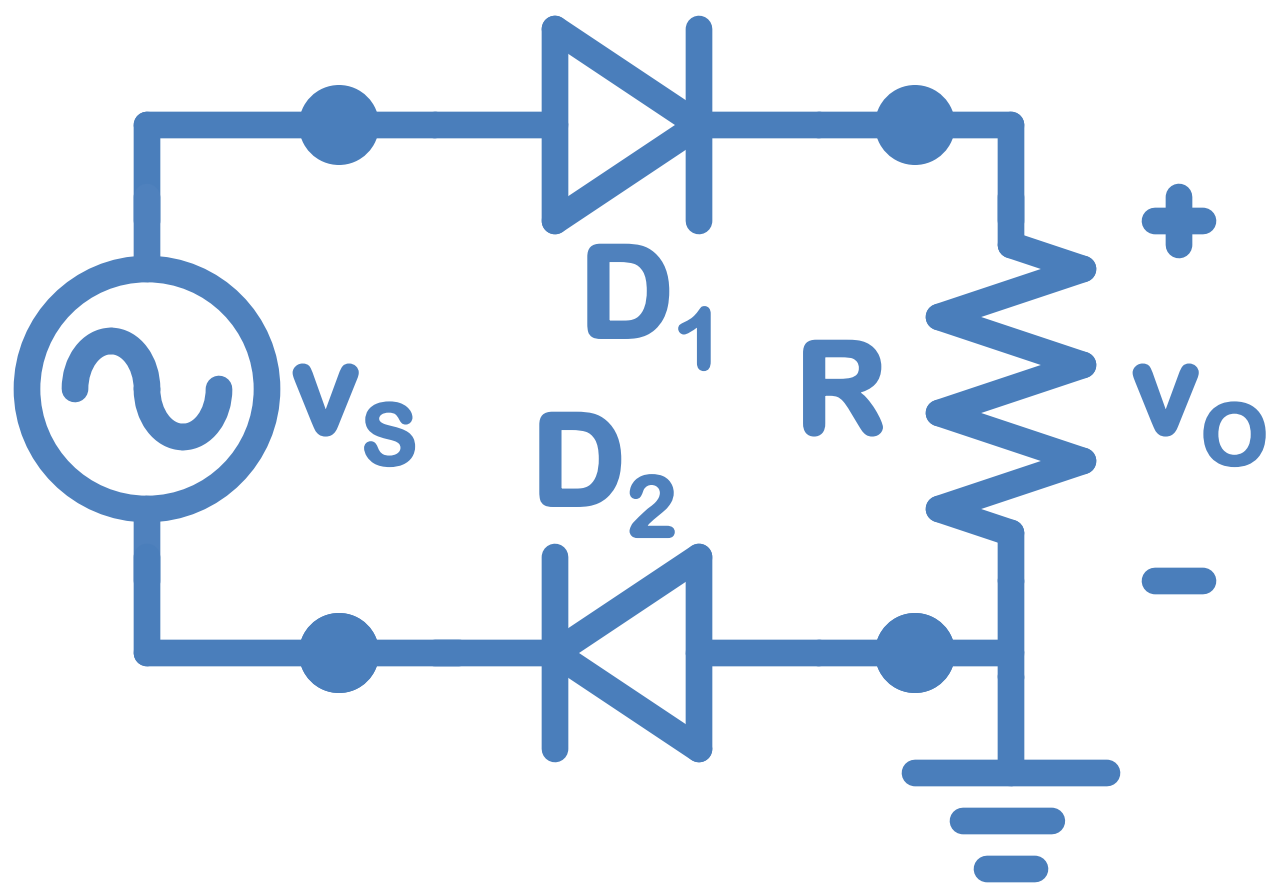
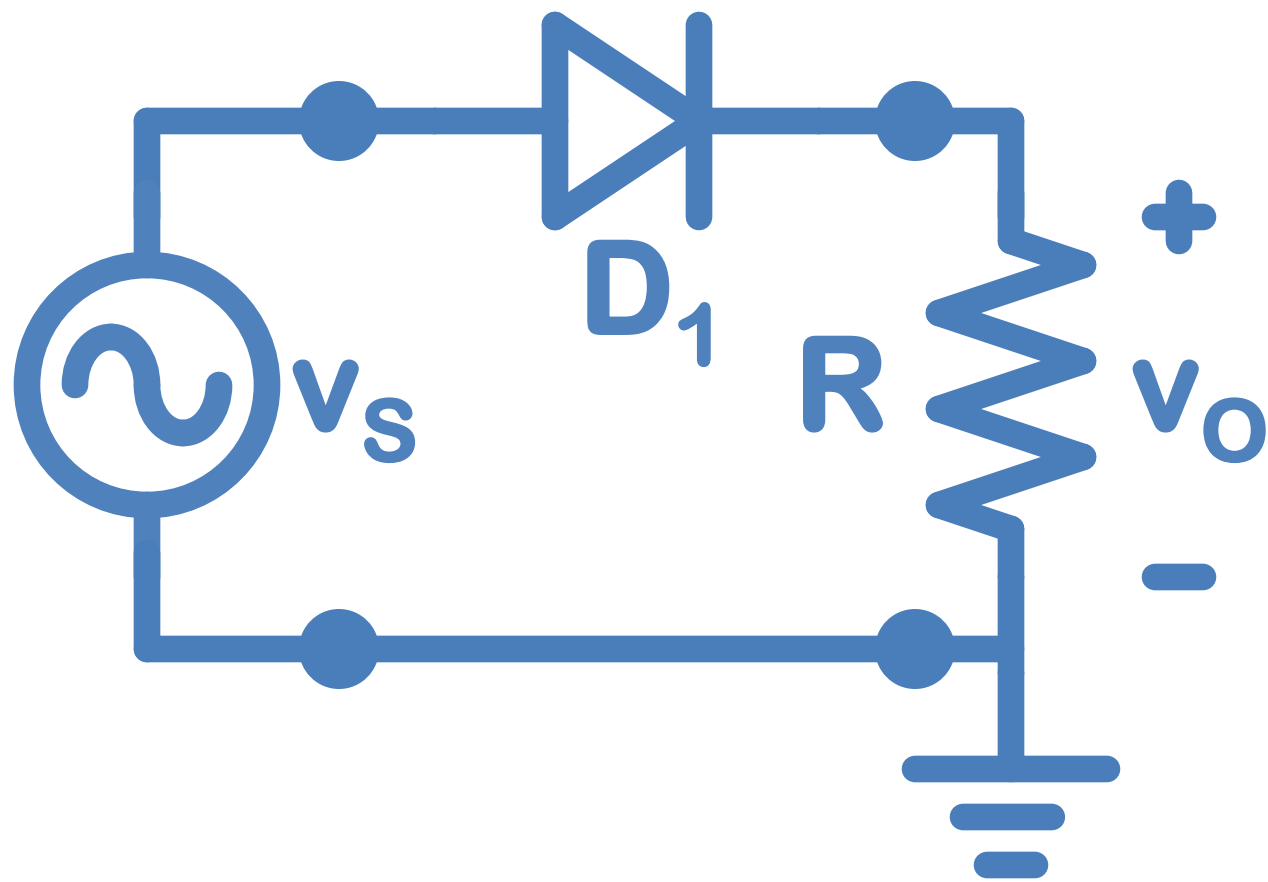
Find the transfer function  $v_o(v_s)$

Find the Peak Inverse Voltage

$$PIV = \max(-v_D(v_s))$$

Plot input & output for a sinusoidal input.

$$v_s(t) = V_p \cos(2\pi t/T)$$



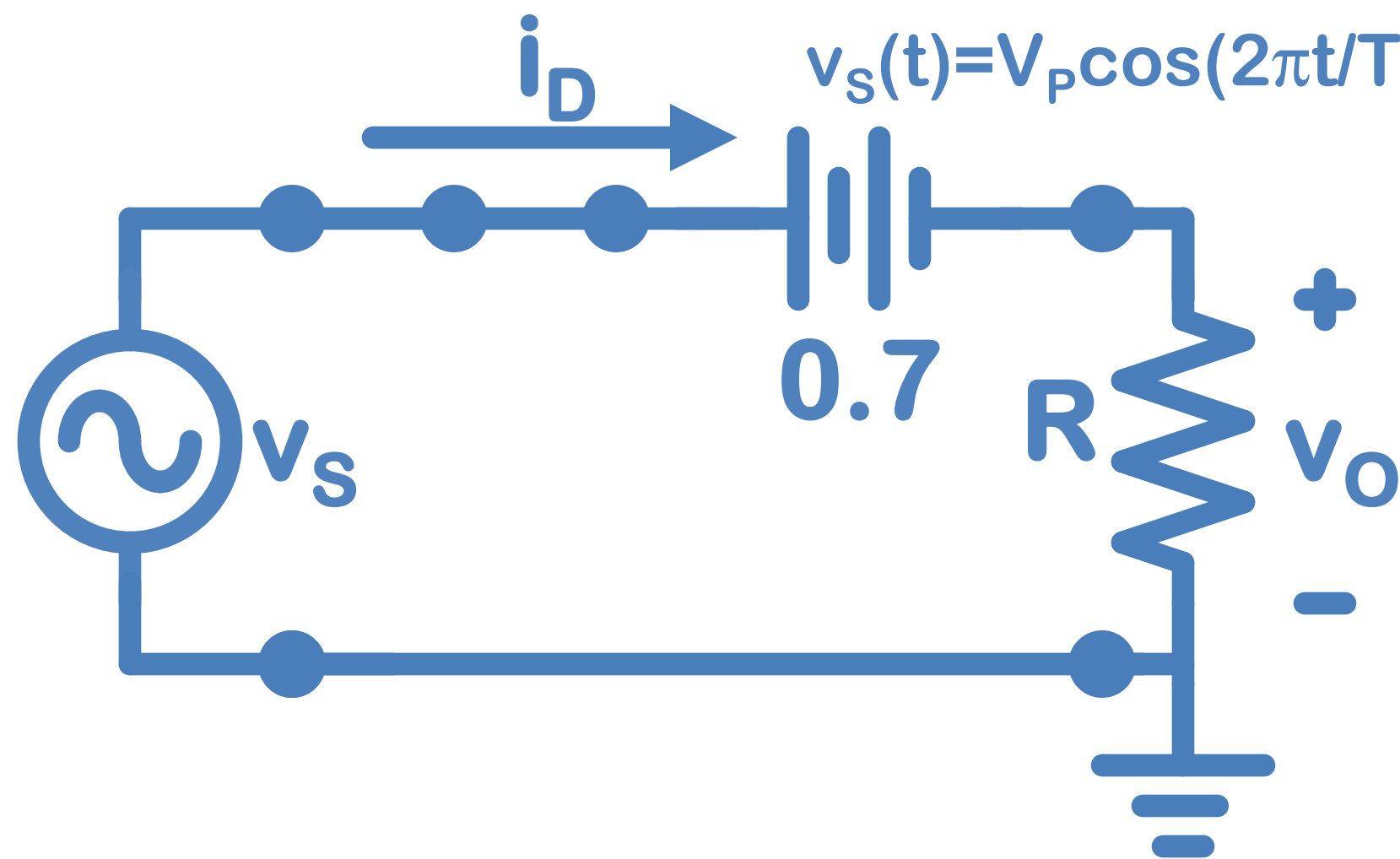
# Positive Half Wave Rectifier

Find the transfer function  $v_O(v_S)$

Find the Peak Inverse Voltage

$$PIV = \max(-v_D(v_S))$$

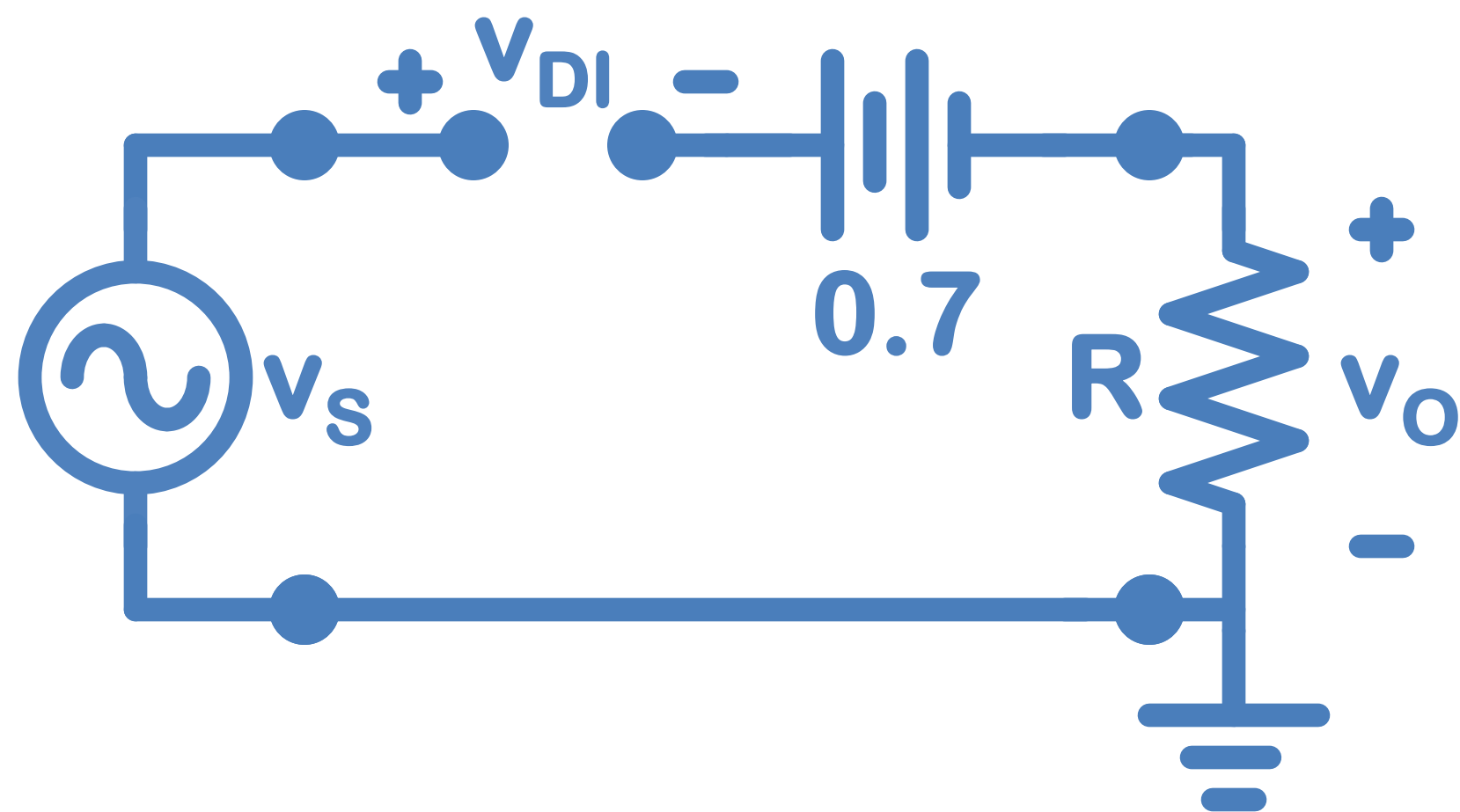
Plot input & output for a sinusoidal input.



FB:  $v_O(v_S) = v_S - 0.7$   
 $i_D(v_S) = (v_S - 0.7)/R > 0; v_S > 0.7$

RB:  $v_O(v_S) = 0$   
 $v_D(v_S) = v_S < 0.7; v_S < 0.7$

$v_O(v_S) = v_S - 0.7$  for  $v_S > 0.7$   
 $v_O(v_S) = 0$  for  $v_S < 0.7$



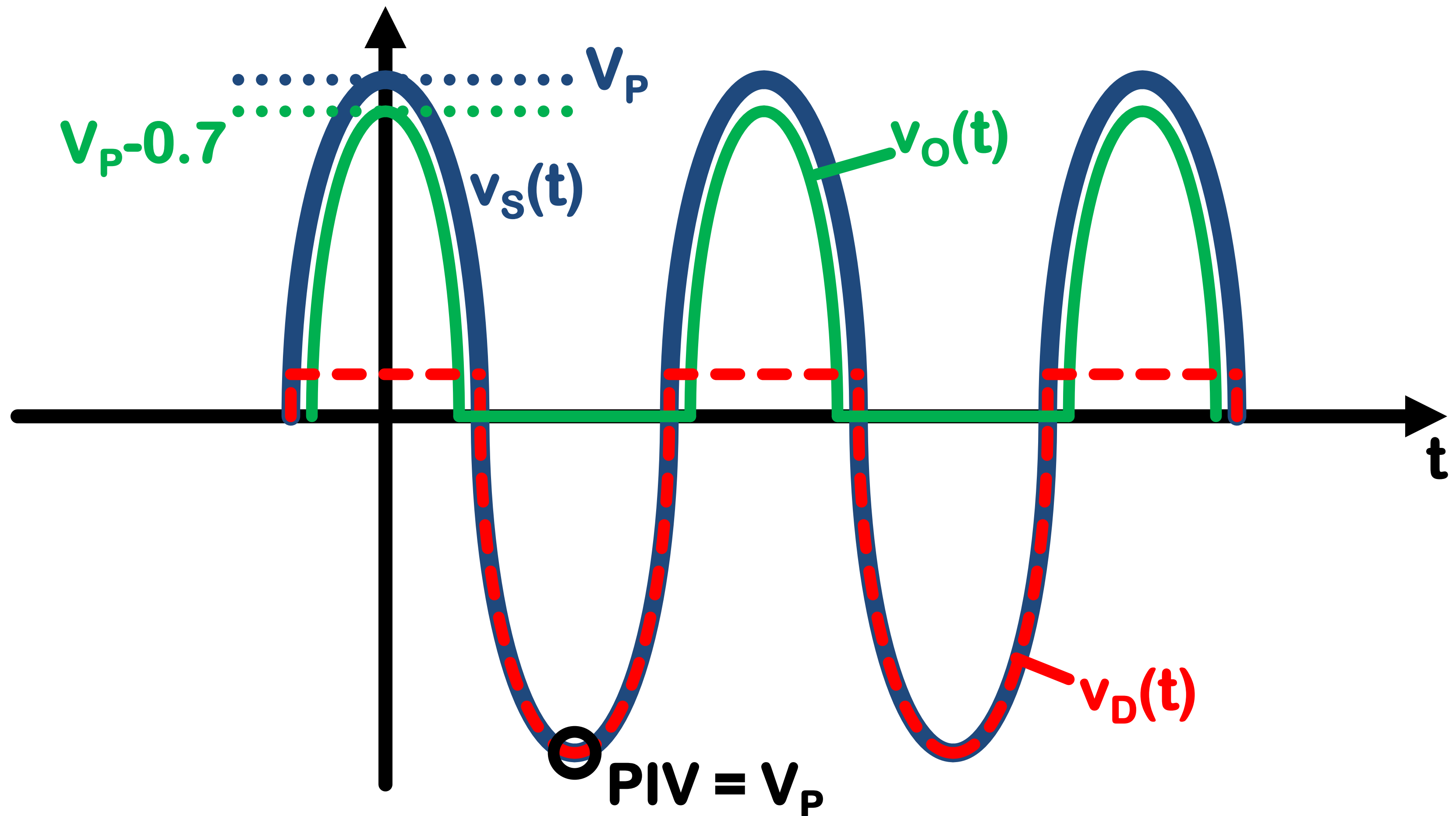
PIV:  $v_D(v_S) = 0.7$  for  $v_S > 0.7$   
 $v_D(v_S) = v_S$  for  $v_S < 0.7$   
 $PIV = \max(-v_D) = V_p$

# Positive Half Wave Rectifier (1-Diode)

$$v_o(v_s) = v_s - 0.7 \text{ for } v_s > 0.7$$

$$v_o(v_s) = 0 \text{ for } v_s < 0.7$$

$$\text{PIV} = V_p$$

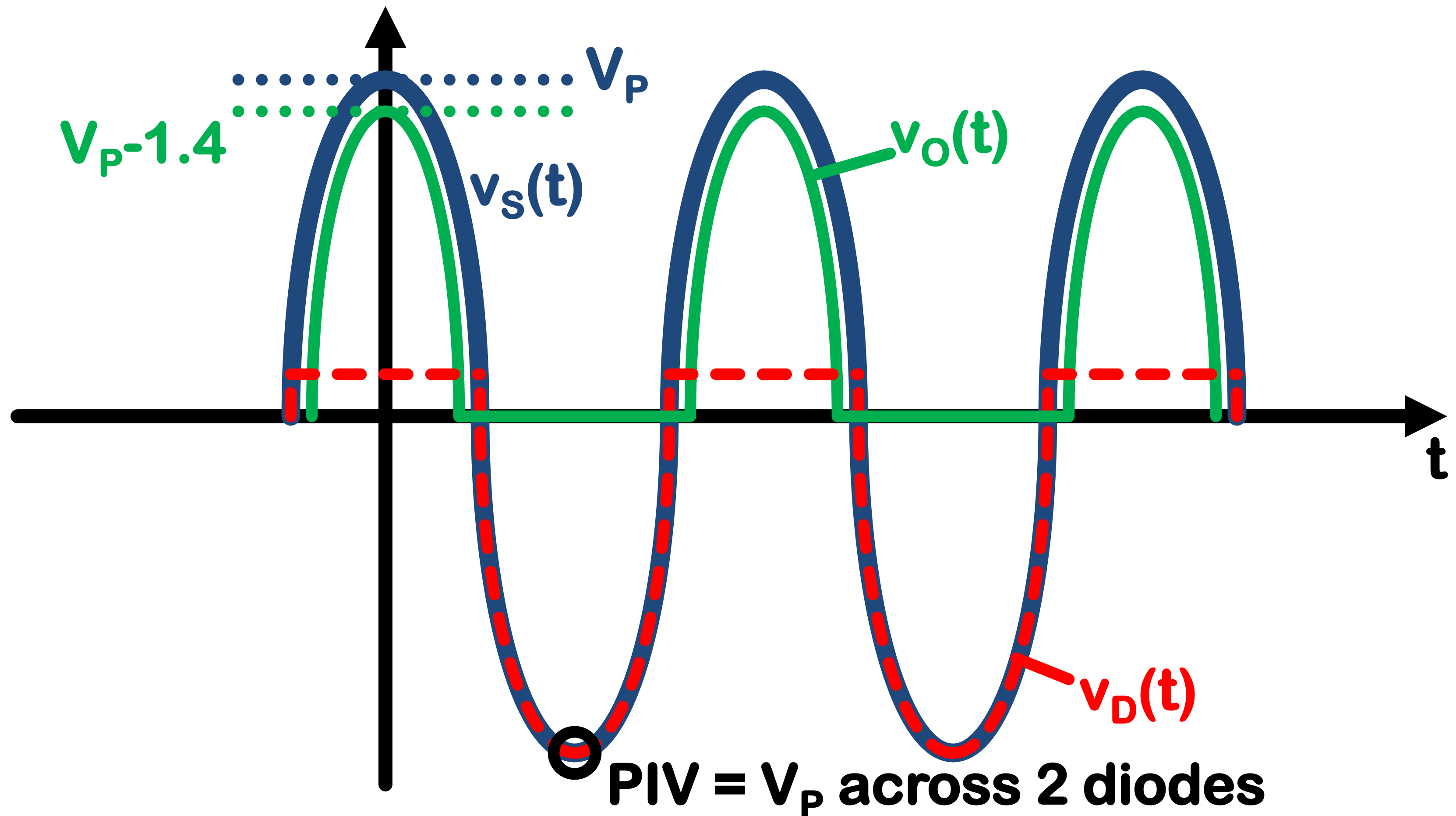


## Positive Half Wave Rectifier (2-Diodes)

$$v_o(v_s) = v_s - 1.4 \text{ for } v_s > 1.4$$

$$v_o(v_s) = 0 \text{ for } v_s < 1.4$$

PIV =  $V_p/2$ ; half across each diode



# Negative Half Wave Rectifier

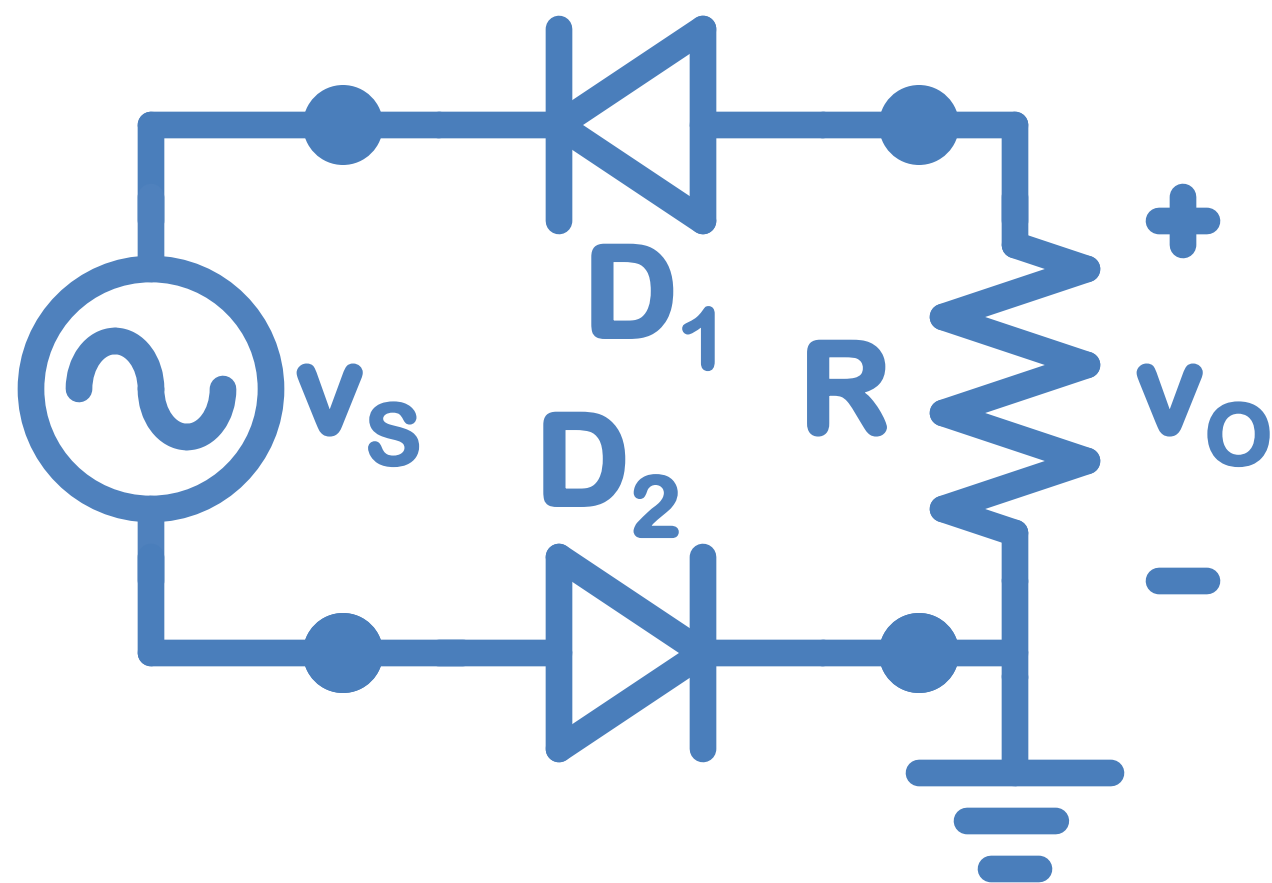
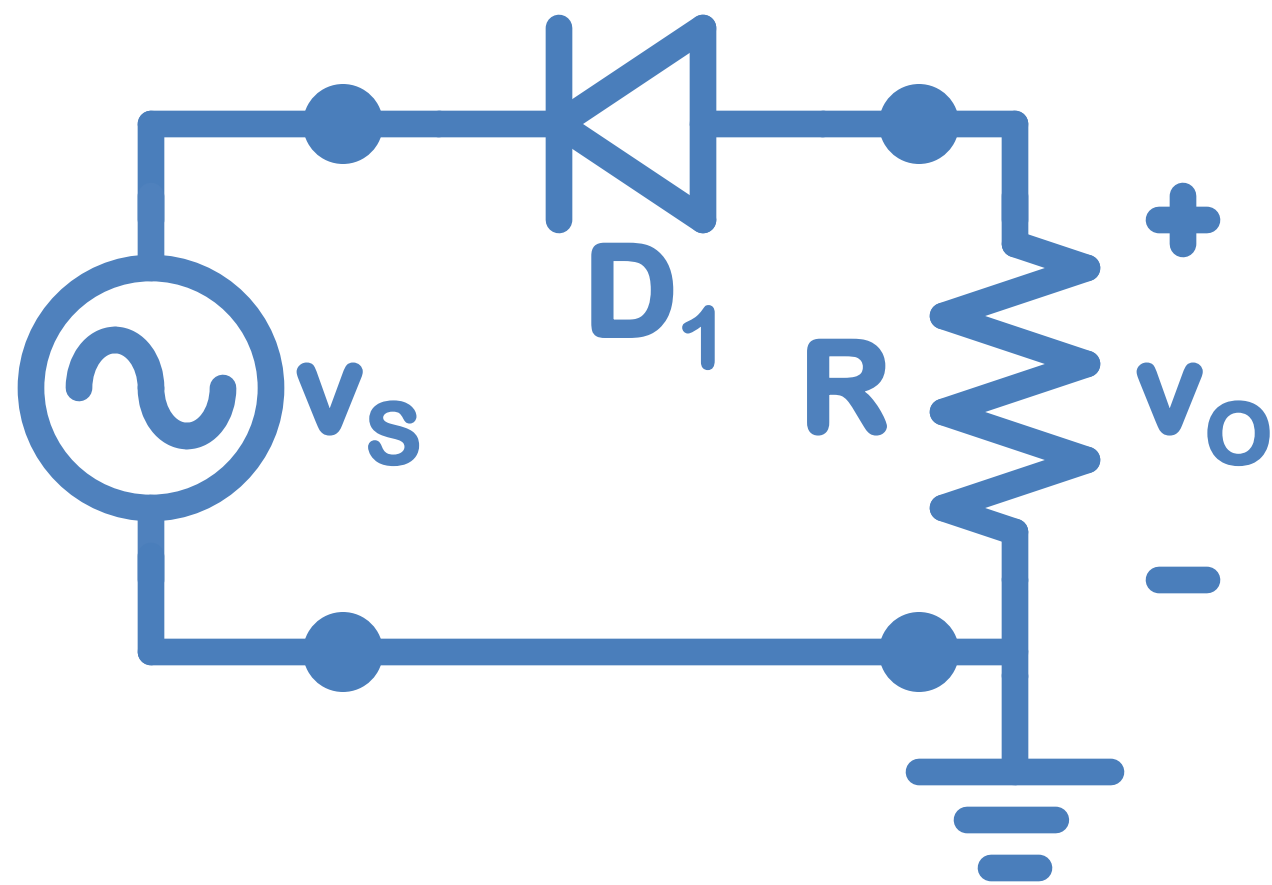
Find the transfer function  $v_o(v_s)$

Find the Peak Inverse Voltage

$$PIV = \max(-v_D(v_s))$$

Plot input & output for a sinusoidal input.

$$v_s(t) = V_p \cos(2\pi t/T)$$



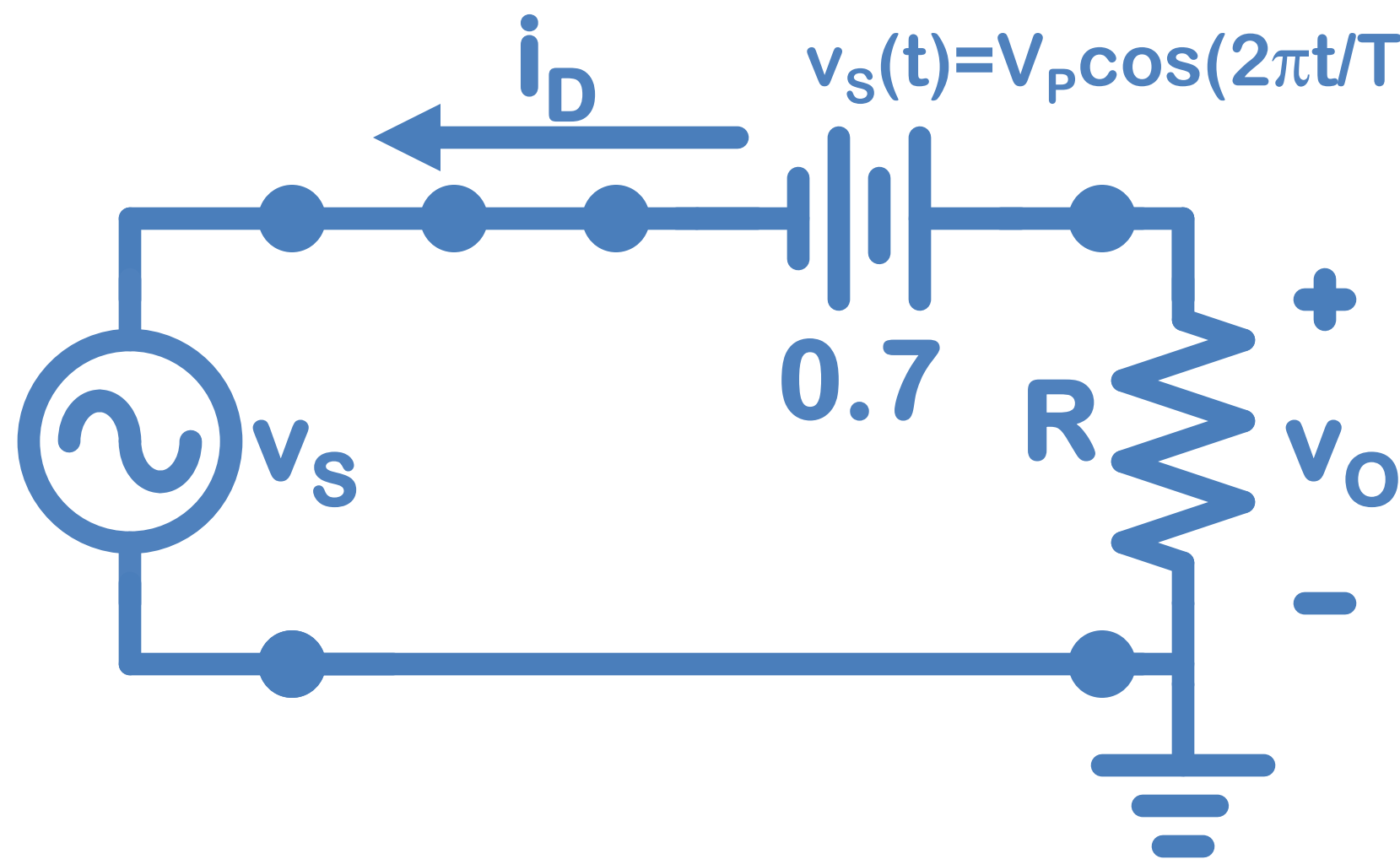
# Negative Half Wave Rectifier

Find the transfer function  $v_O(v_S)$

Find the Peak Inverse Voltage

$$\text{PIV} = \max(-v_D(v_S))$$

Plot input & output for a sinusoidal input.



FB:  $v_O(v_S) = v_S + 0.7$

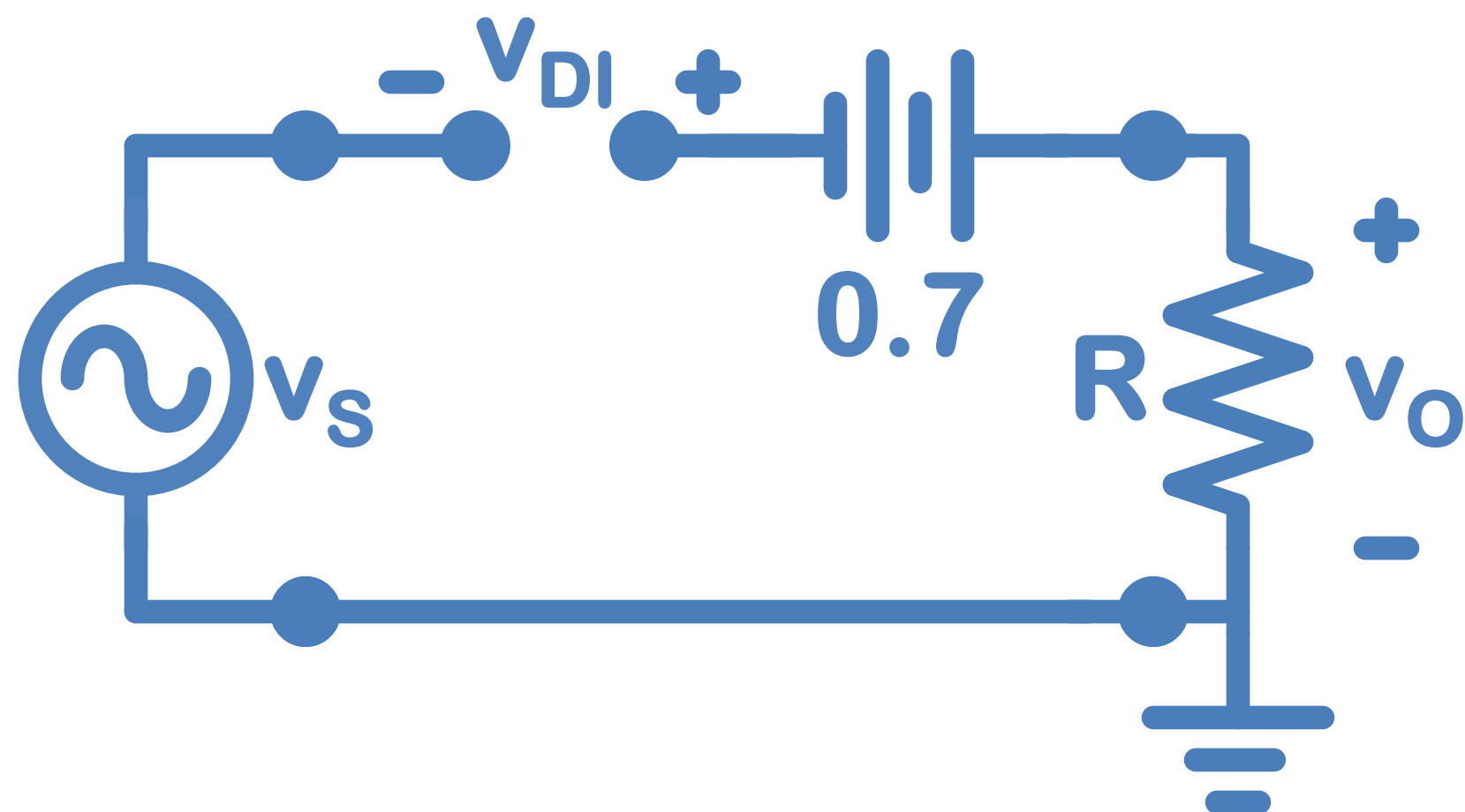
$$i_D(v_S) = -(v_S + 0.7)/R > 0; v_S < -0.7$$

RB:  $v_O(v_S) = 0$

$$v_D(v_S) = -v_S < 0; v_S > -0.7$$

$$v_O(v_S) = v_S + 0.7 \text{ for } v_S < -0.7$$

$$v_O(v_S) = 0 \text{ for } v_S > -0.7$$



PIV:  $v_D(v_S) = 0.7$  for  $v_S < -0.7$

$$v_D(v_S) = -v_S \text{ for } v_S > -0.7$$

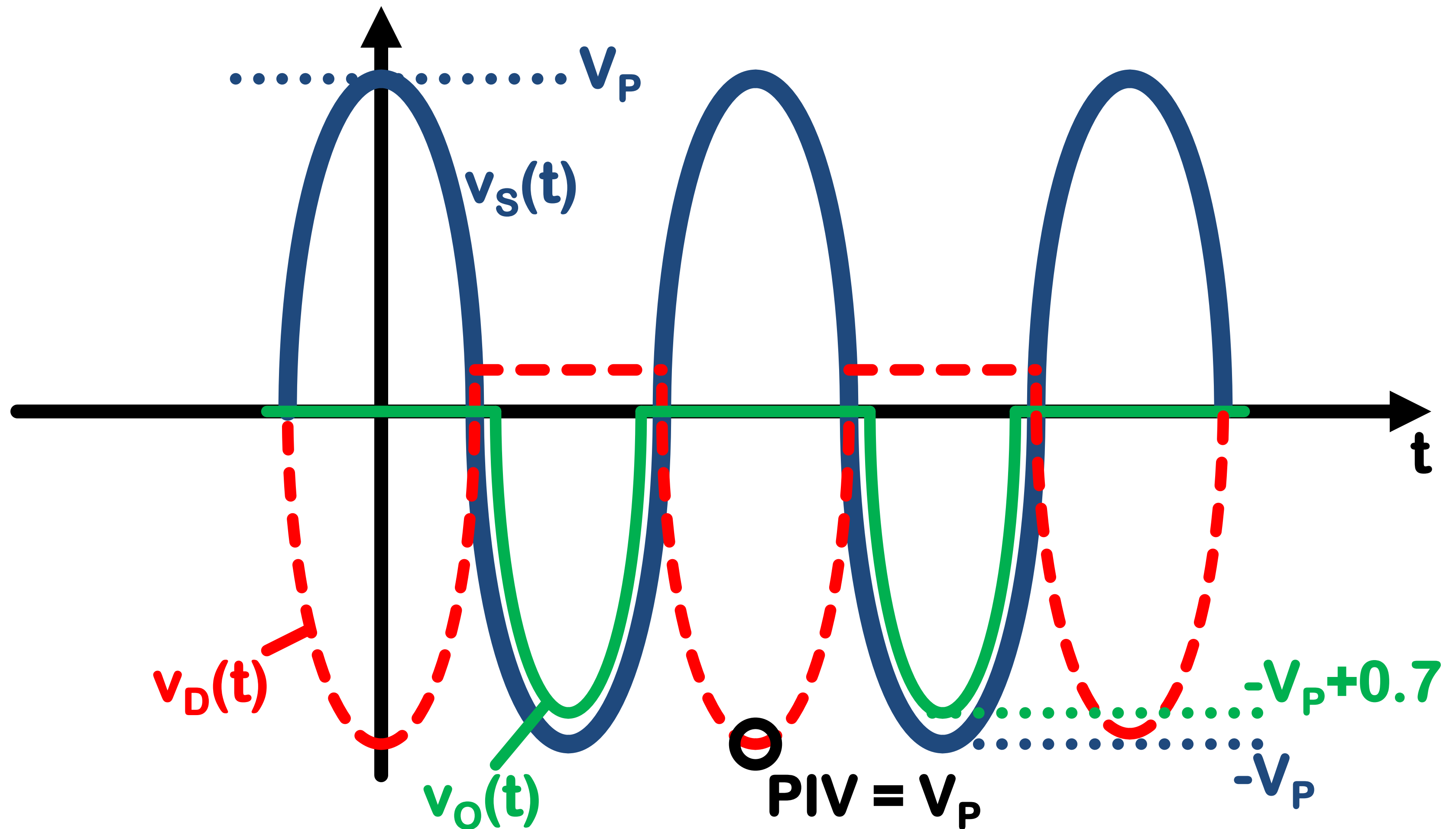
$$\text{PIV} = \max(-v_D) = V_p$$

# Negative Half Wave Rectifier (1-Diode)

$$v_o(v_s) = v_s + 0.7 \text{ for } v_s < -0.7$$

$$v_o(v_s) = 0 \text{ for } v_s > -0.7$$

$$\text{PIV} = V_P$$

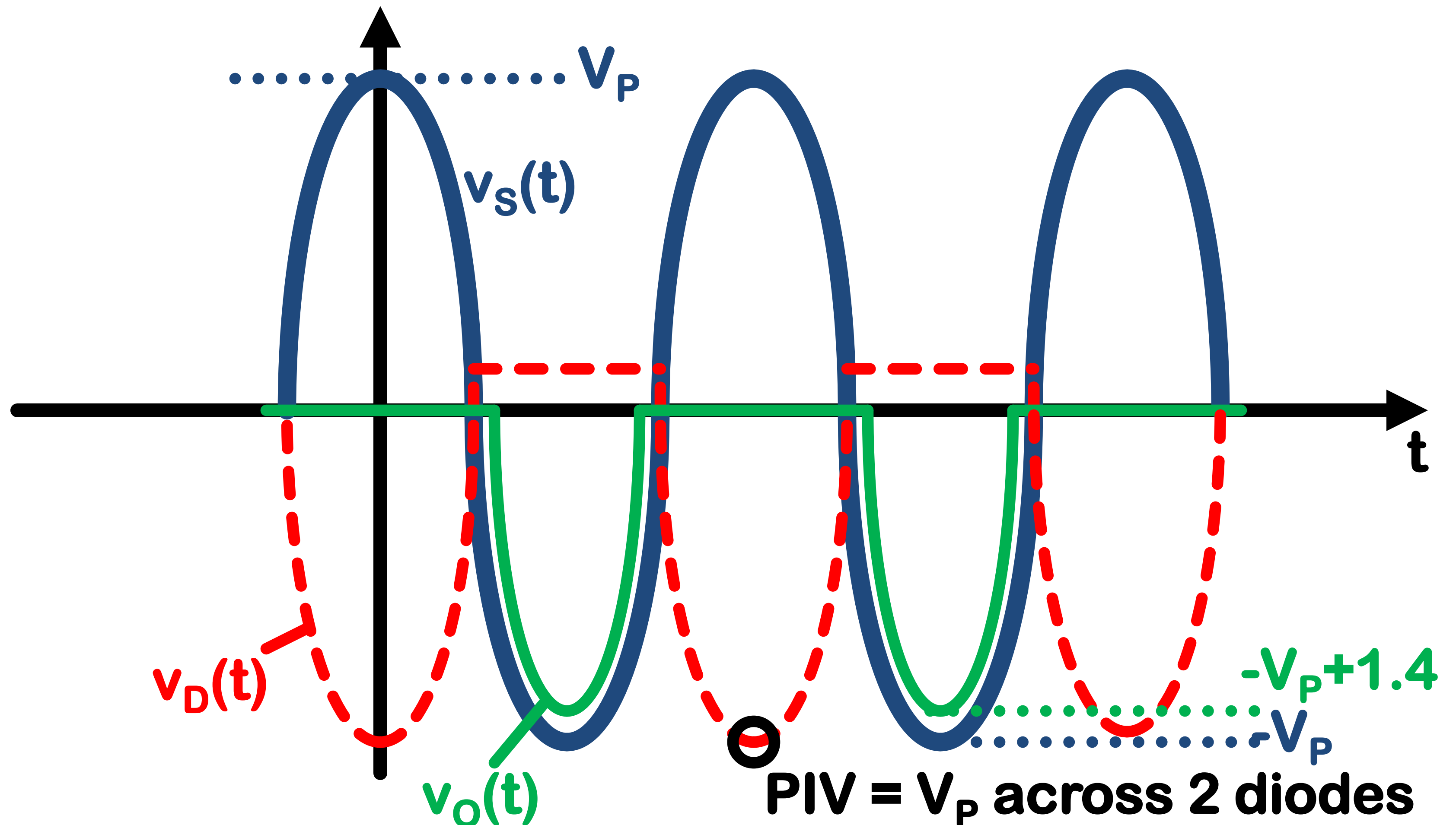


## Negative Half Wave Rectifier (2-Diodes)

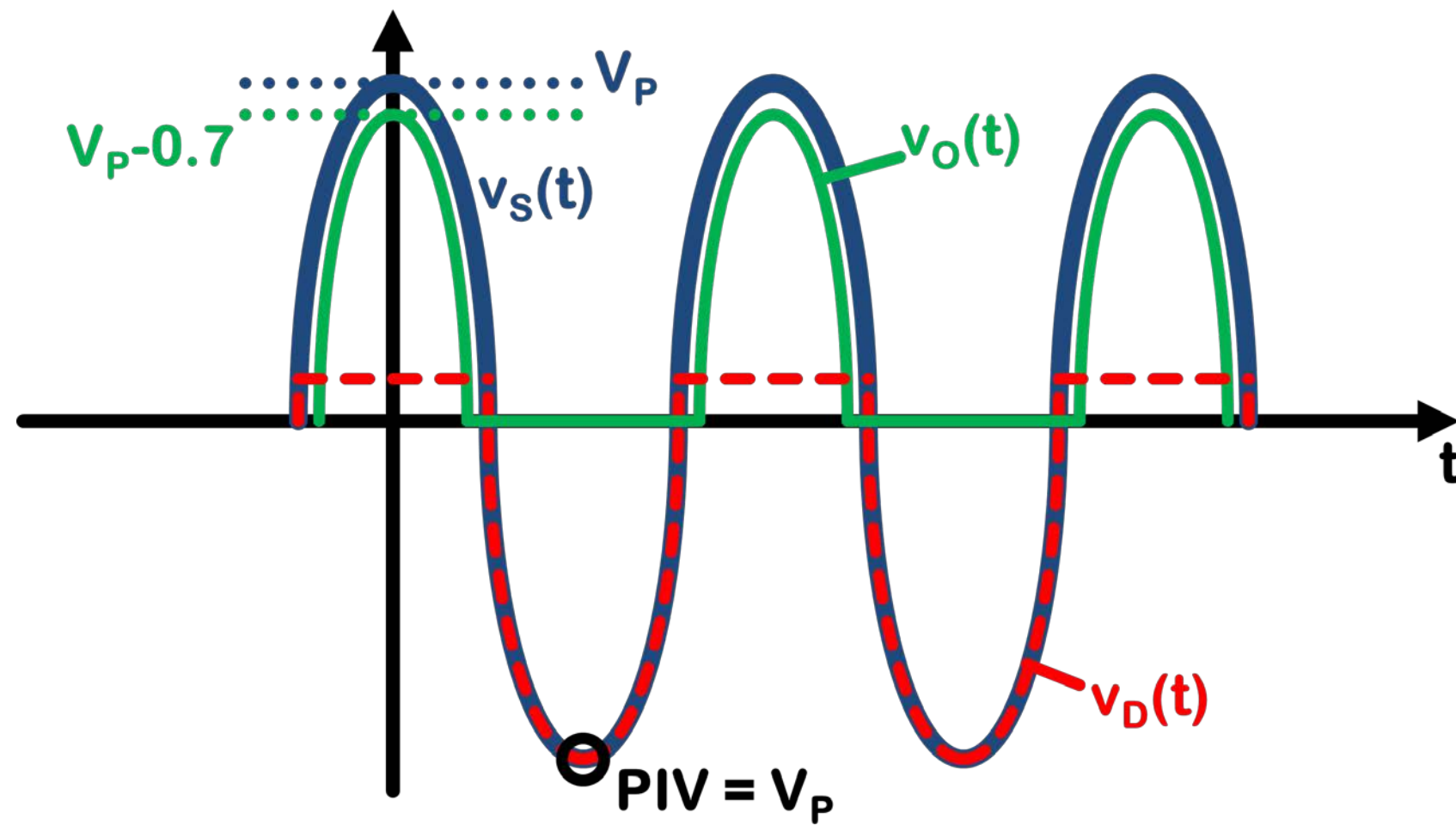
$$v_o(v_s) = v_s + 1.4 \text{ for } v_s < -1.4$$

$$v_o(v_s) = 0 \text{ for } v_s > -1.4$$

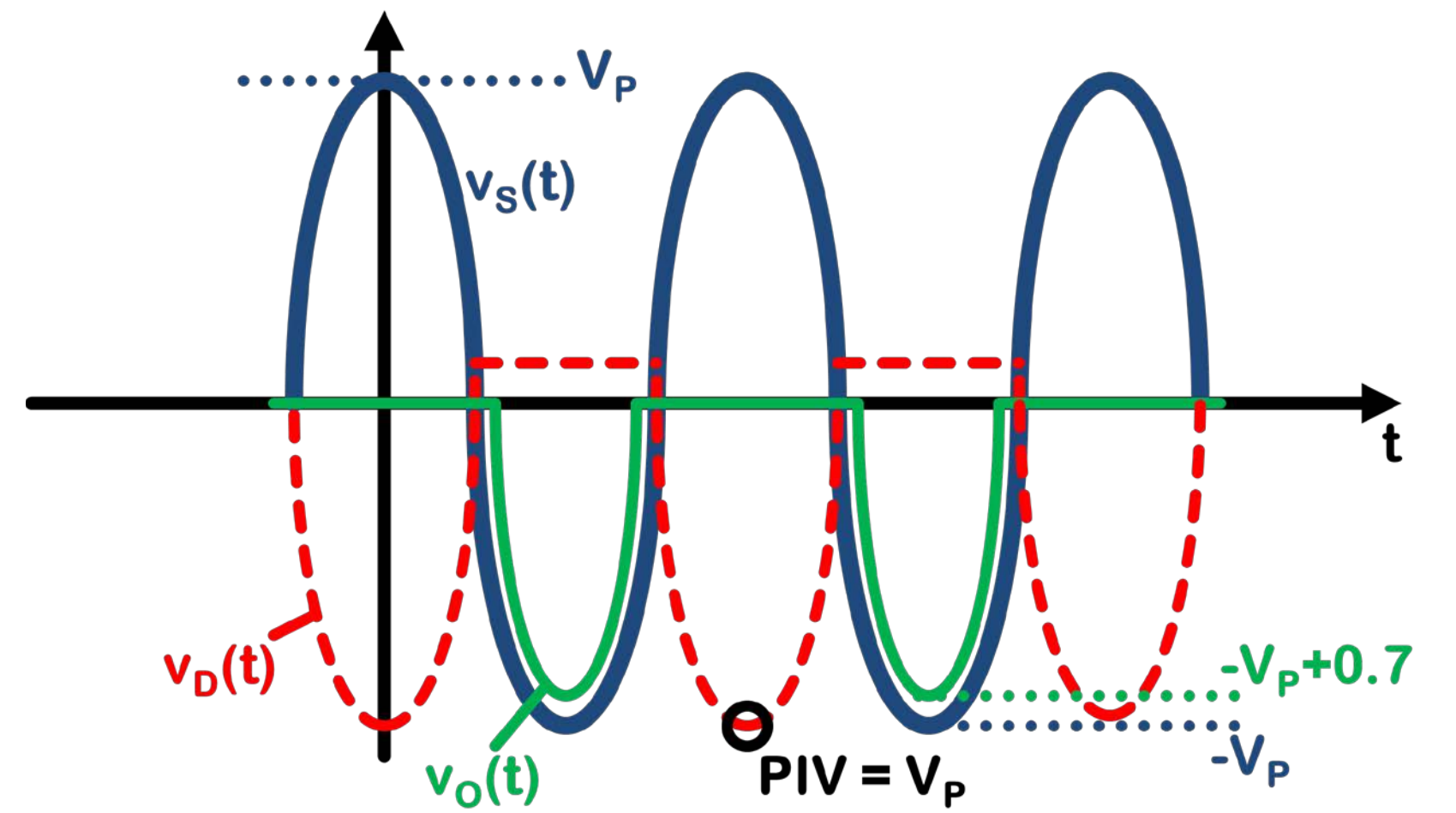
PIV =  $V_p/2$ ; half across each diode



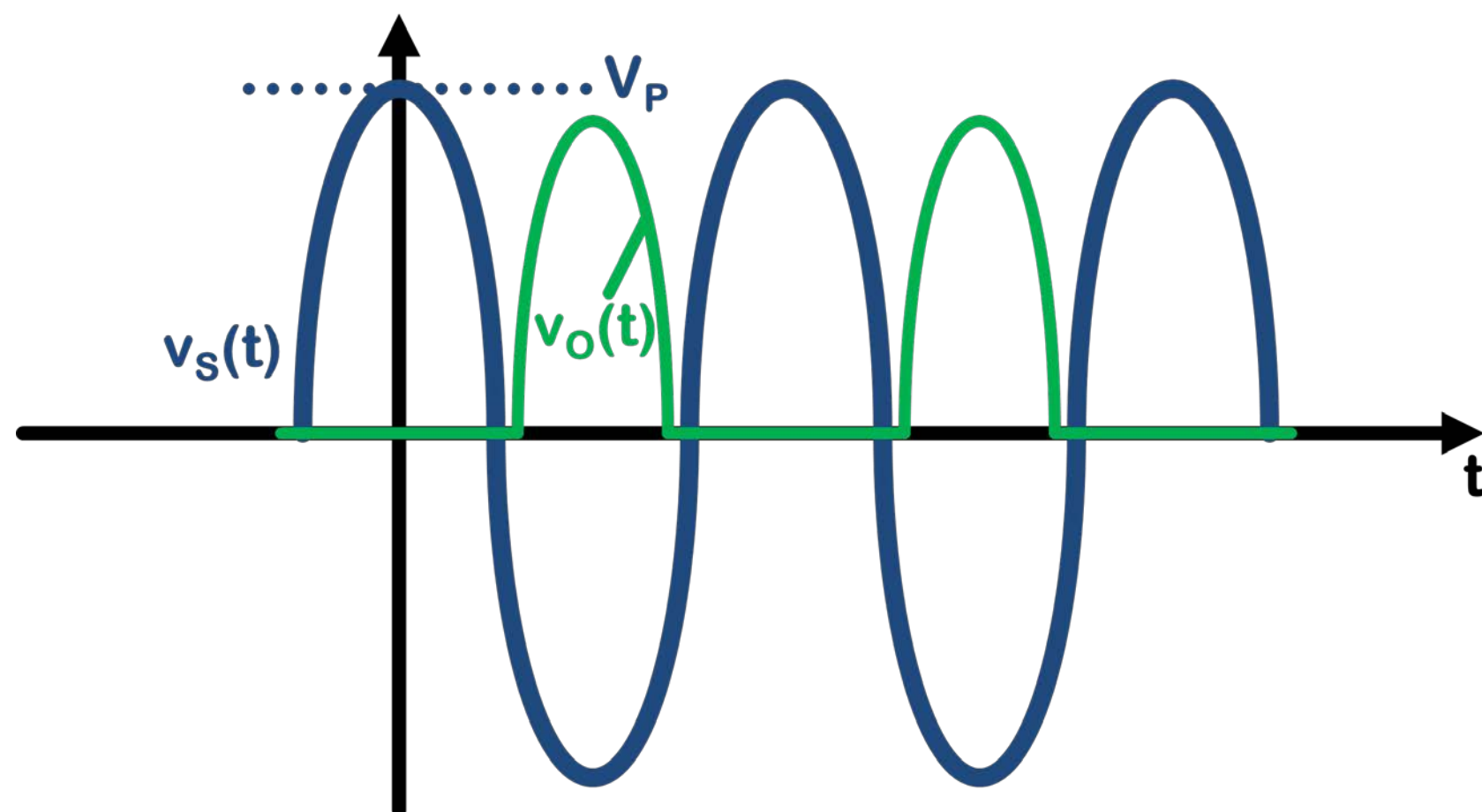
# Positive Half Wave



# Negative Half Wave



???



# Negative Half Wave Rectifier

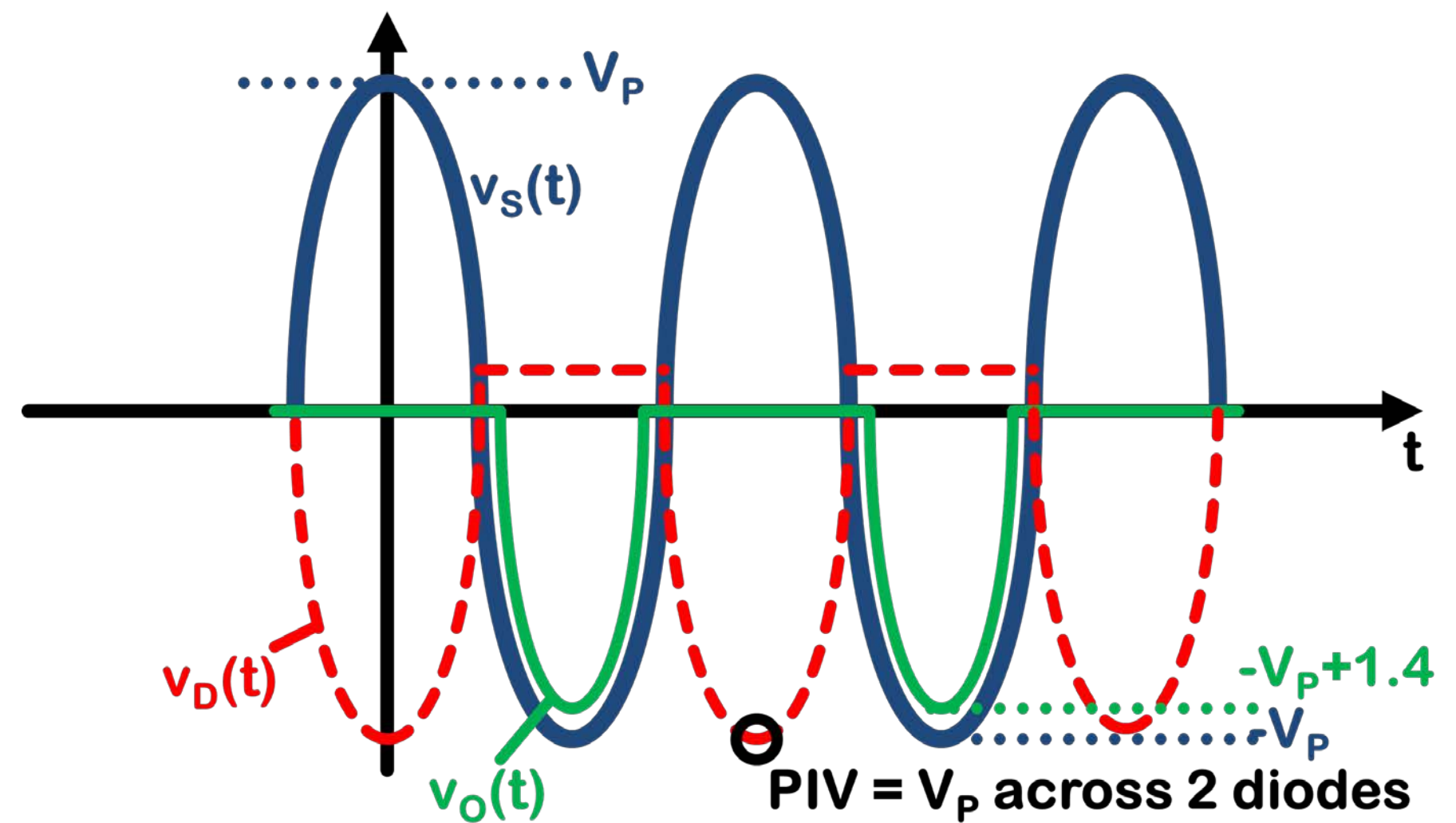
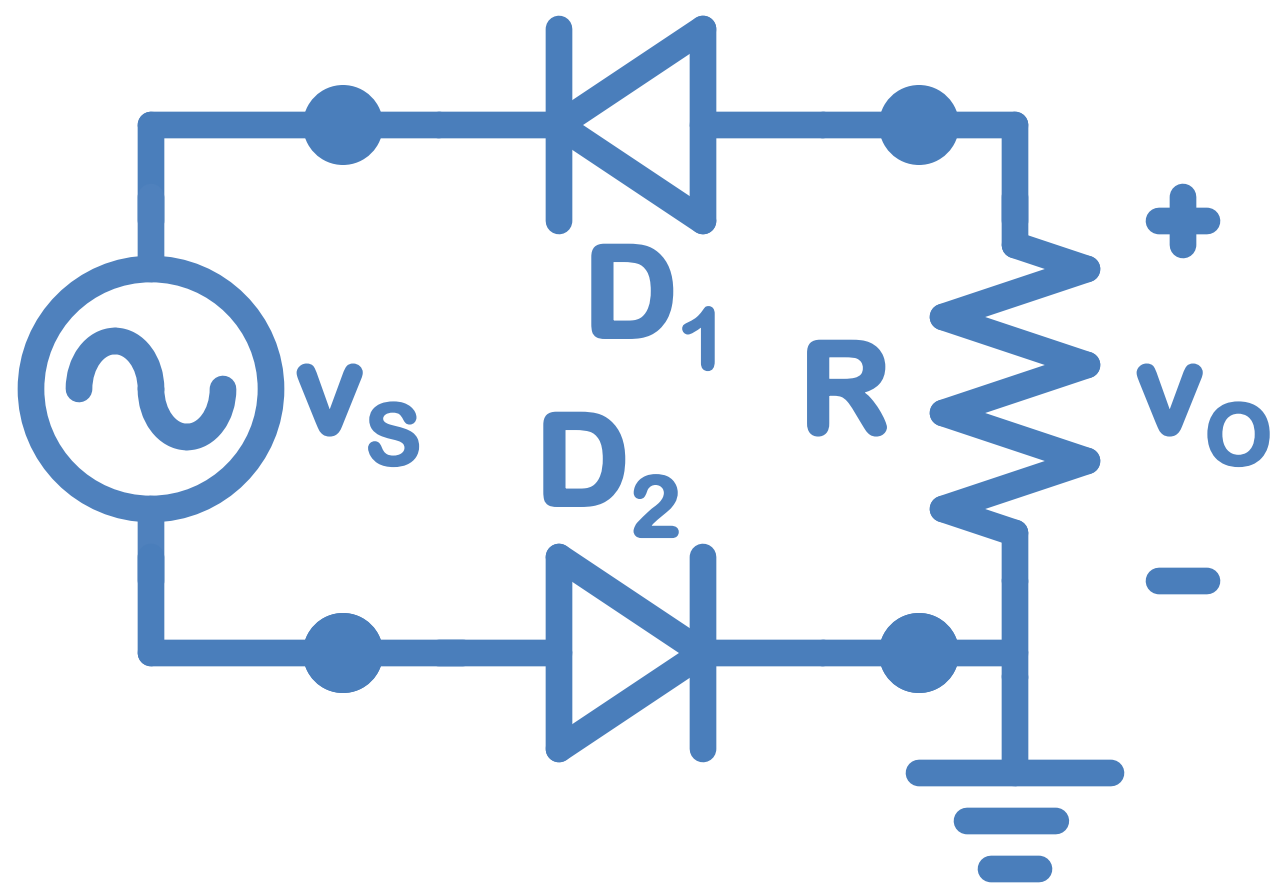
Find the transfer function  $v_o(v_s)$

Find the Peak Inverse Voltage

$$PIV = \max(-v_D(v_s))$$

Plot input & output for a sinusoidal input.

$$v_s(t) = V_p \cos(2\pi t/T)$$



# Inverting Half Wave Rectifier

Find the transfer function  $v_o(v_s)$

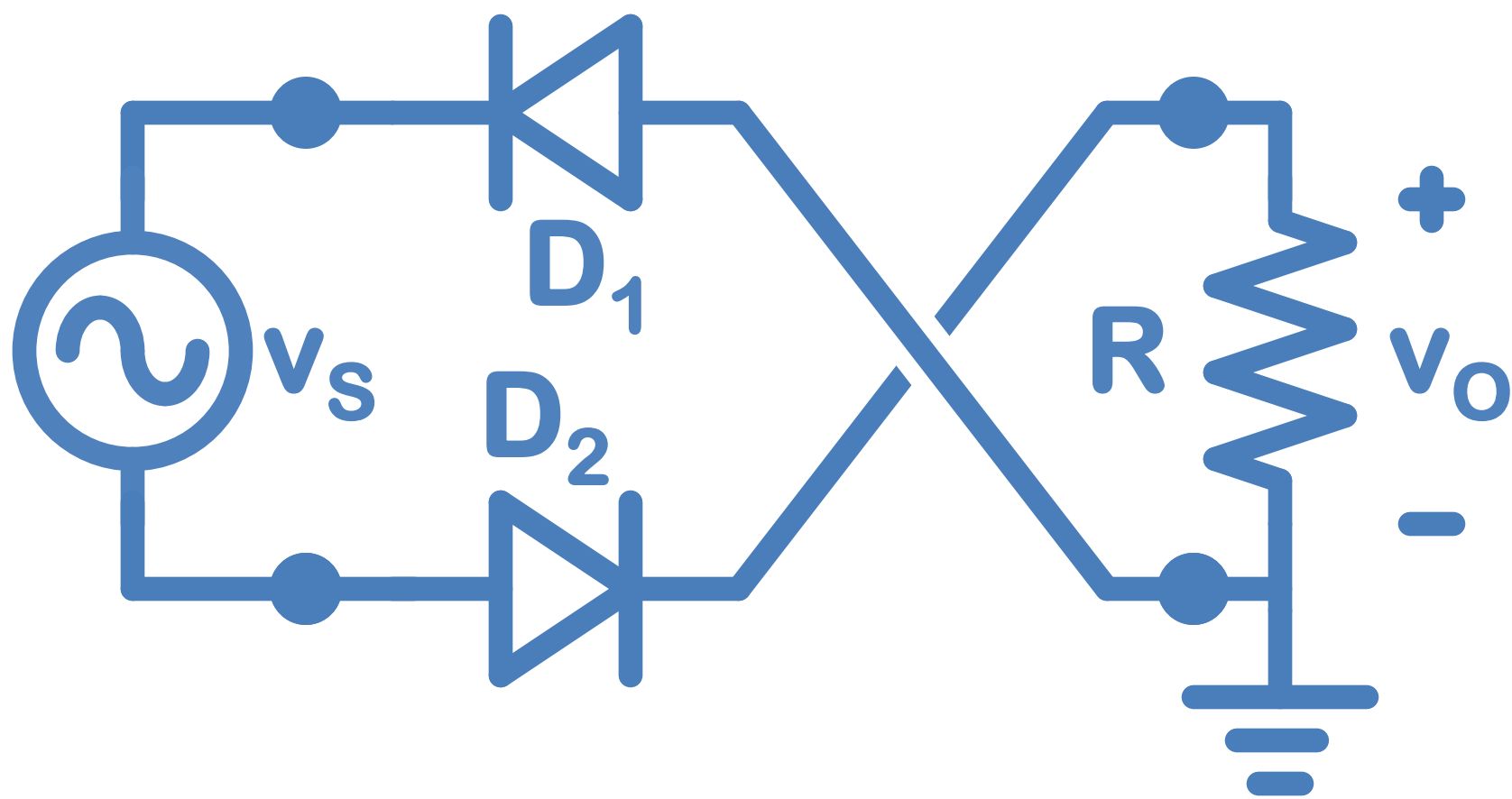
Find the Peak Inverse Voltage

$$PIV = \max(-v_D(v_s))$$

Plot input & output for a sinusoidal input.

$$v_s(t) = V_p \cos(2\pi t/T)$$

Same Analysis as Negative Half Wave with new  $(v_o) = -\text{old}(v_o)$

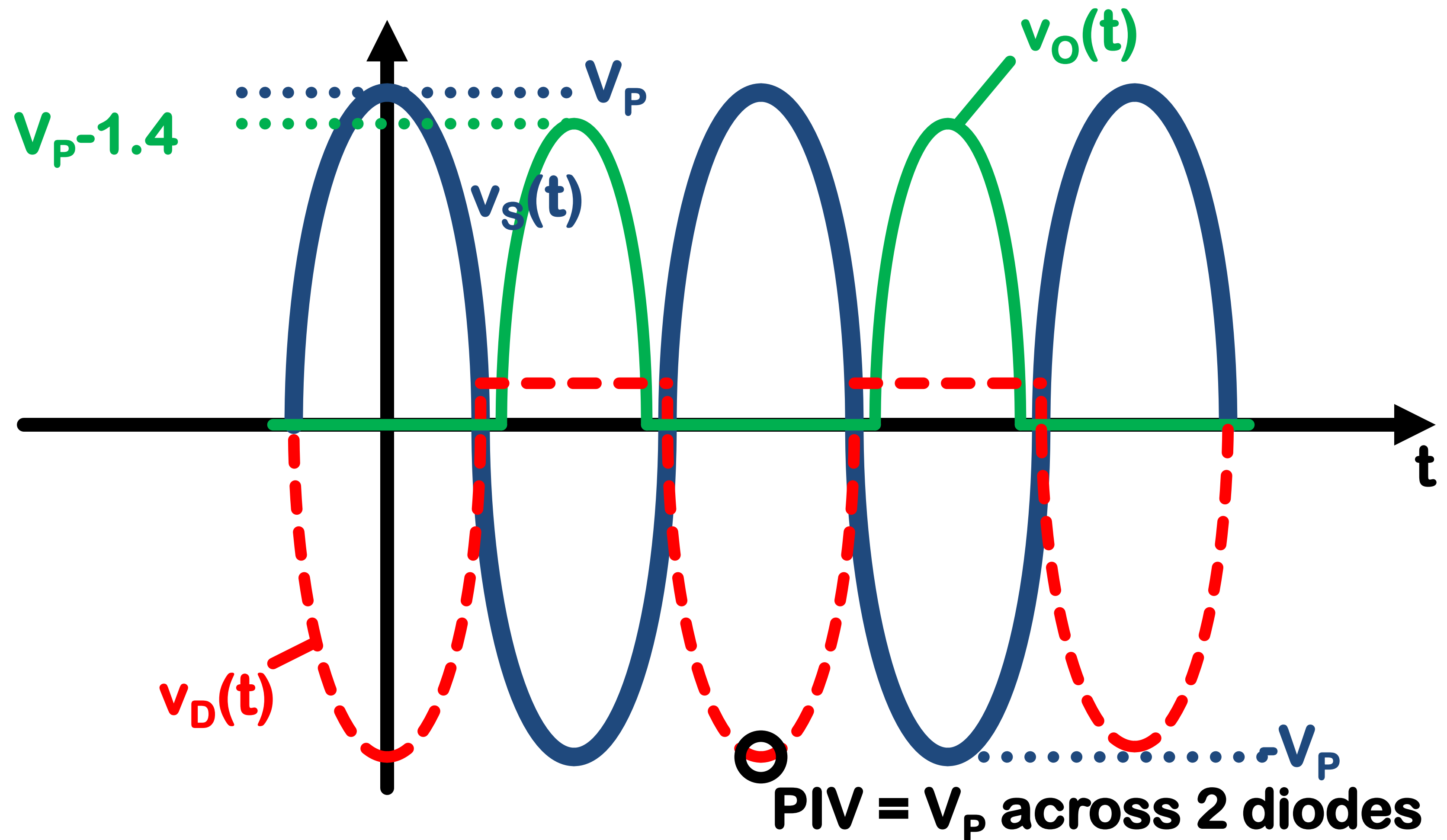


## Negative Half Wave Rectifier (2-Diodes)

$$v_o(v_s) = -v_s - 1.4 \text{ for } v_s < -1.4$$

$$v_o(v_s) = 0 \text{ for } v_s > -1.4$$

PIV =  $V_p/2$ ; half across each diode



# Full Wave "Bridge" Rectifier

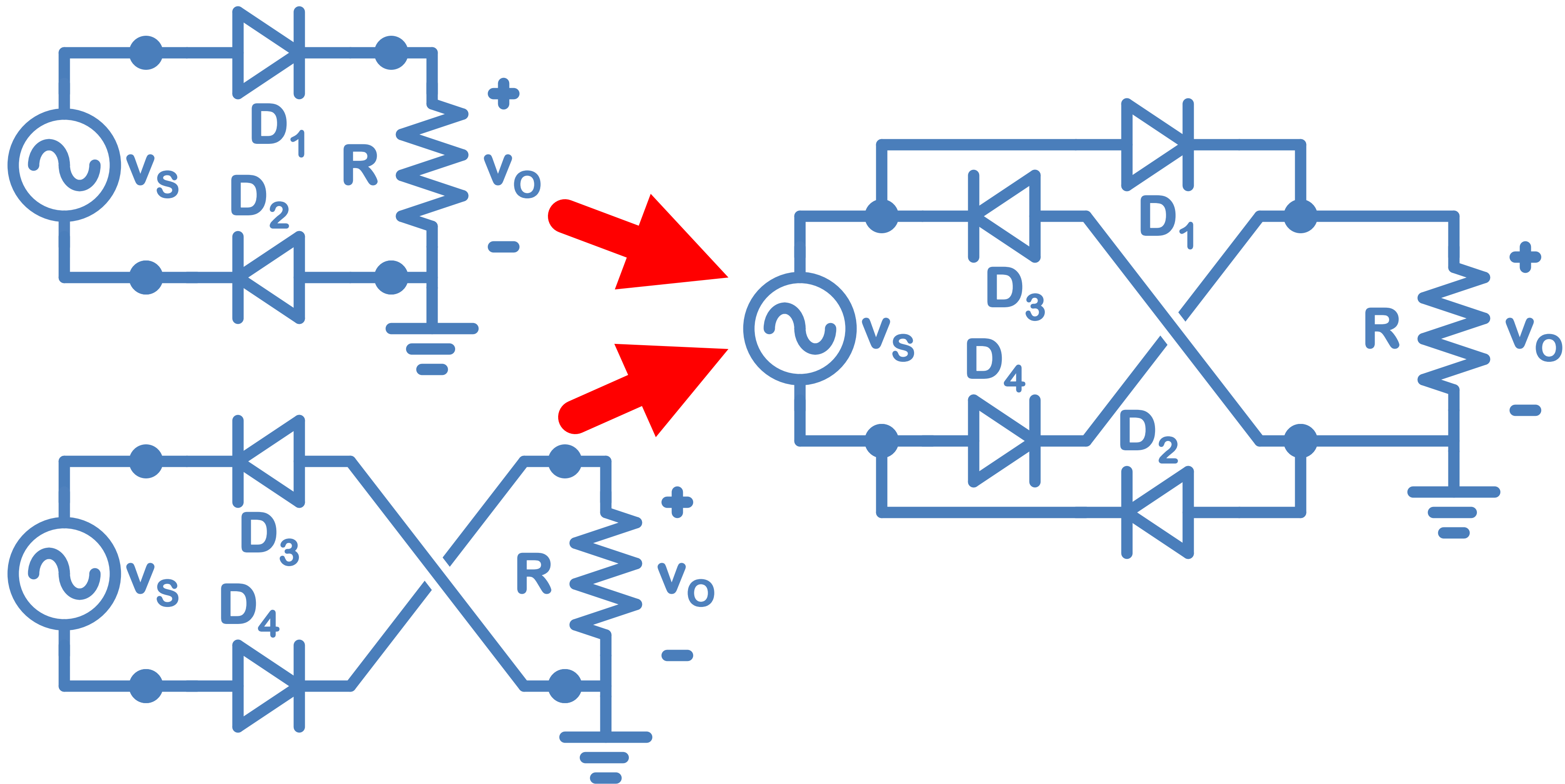
Find the transfer function  $v_o(v_s)$

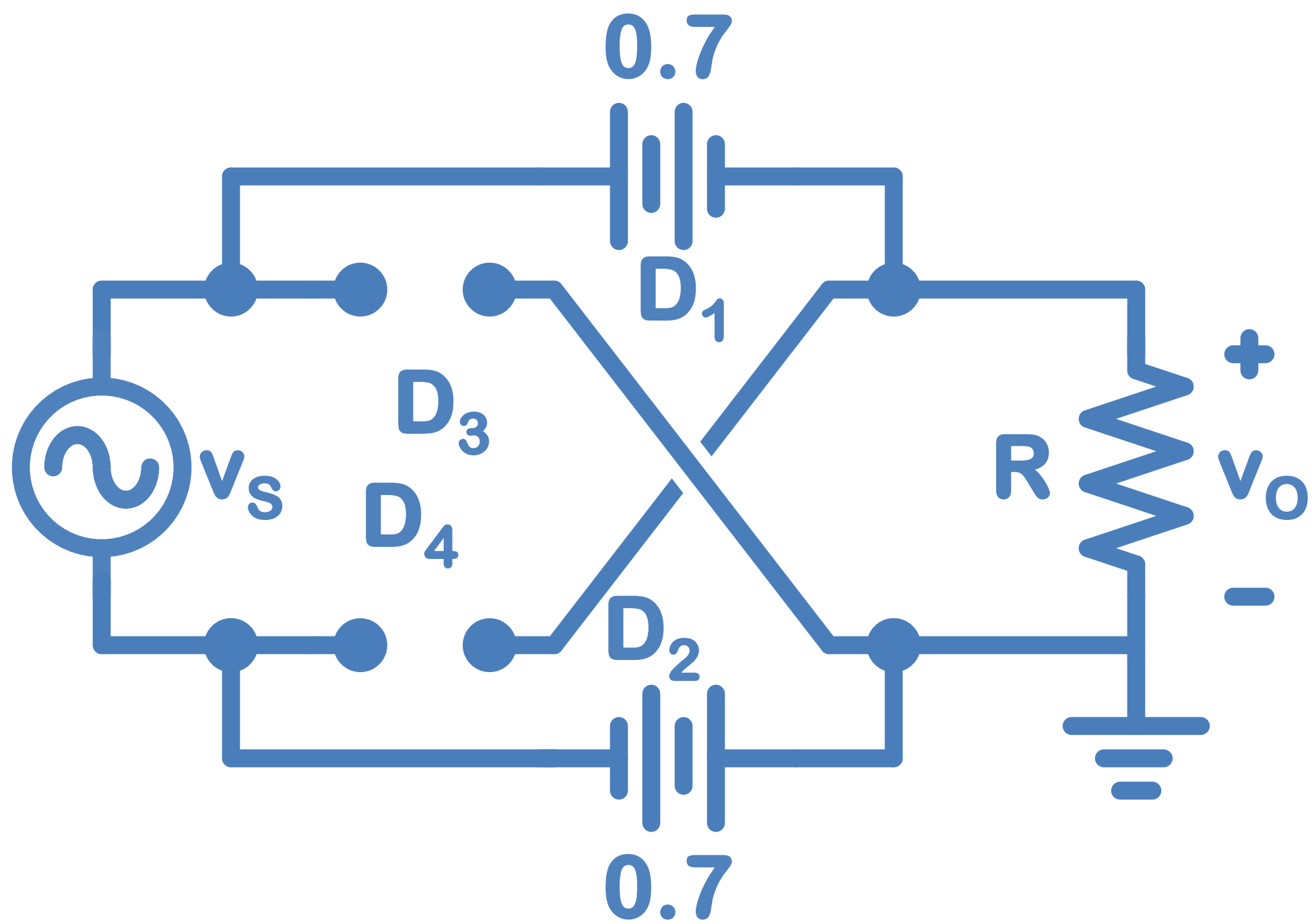
Find the Peak Inverse Voltage

$$PIV = \max(-v_D(v_s))$$

Plot input & output for a sinusoidal input.

$$v_s(t) = V_p \cos(2\pi t/T)$$

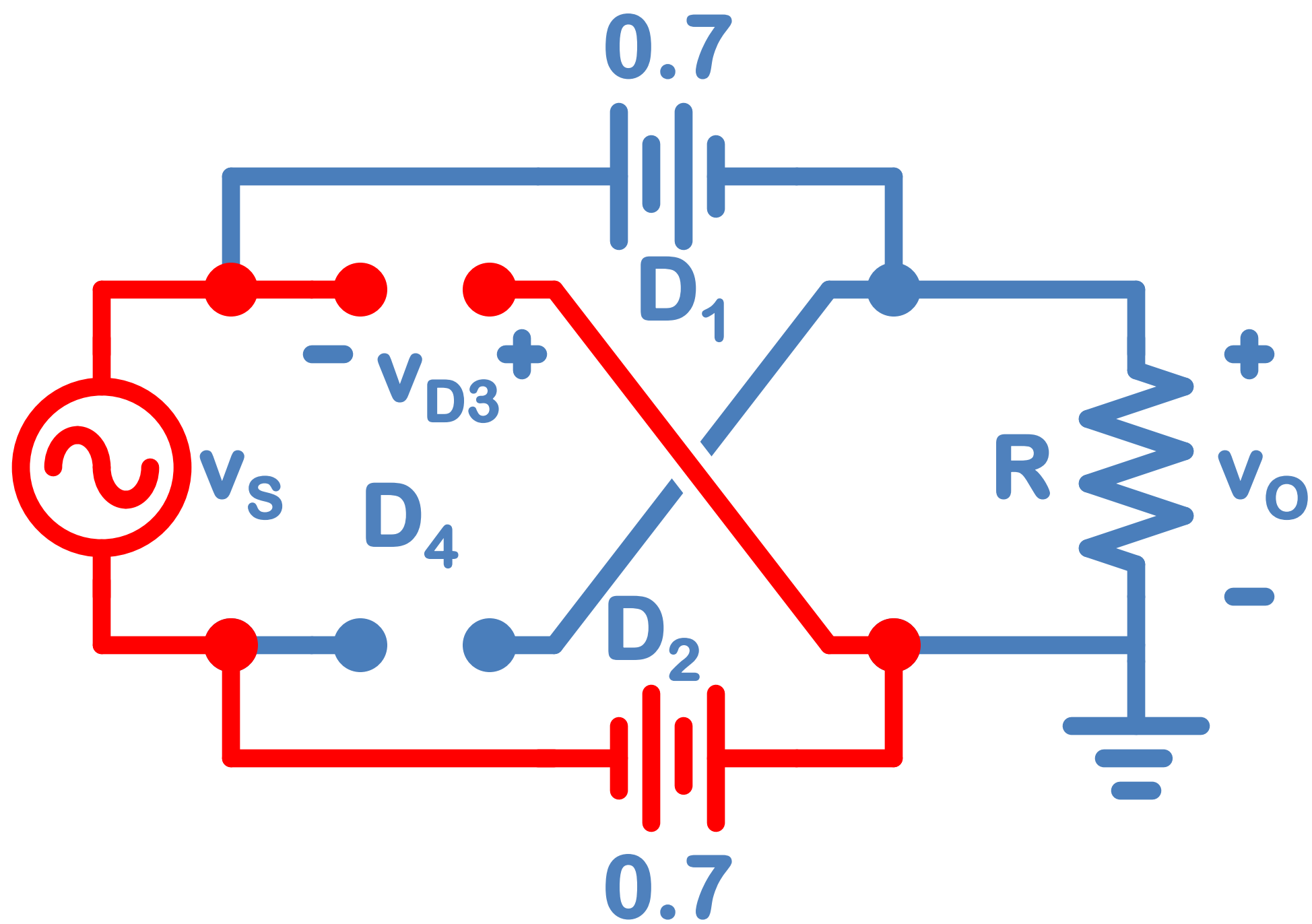




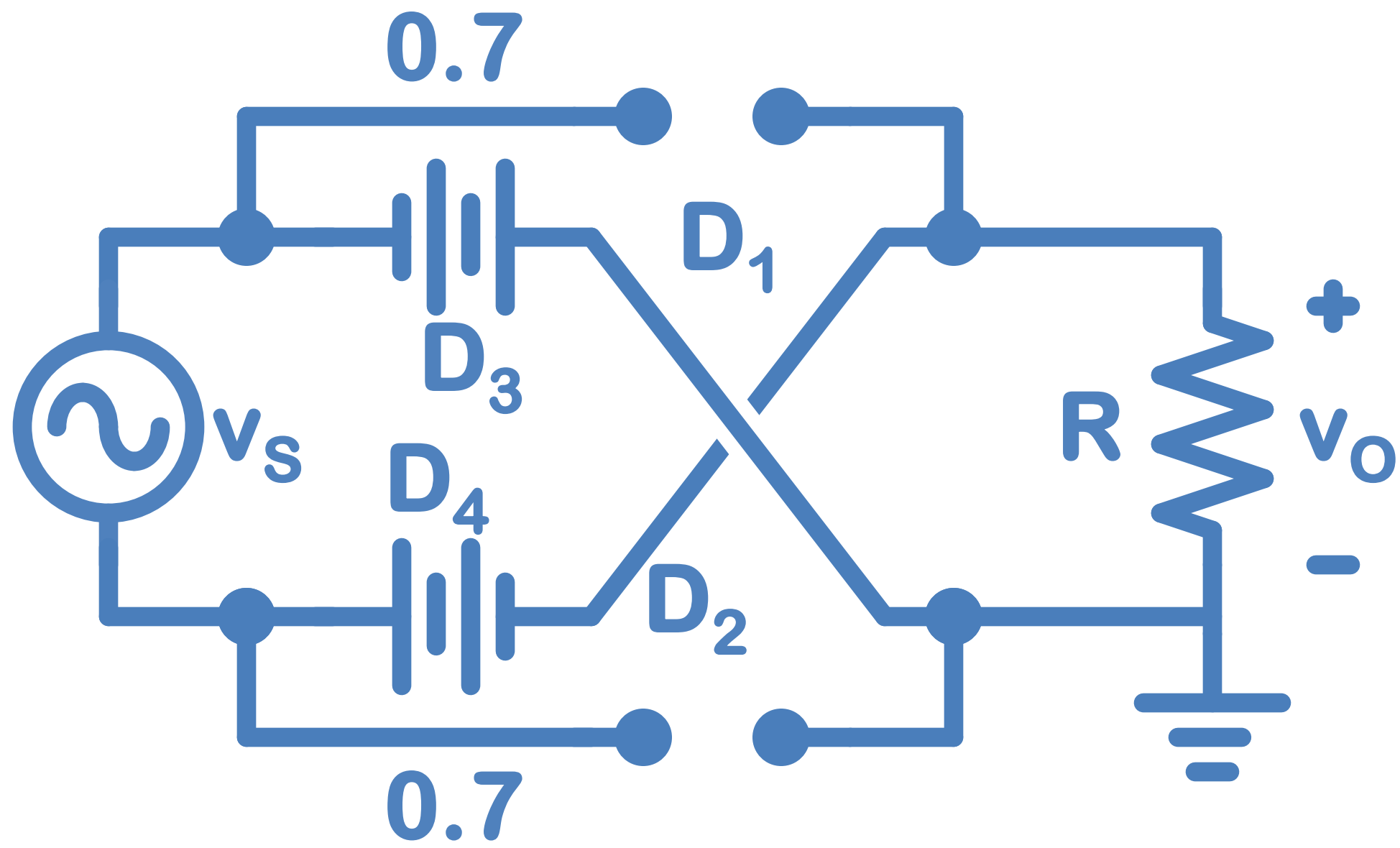
FB,FB,RB,RB:

$$v_o(v_s) = v_s - 1.4$$

$$i_D(v_s) = (v_s - 1.4)/R > 0; v_s > 1.4$$



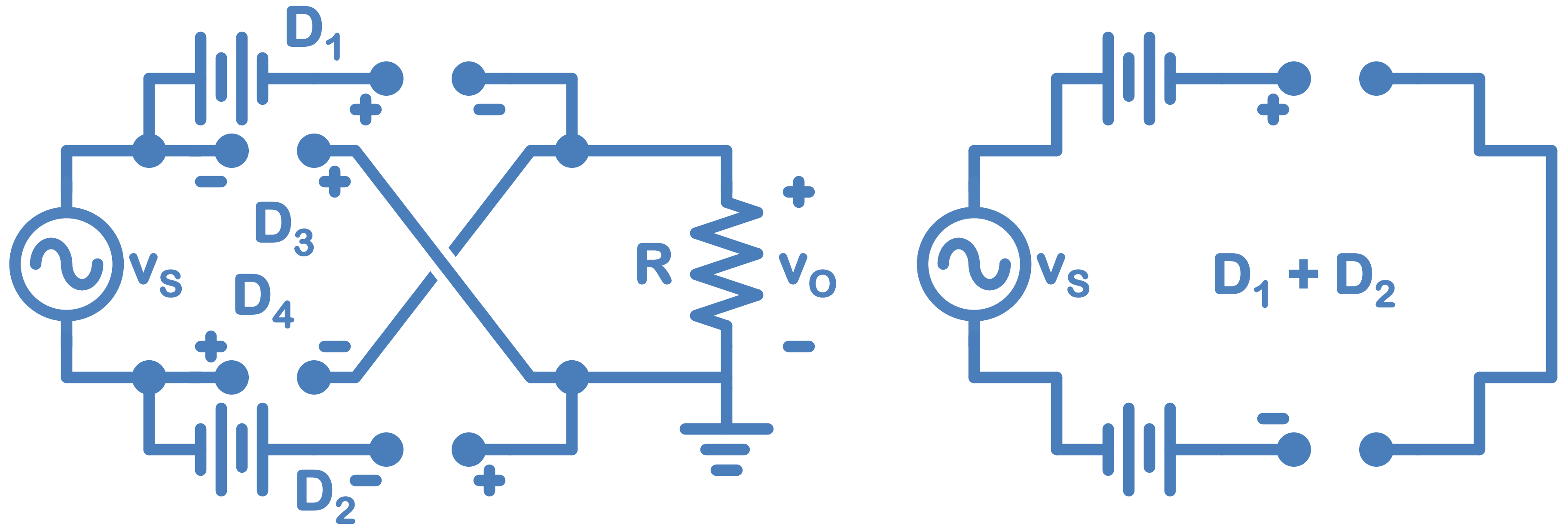
PIV: when  $v_s = V_p$   
 $-V_p - v_{D3} + 0.7 = 0$   
 $v_{D3} = 0 - (V_p - 0.7)$   
 $PIV = \max(-v_D) = V_p - 0.7$



RB, RB, FB, FB:

$$v_o(v_s) = -v_s - 1.4$$

$$i_D(v_s) = -(v_s + 1.4)/R > 0; v_s < -1.4$$



RB,RB,RB,RB:

$$v_o(v_s) = 0$$

$$v_{D1} + v_{D2} = 2v_D = v_s - 1.4 < 0; v_s < 1.4$$

Similarly for D3, and D4:  $v_s > -1.4$

$$v_o(v_s) = 0 \text{ for } -1.4 < v_s < 1.4$$

# Full Wave "Bridge" Rectifier

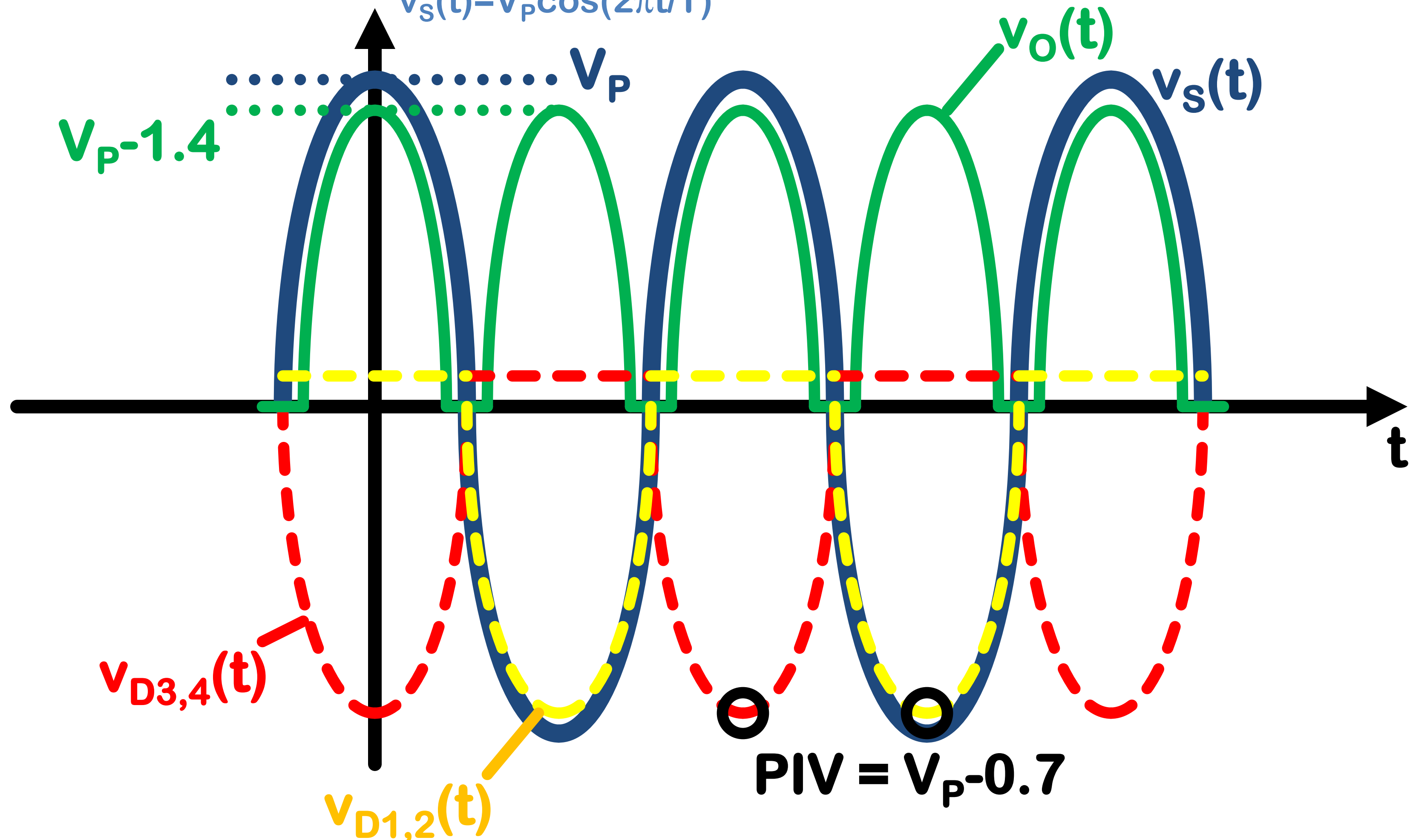
Find the transfer function  $v_o(v_s)$

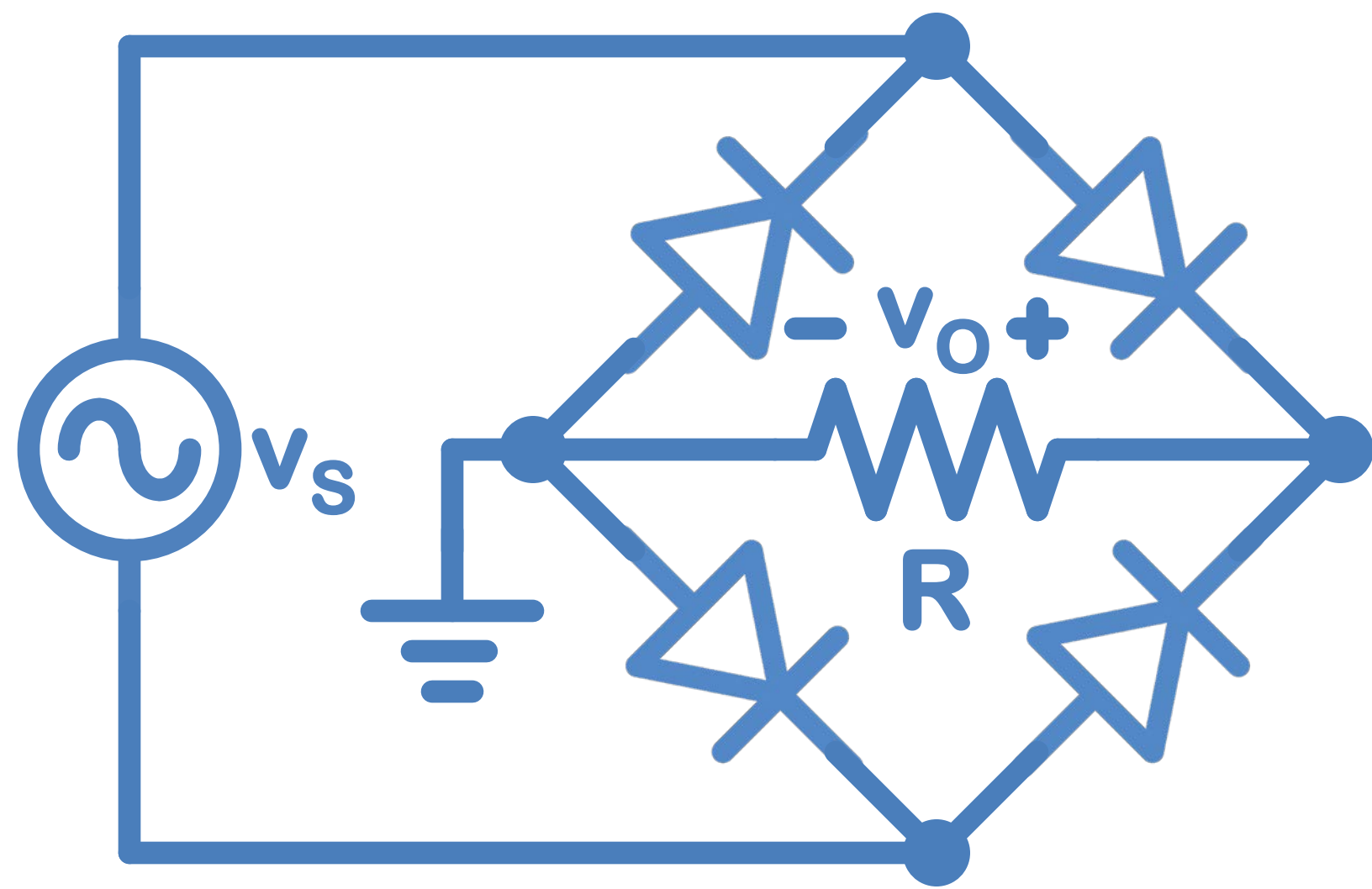
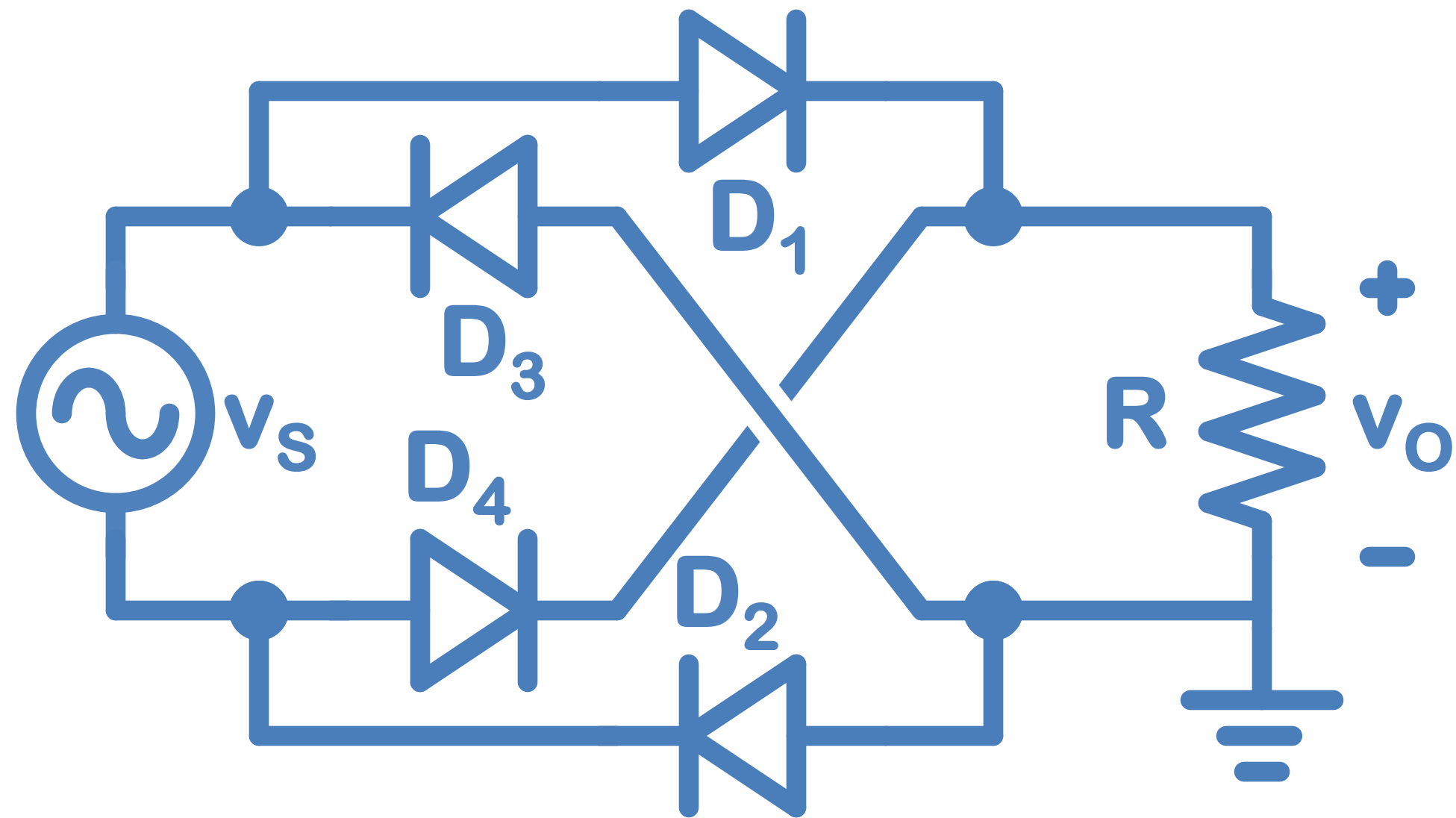
Find the Peak Inverse Voltage

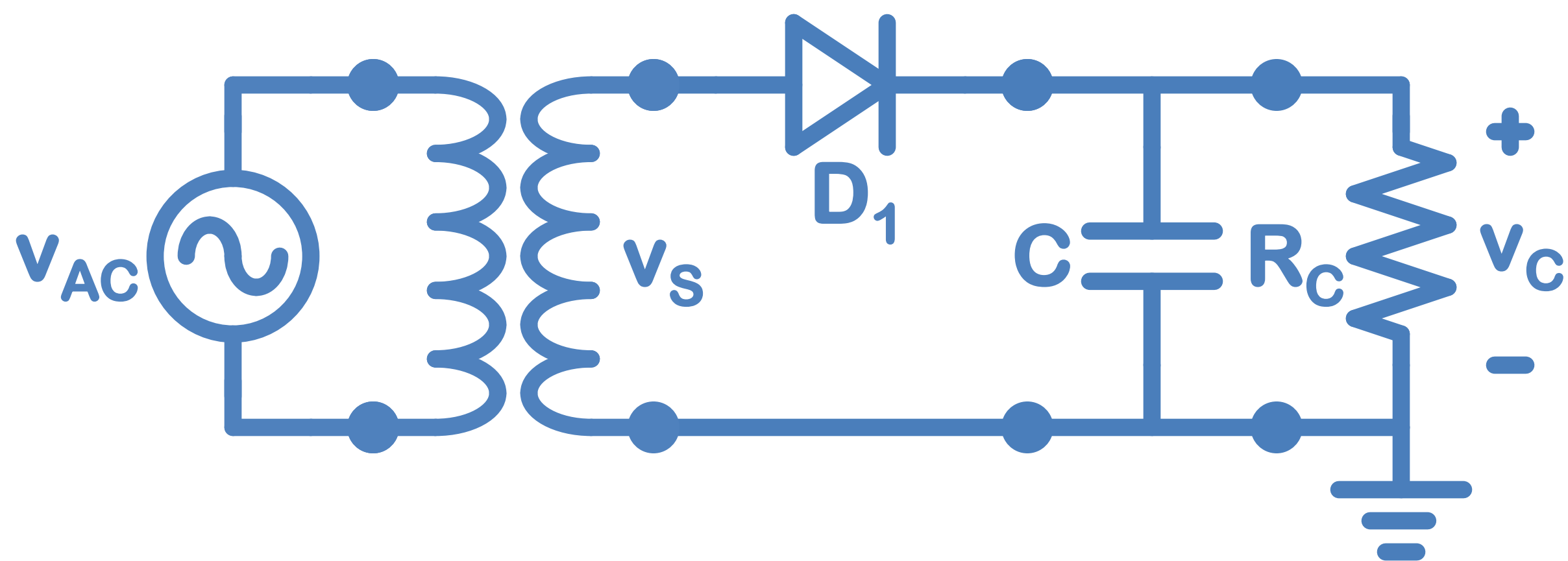
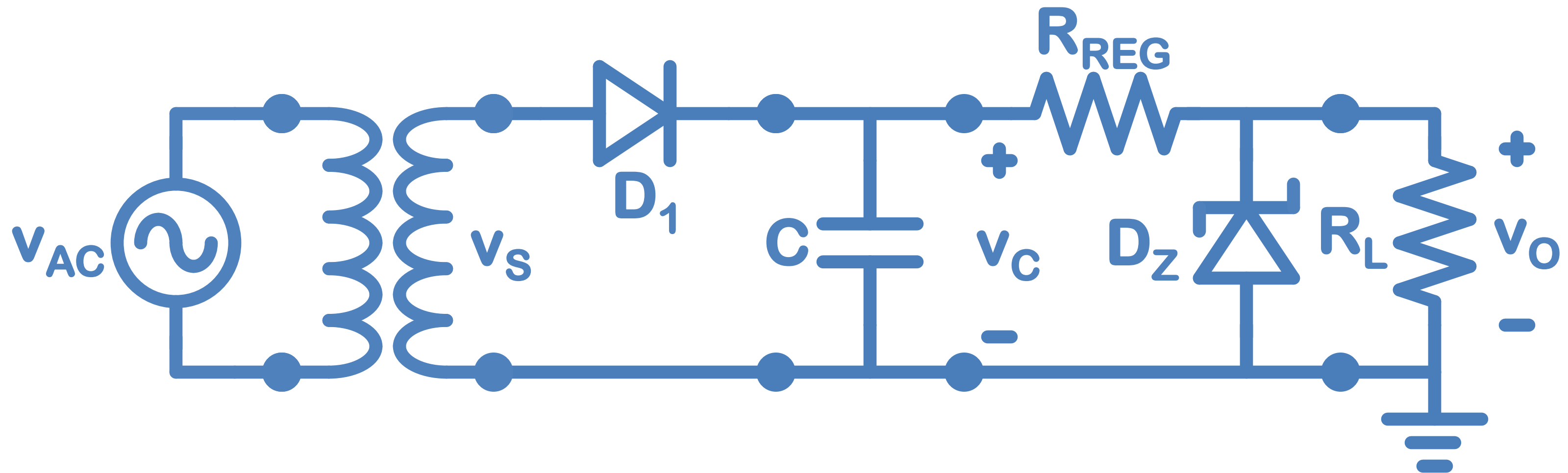
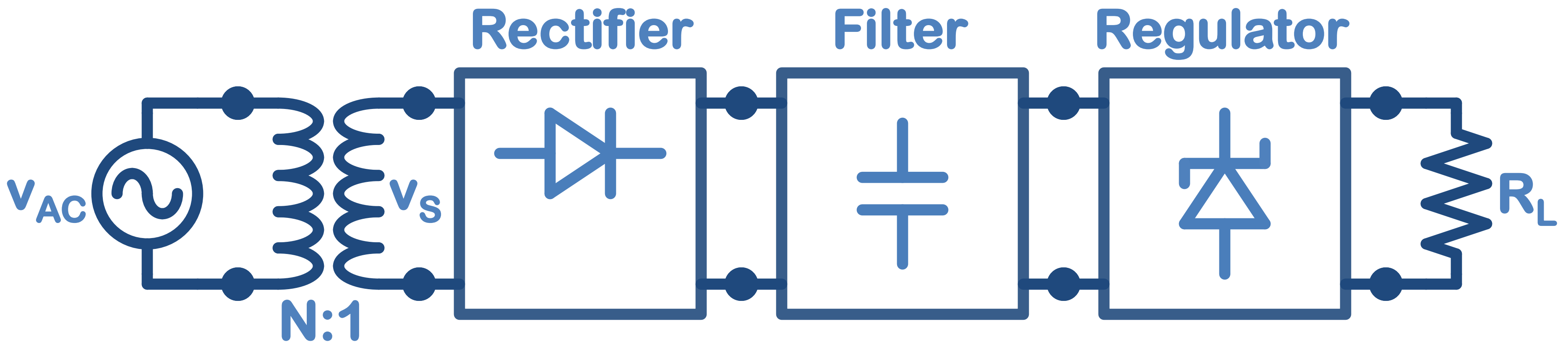
$$\text{PIV} = \max(-v_D(v_s))$$

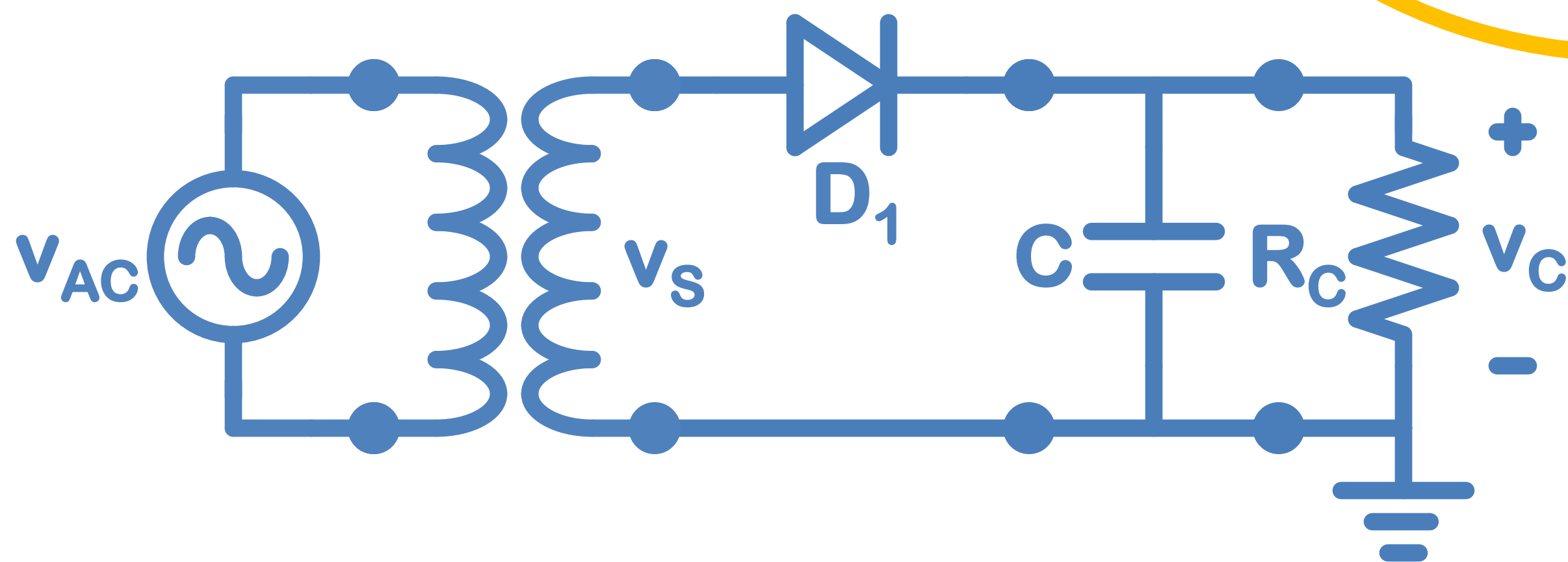
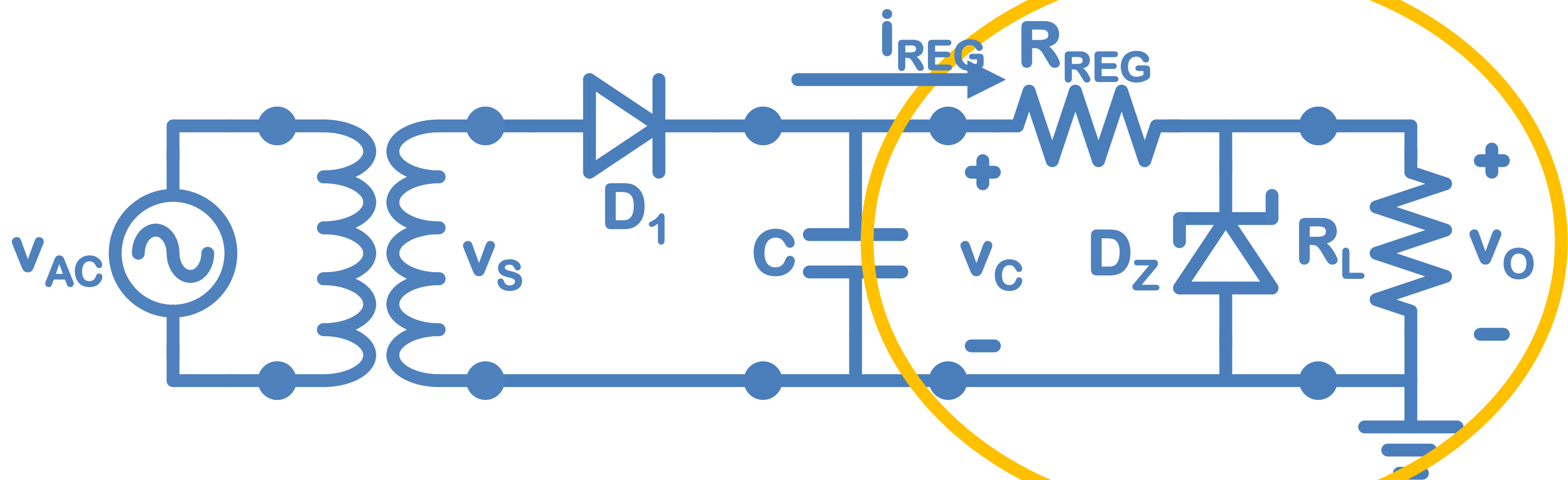
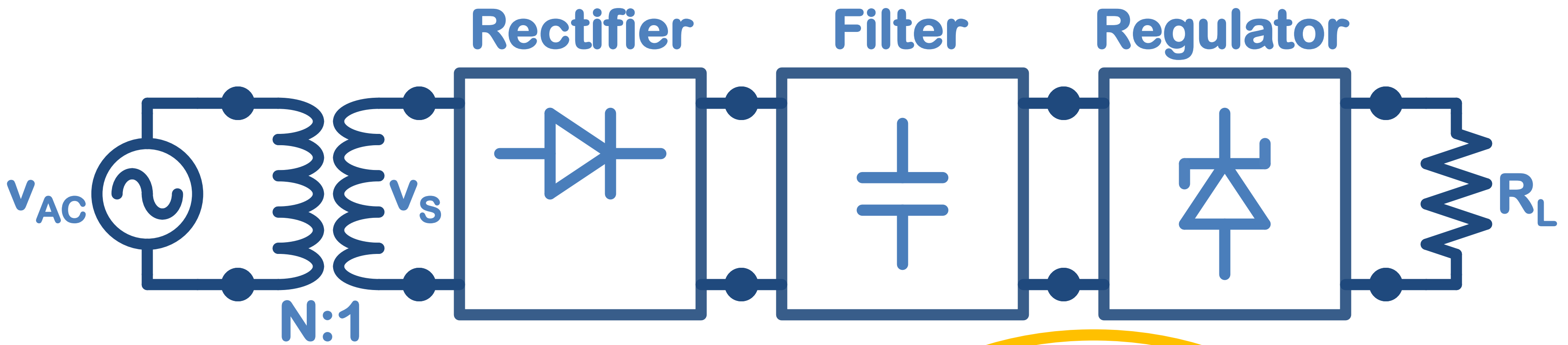
Plot input & output for a sinusoidal input.

$$v_s(t) = V_p \cos(2\pi t/T)$$





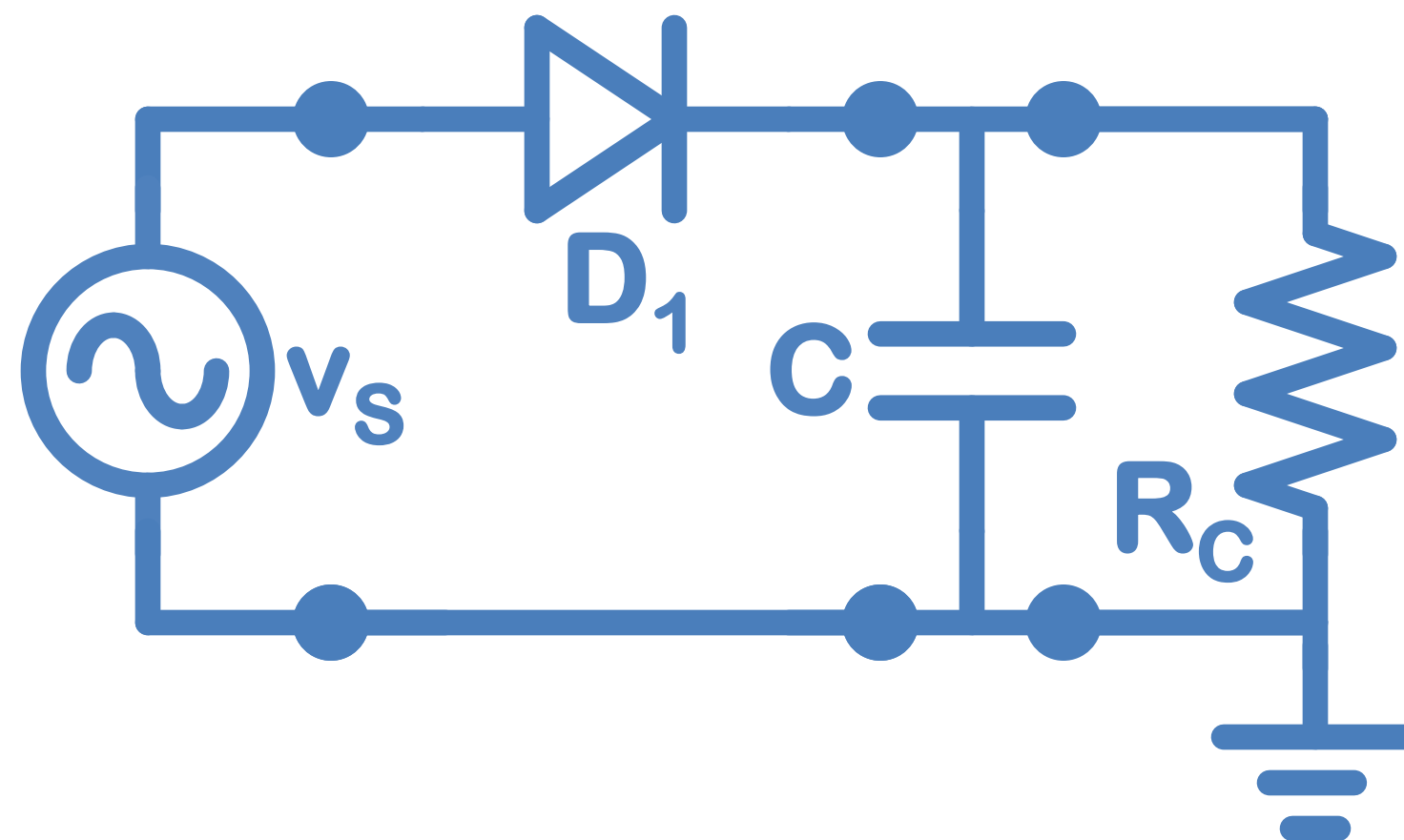
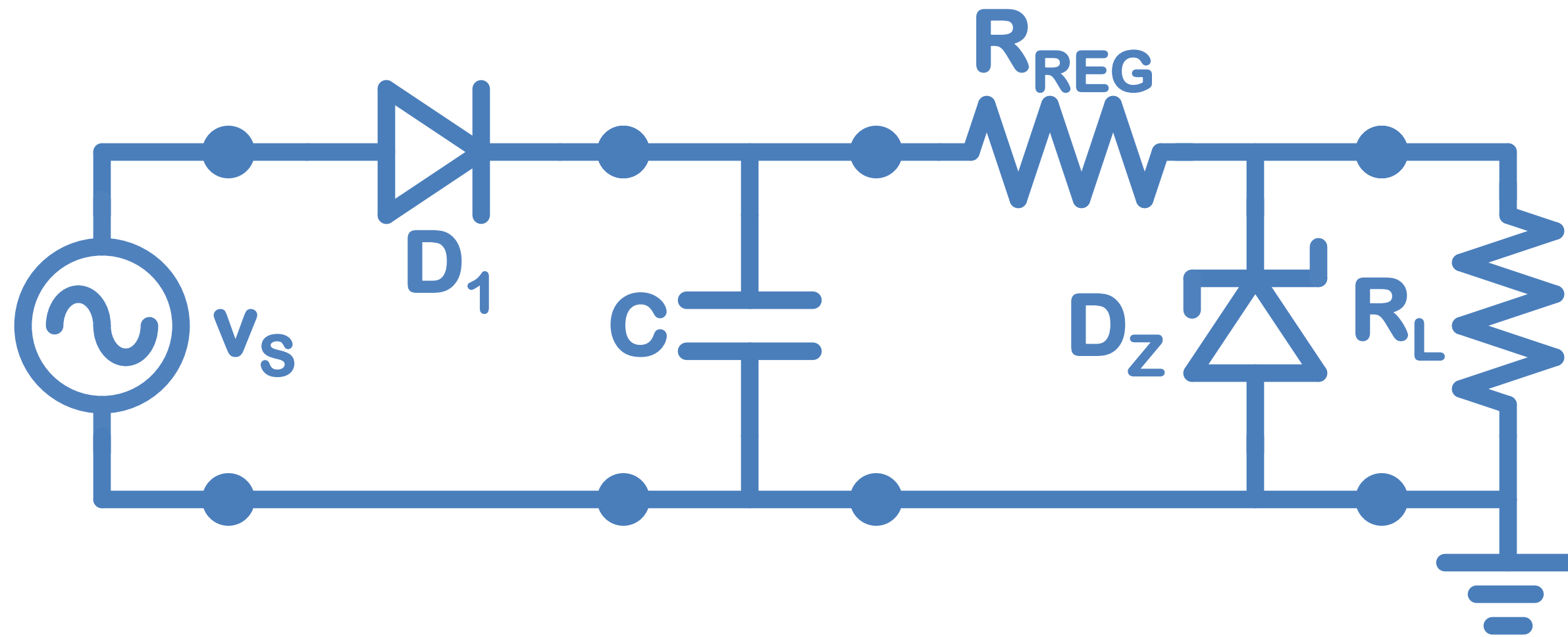




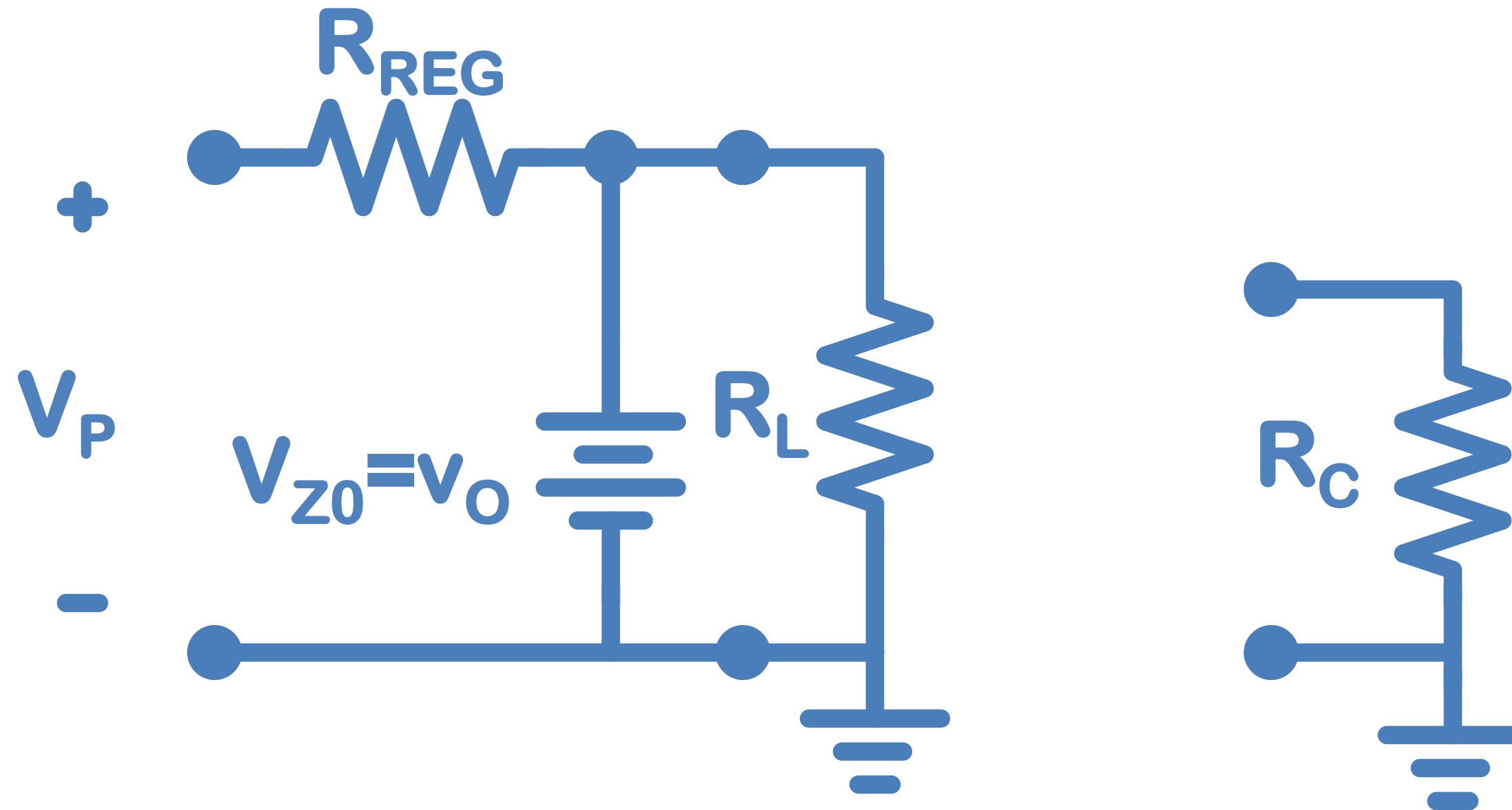
**Model Regulator as a resistor.**

$$R_C = v_C / i_{REG}$$

## How can we model the Regulator and Load as a single Resistor, $R_C$ ?



## How can we model the Regulator and Load as a single Resistor?



$V_P$  is the peak output of the Rectifier.

Use a simple CVD model for the Zener Diode.

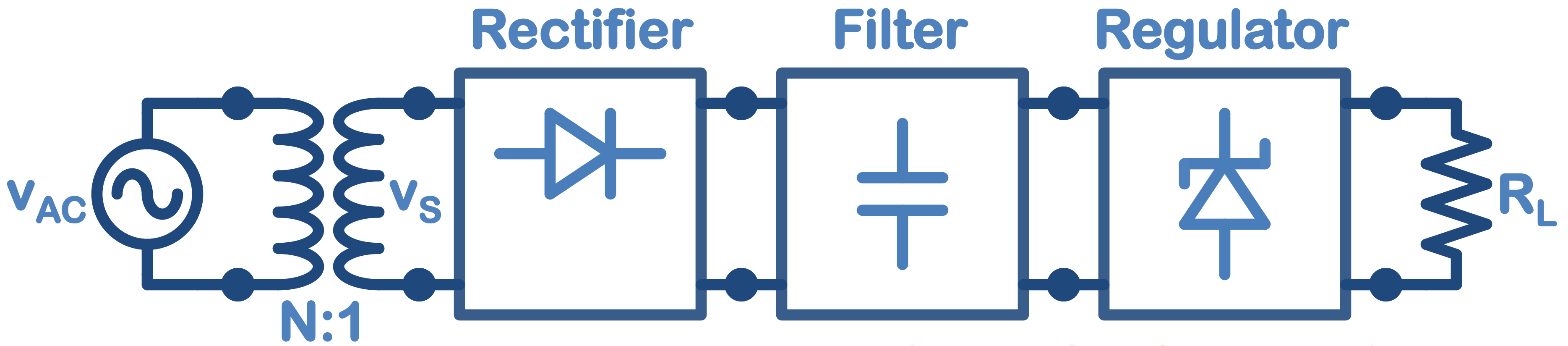
$$I_{REG} = (V_P - V_{Z0})/R_{REG}$$

The effective resistance that is in parallel with the capacitor filter can be approximated as.

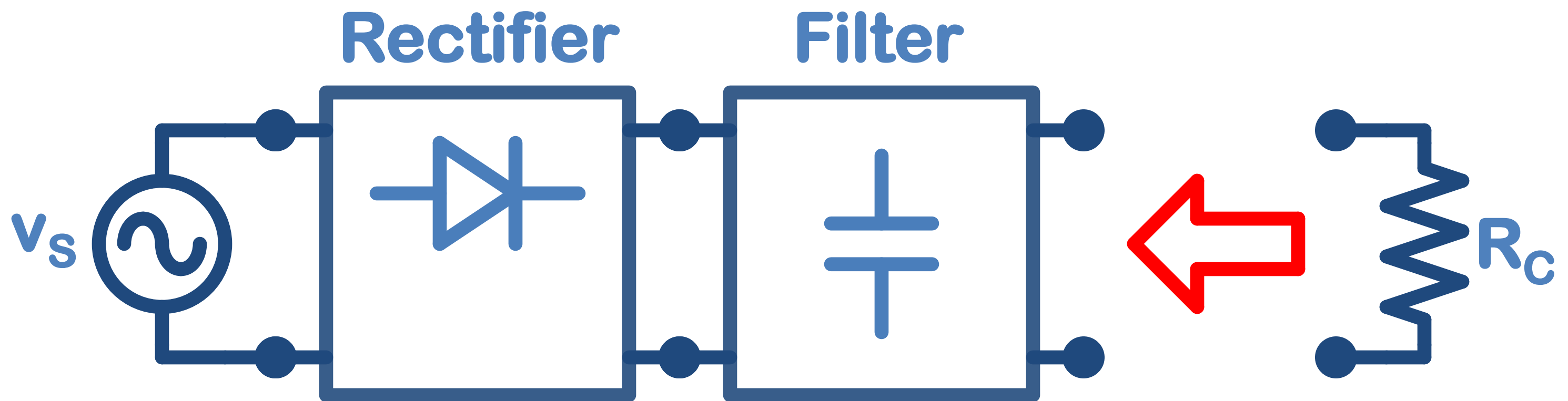
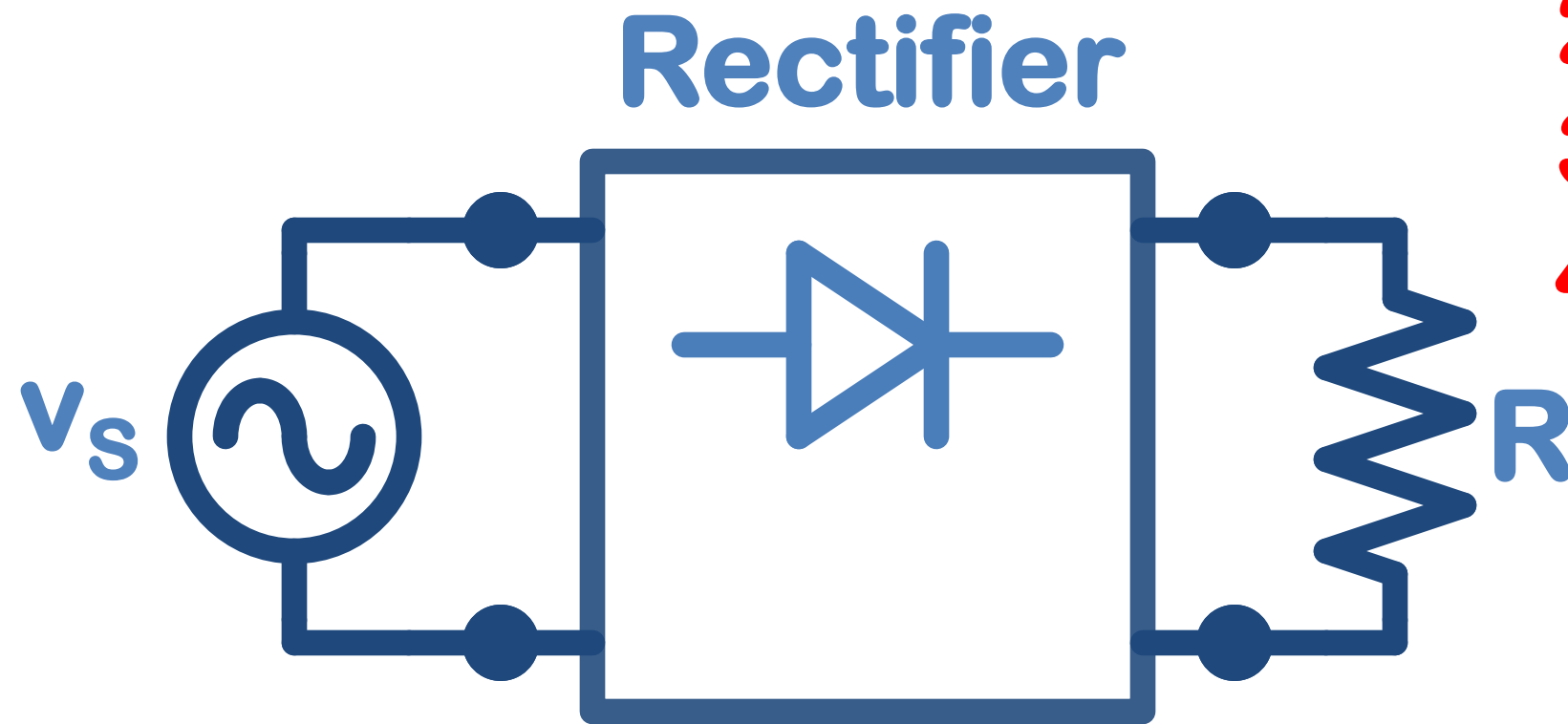
$$R_C = V_P/I_{REG} = V_P R_{REG}/(V_P - V_{Z0})$$

To ensure FB  $I_{REG} > I_L$ , or  $(V_P - V_{Z0})/R_{REG} > V_{Z0}/R_L$

$$R_{REG} < R_L (V_P - V_{Z0})/V_{Z0}$$

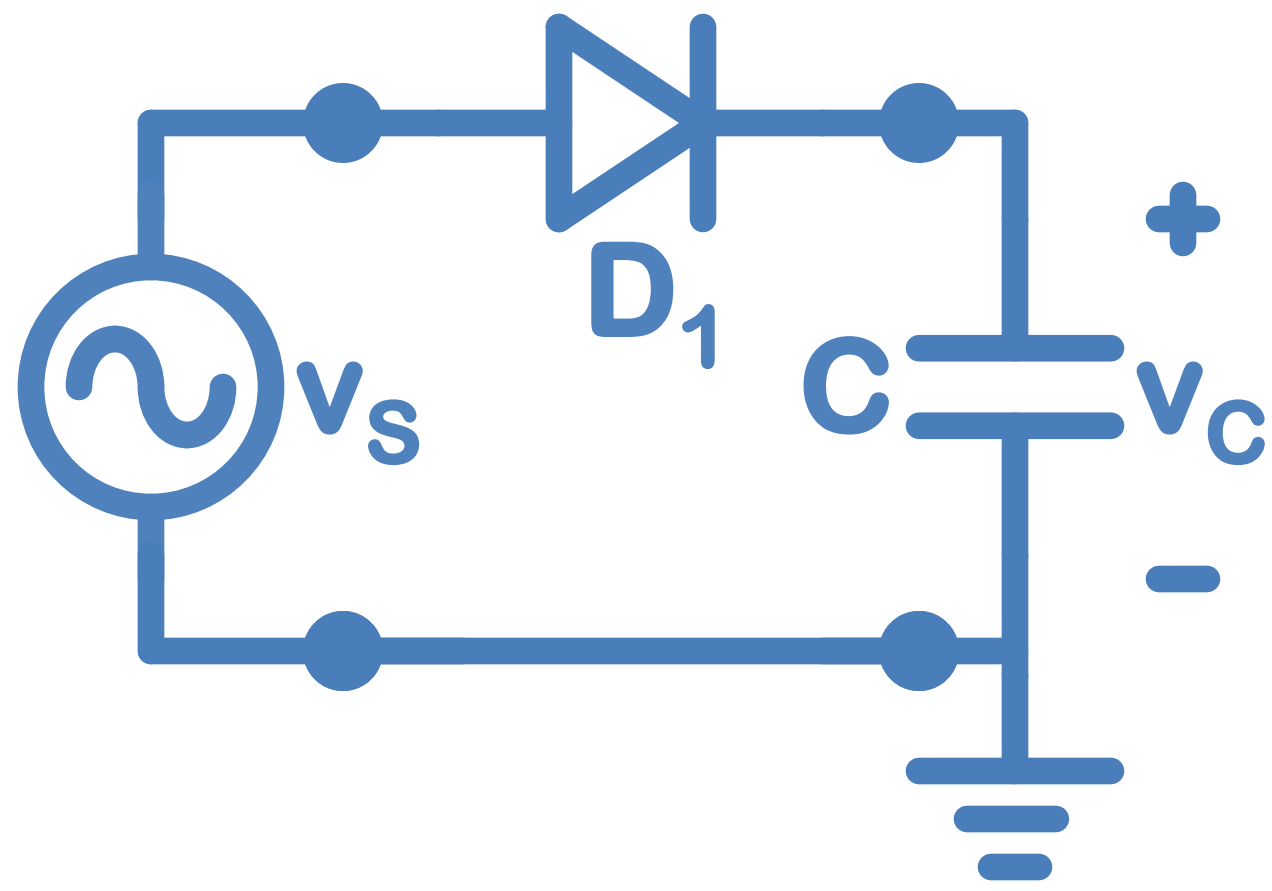


- 1) Rectifier Circuits w/Resistor.
- 2) Filter Capacitor.
- 3) Capacitor & Resistor.
- 4) Regulator.



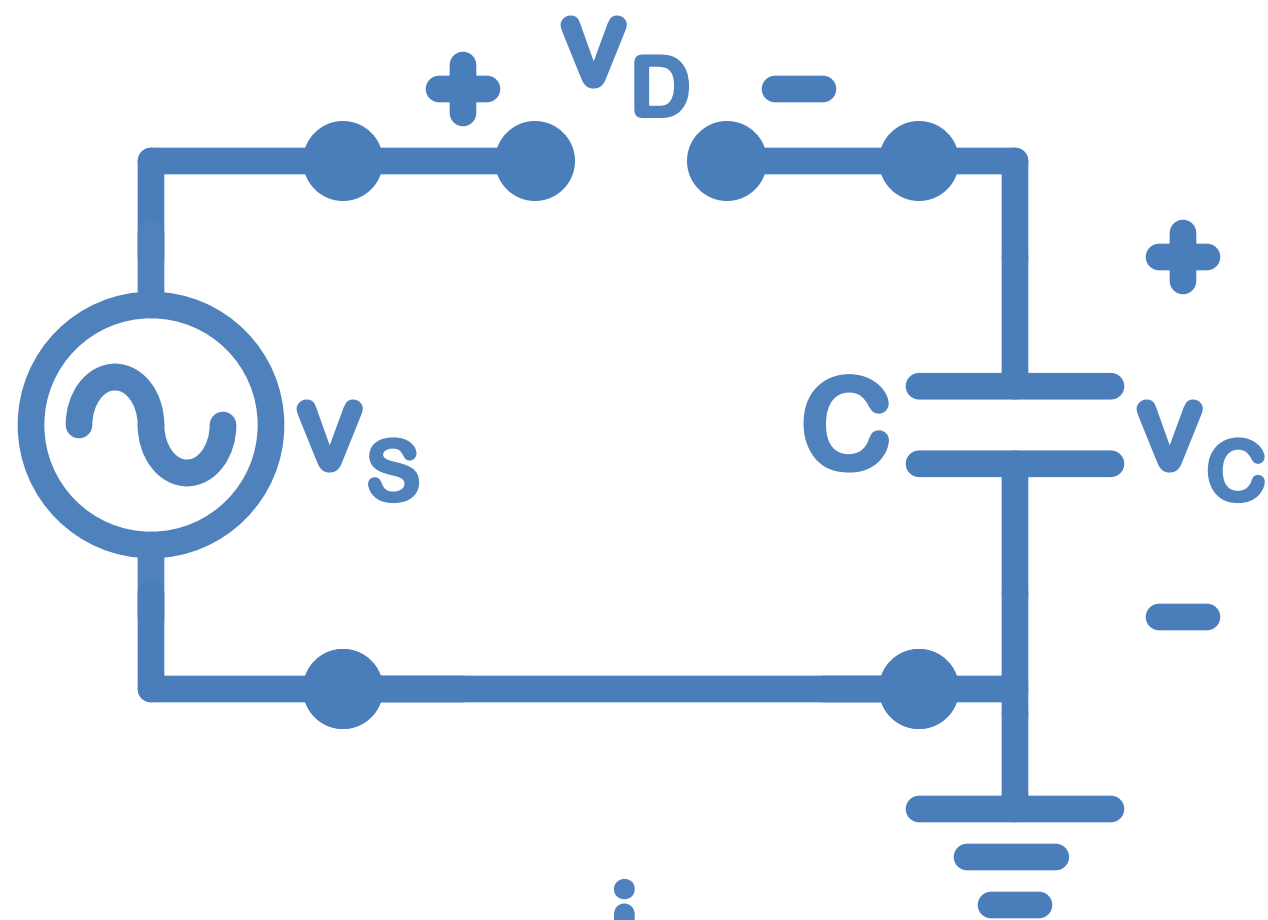
# Positive Half Wave Rectifier w Capacitor

Find the transfer function  $v_C(v_S)$  using ideal Diode



# Positive Half Wave Rectifier w Capacitor

Find the transfer function  $v_C(v_S)$  using ideal Diode



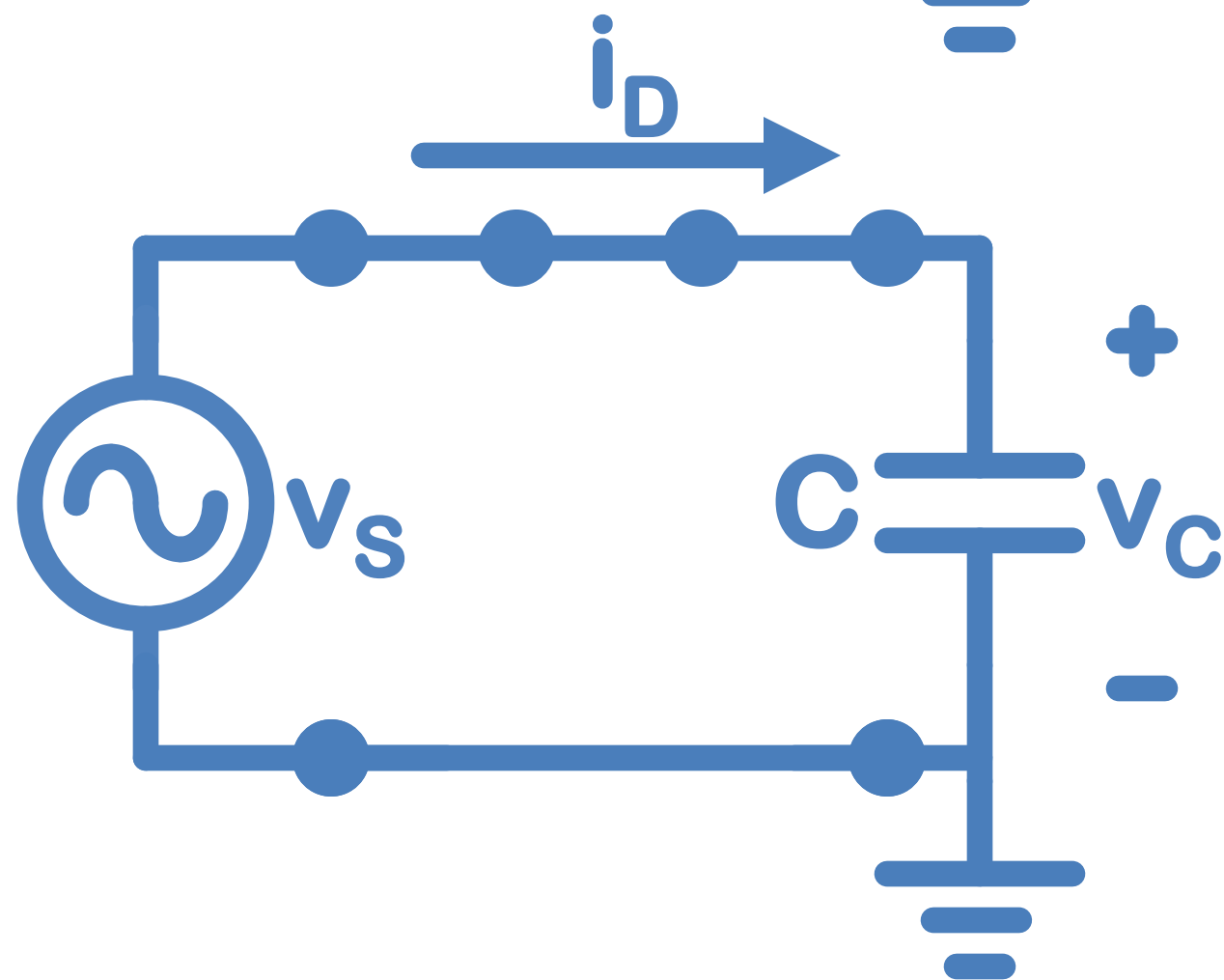
RB:  $i_D = i_C = 0$

$$i_C = C \cdot v_C'(t) = 0; v_C'(t) = 0$$

the voltage stays constant

$$v_D(v_S) = v_S - v_C < 0; v_S < v_C$$

\*as long as  $v_S < v_C$ ,  $v_C$  stays constant\*



FB:  $v_C(v_S) = v_S$

$$i_D(v_S) = i_C = C \cdot v_C'(t) = C \cdot v_S'(t) > 0;$$

$$v_S'(t) > 0; v_S(t) > v_C \text{ and increasing}$$

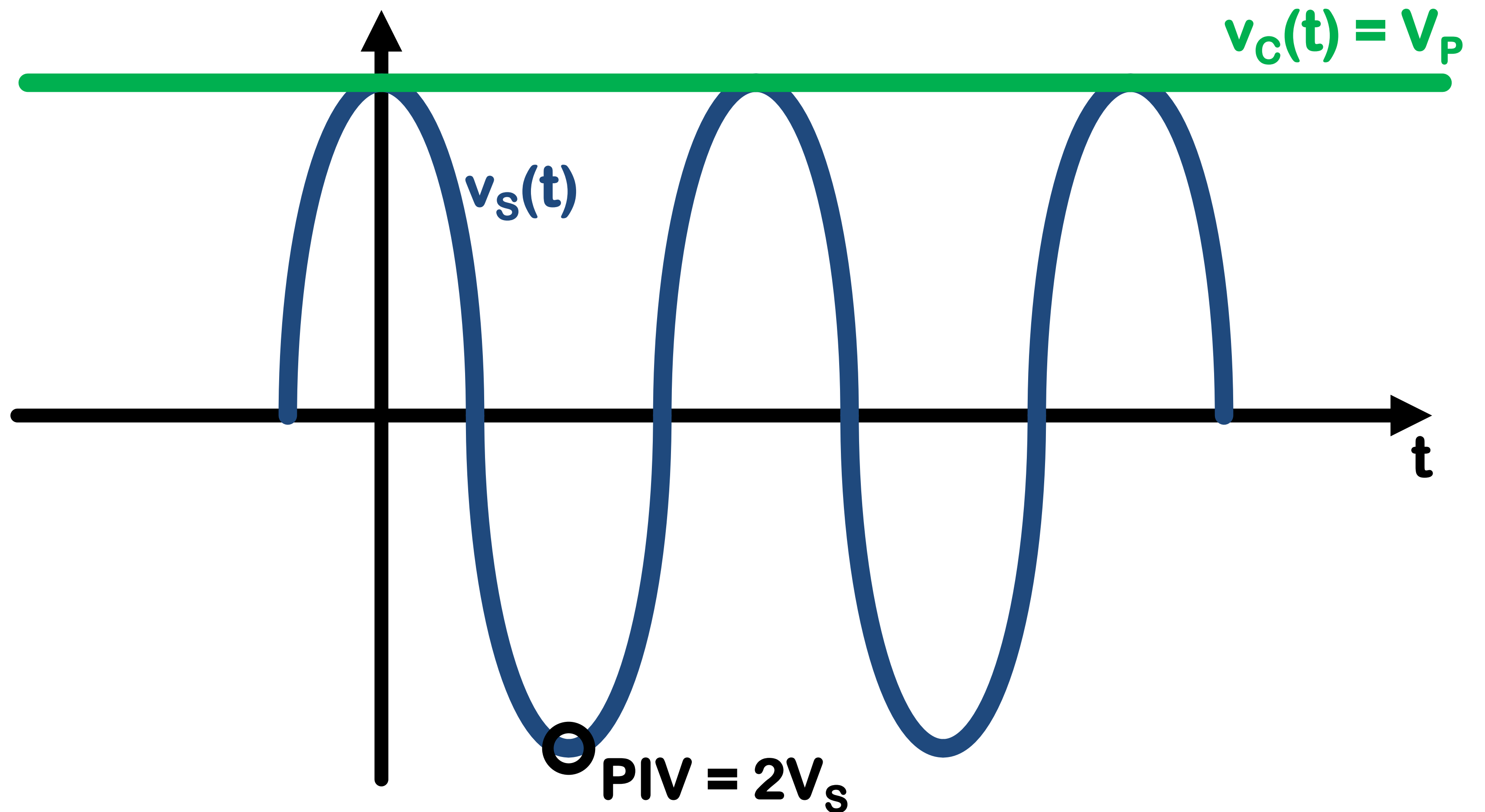
\*if  $v_S > v_C$  and increasing,  $v_C = v_S$ \*

## Summary

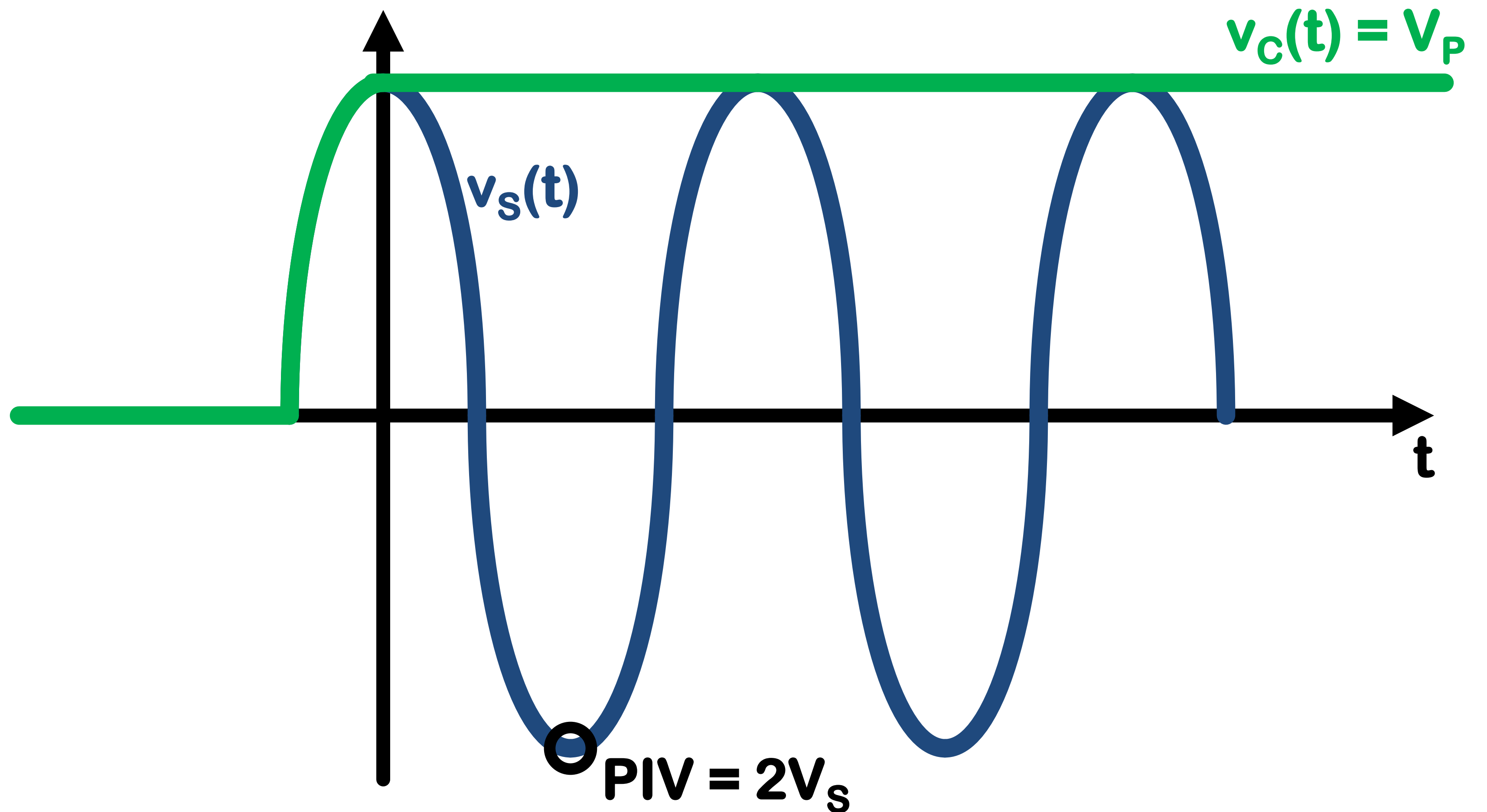
$v_C(t)$  stays constant for  $v_S < v_C$

$v_C(t) = v_S(t)$  for  $v_S > v_C$  and increasing

# Half Wave Rectifier with Capacitor (Ideal)

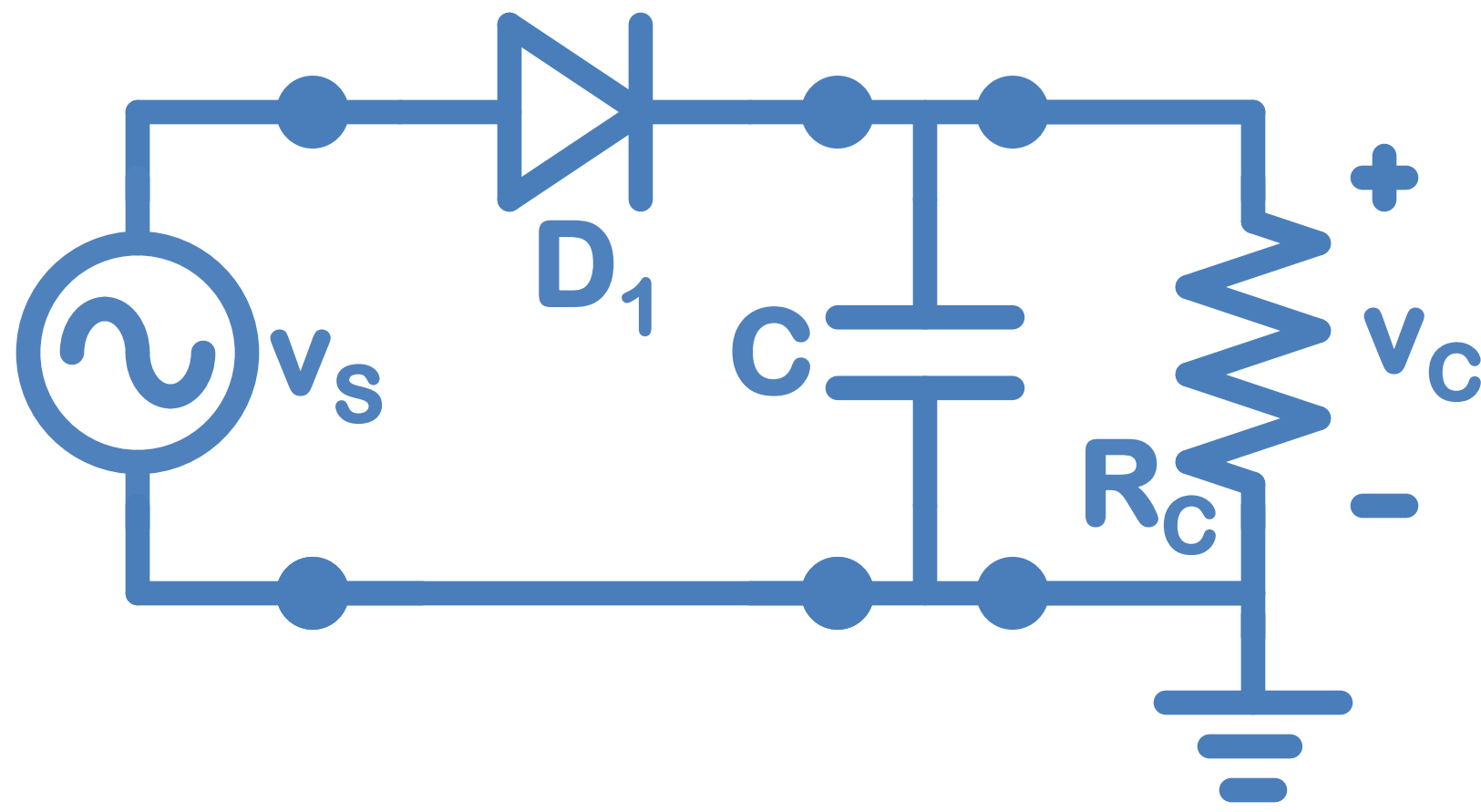


# Half Wave Rectifier with Capacitor (Ideal)



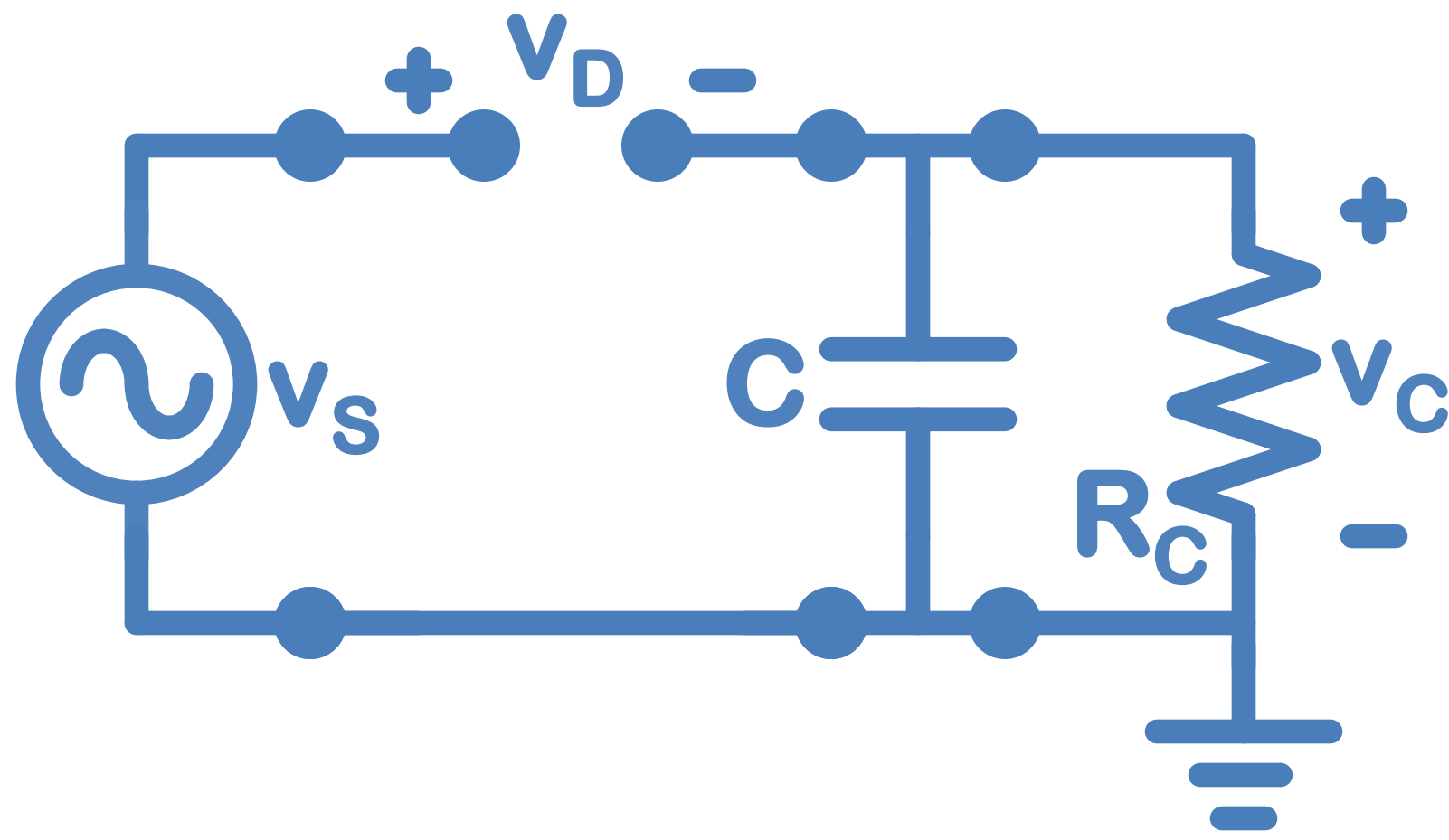
## Positive Half Wave Rectifier w Cap & Res

Find the transfer function  $v_C(v_S)$  using ideal diode



# Positive Half Wave Rectifier w Cap & Res

Find the transfer function  $v_C(v_S)$  using ideal diode



$$\text{RB: } i_D = 0; i_C = -i_R$$
$$C \cdot v_C'(t) + v_C(t)/R_C = 0$$

This is a linear 1<sup>st</sup> order diff. eq.

$$v_C(t) = A \exp(-t/R_C C) + B$$

Boundary Conditions

$$v_C(\infty) = 0; B = 0; A = v_C(0)$$

Assume  $t=0$  is when diode FB to RB.

$$v_D = v_S - v_C < 0; v_S < v_C$$

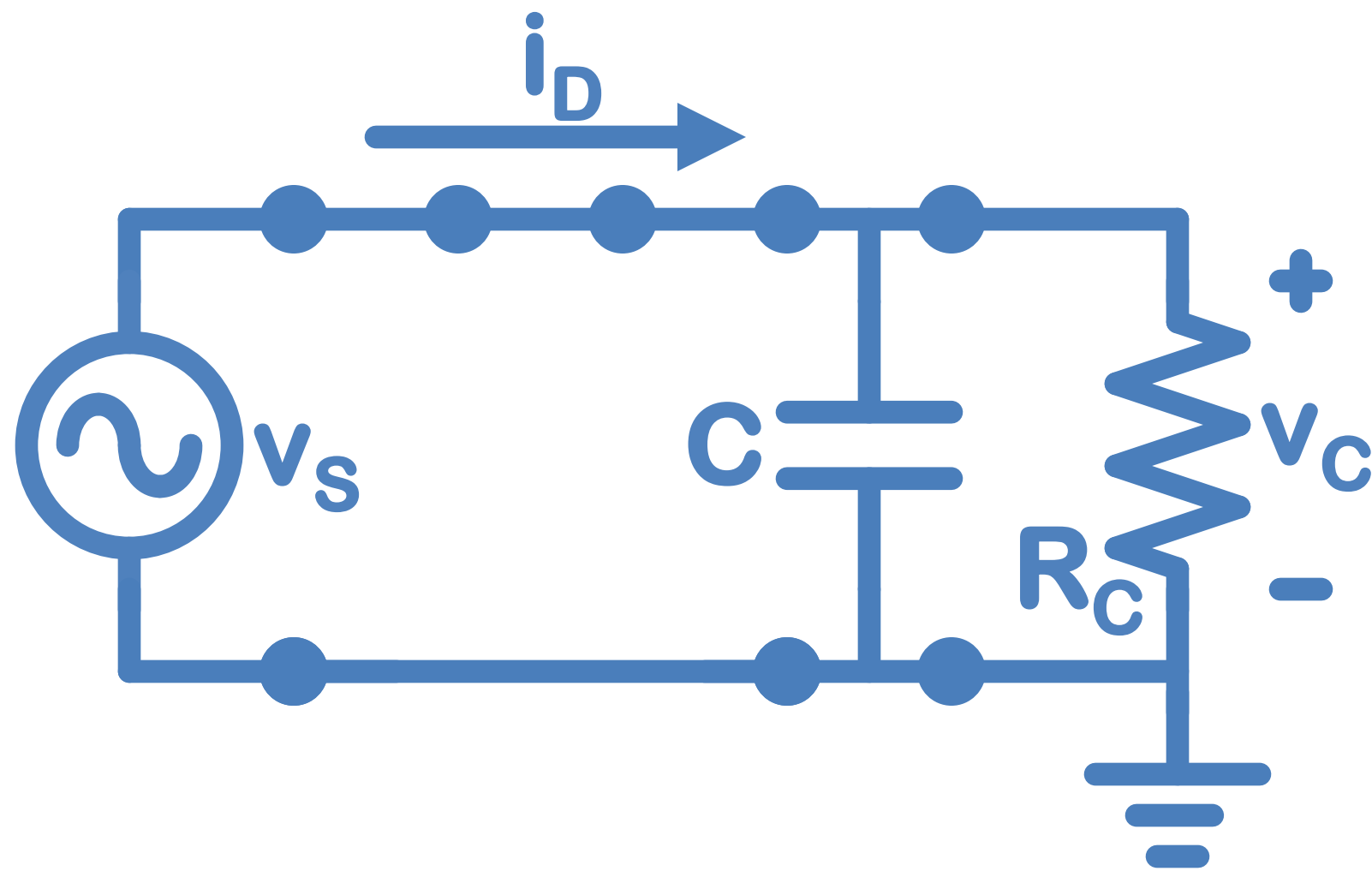
## Summary

For diode going FB to RB at  $t=0$ .

$$v_C(t) = v_C(t=0) \cdot \exp(-t/R_C C)$$

# Positive Half Wave Rectifier w Cap & Res

Find the transfer function  $v_C(v_S)$  using ideal diode



FB:  $v_C(t) = v_S(t)$

$$i_D(t) = i_R + i_C = v_S(t)/R_C + C \cdot v_S'(t) > 0$$

$$v_S'(t) > -v_S(t)/R_C C$$

\*assuming  $R_C C \gg T = 1/f$ , this can be approximated as:

$$v_S'(t) > 0$$

## Summaries

FB:  $v_C(t) = v_S$

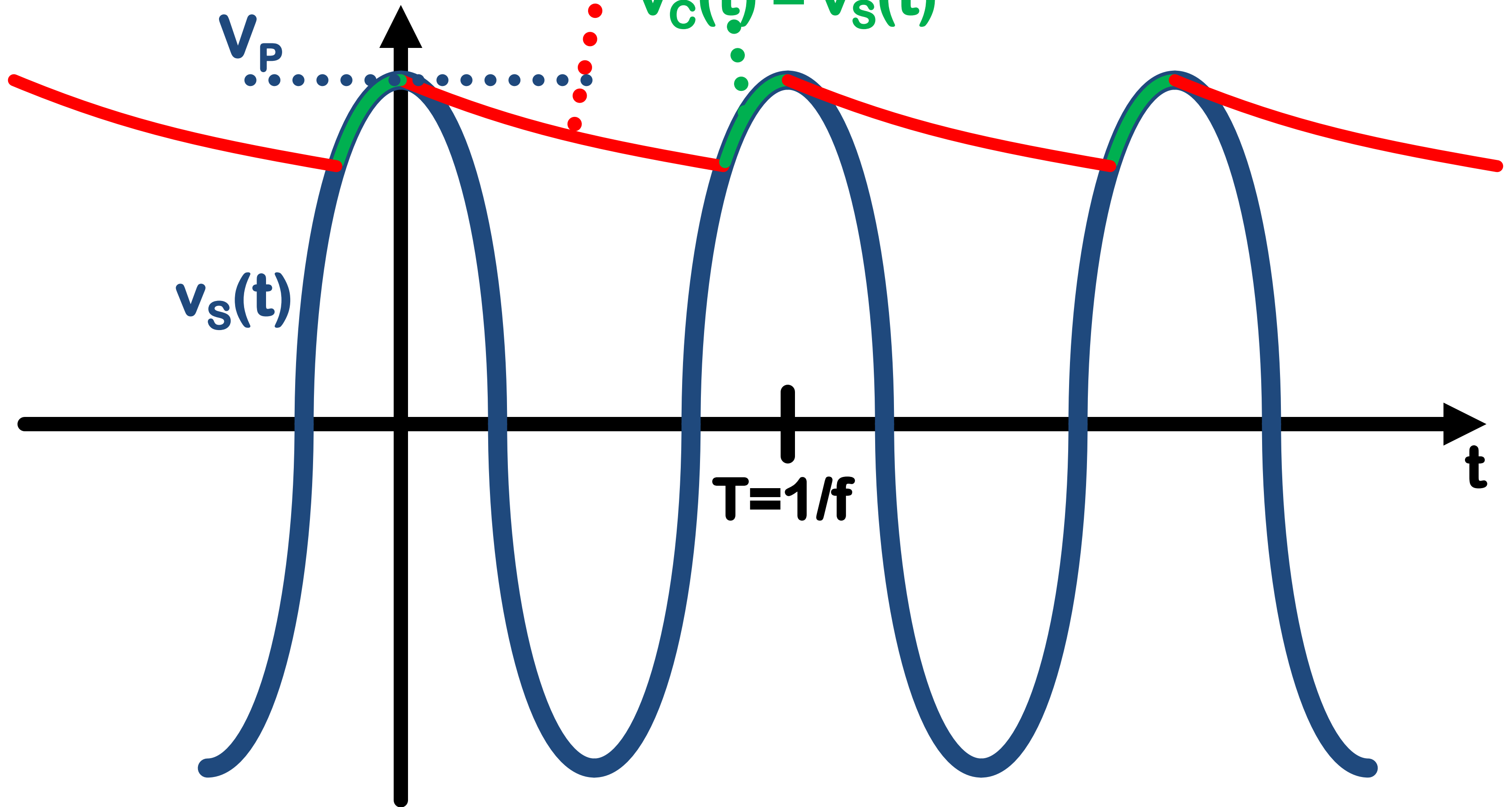
RB:  $v_C(t) = v_C(t=0) \cdot \exp(-t/R_C C)$

Switches between FB and RB when  $v_S'(t)$  goes from pos to neg (peak)  
assume this happens at  $t=0$

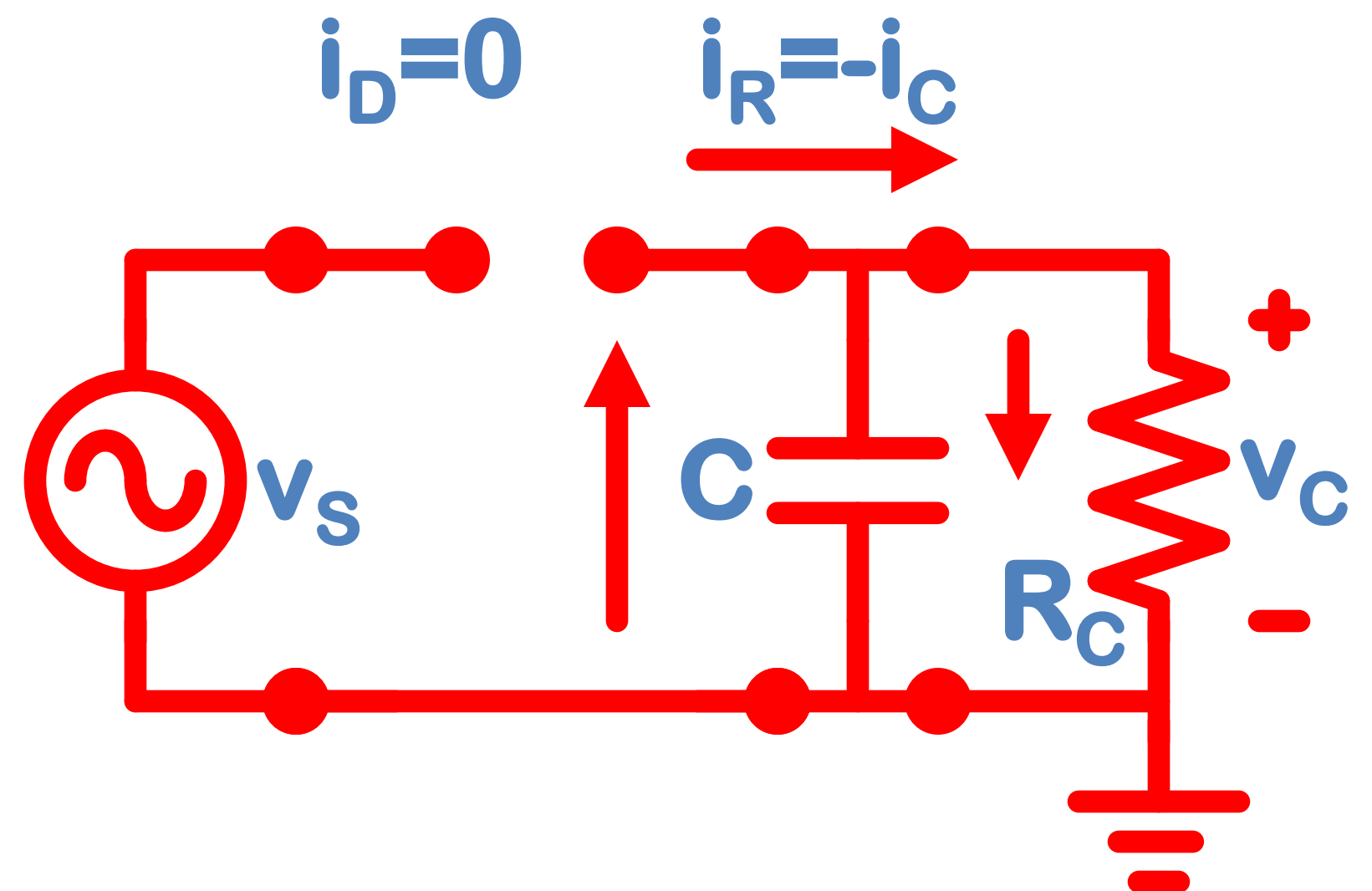
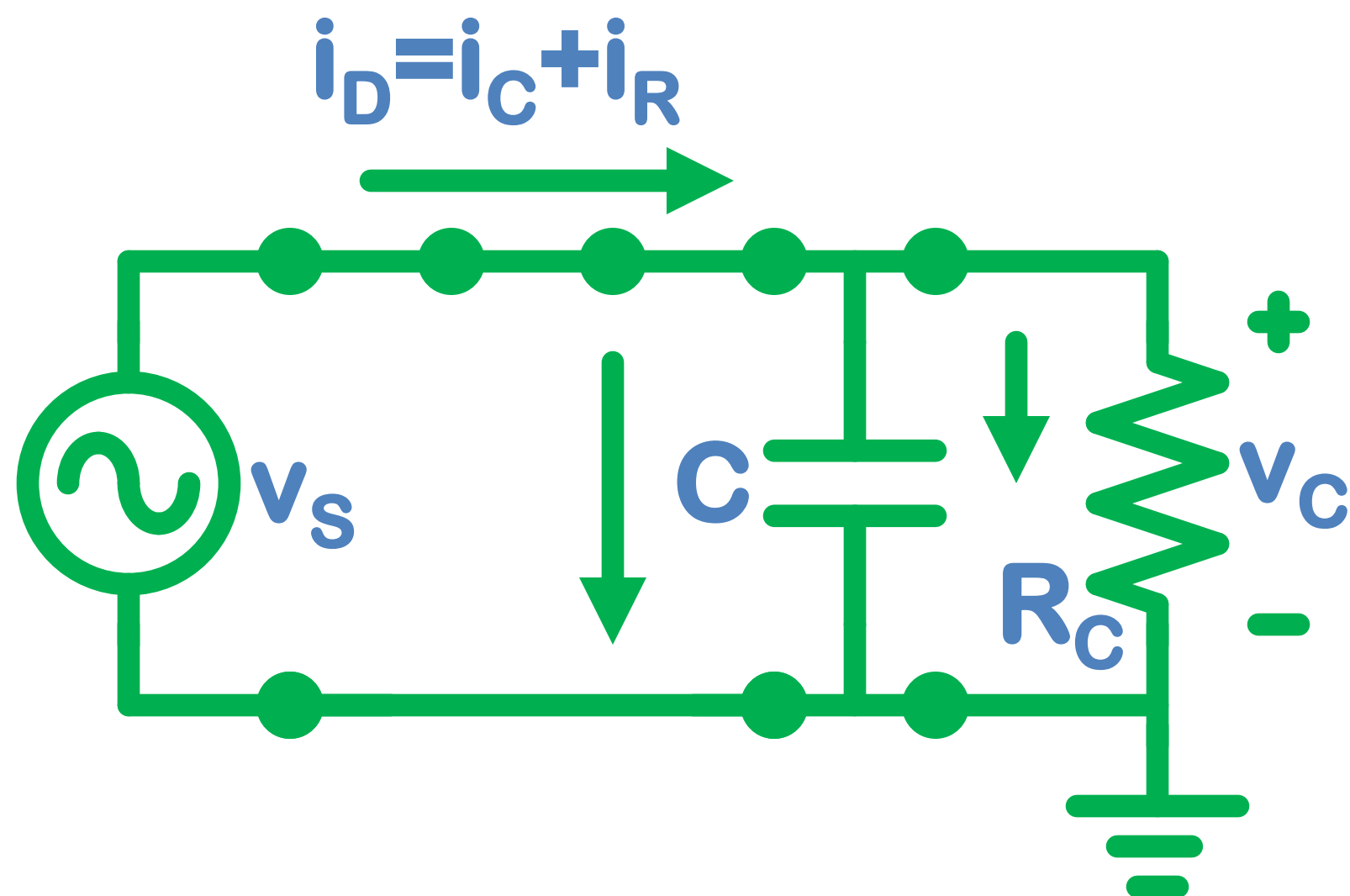
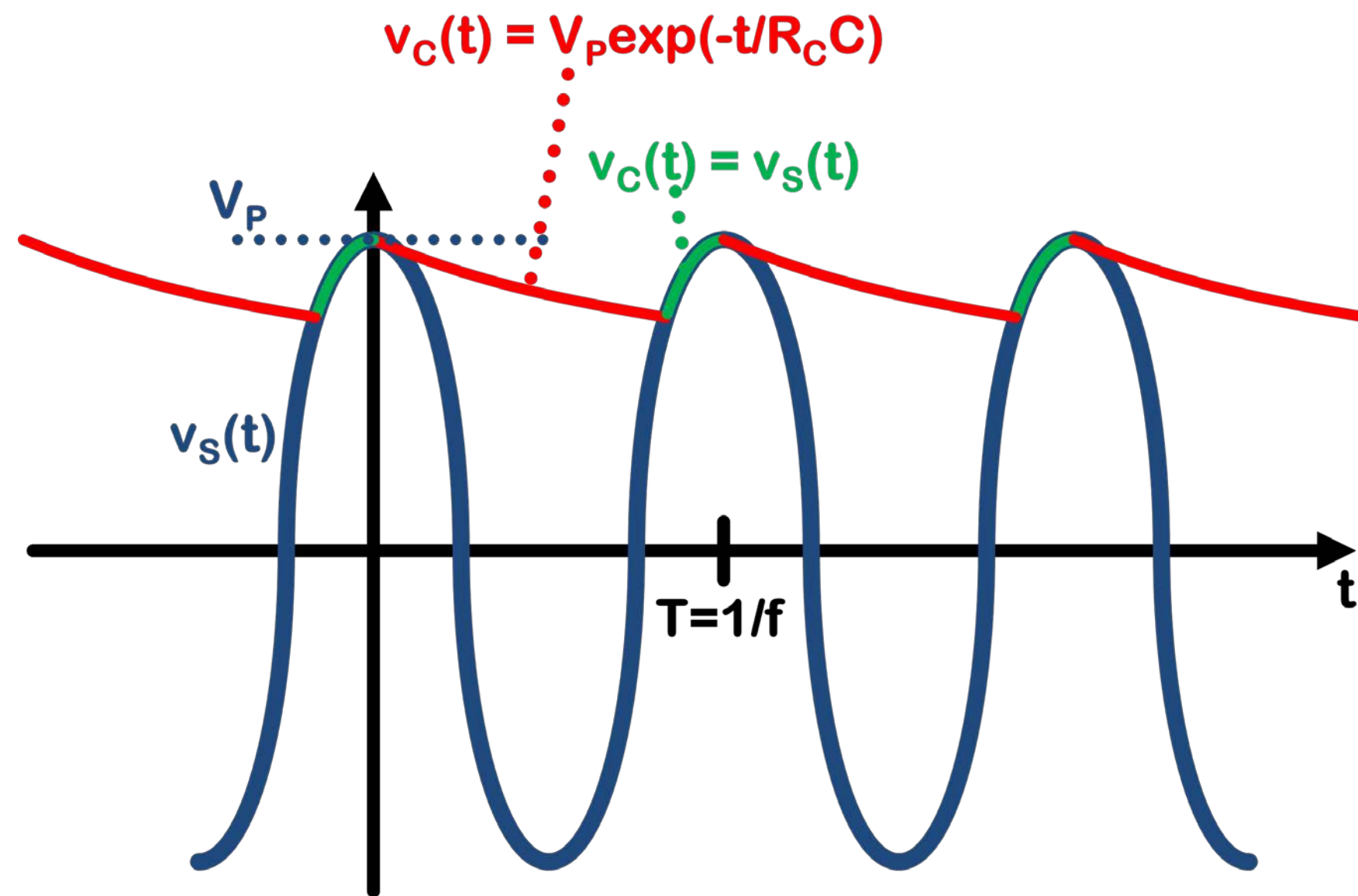
# Half Wave Rectifier with Capacitor And Resistor (Ideal)

$$v_C(t) = V_P \exp(-t/R_C C)$$

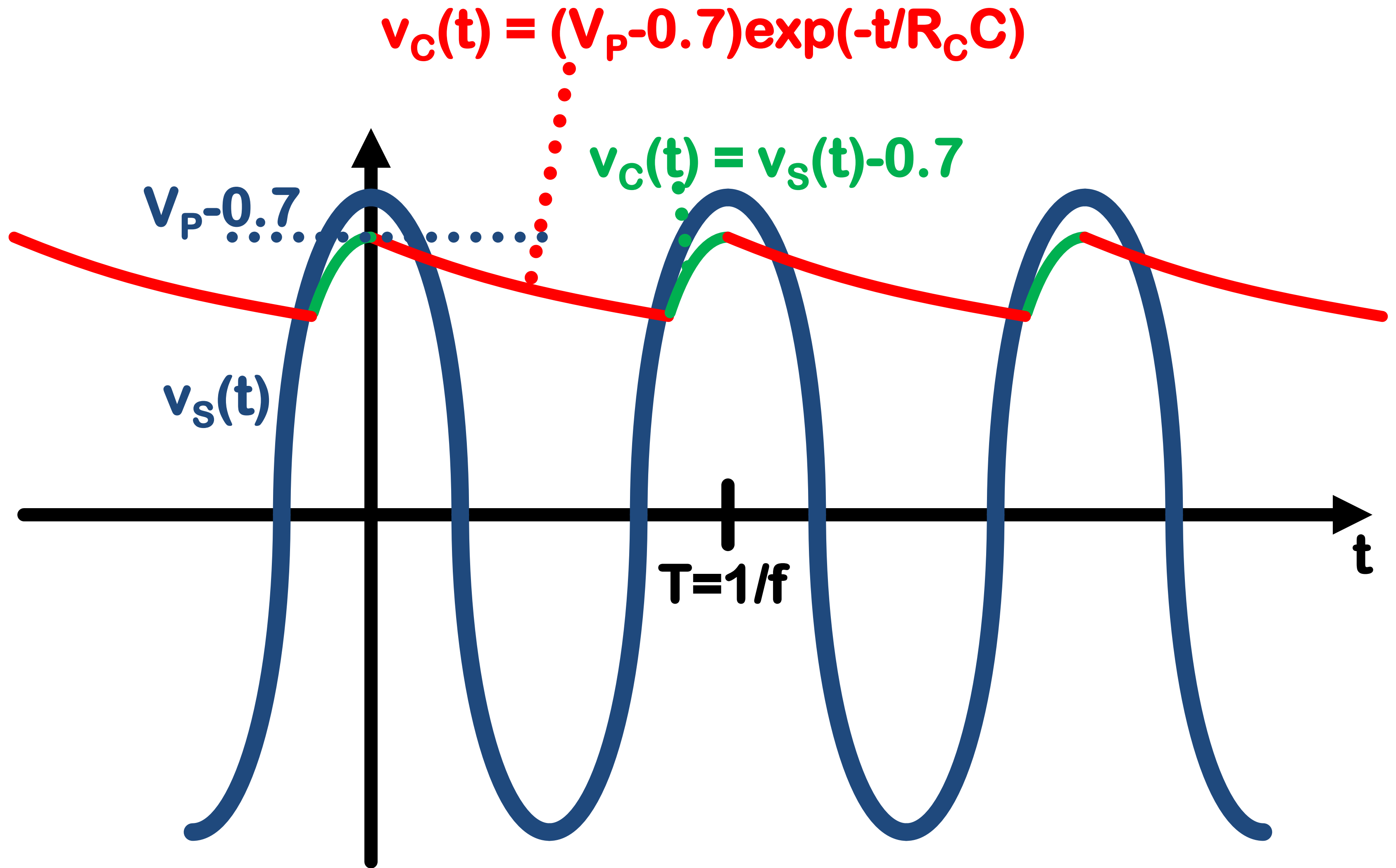
$$v_C(t) = v_S(t)$$



# Half Wave Rectifier with Capacitor And Resistor (Ideal)

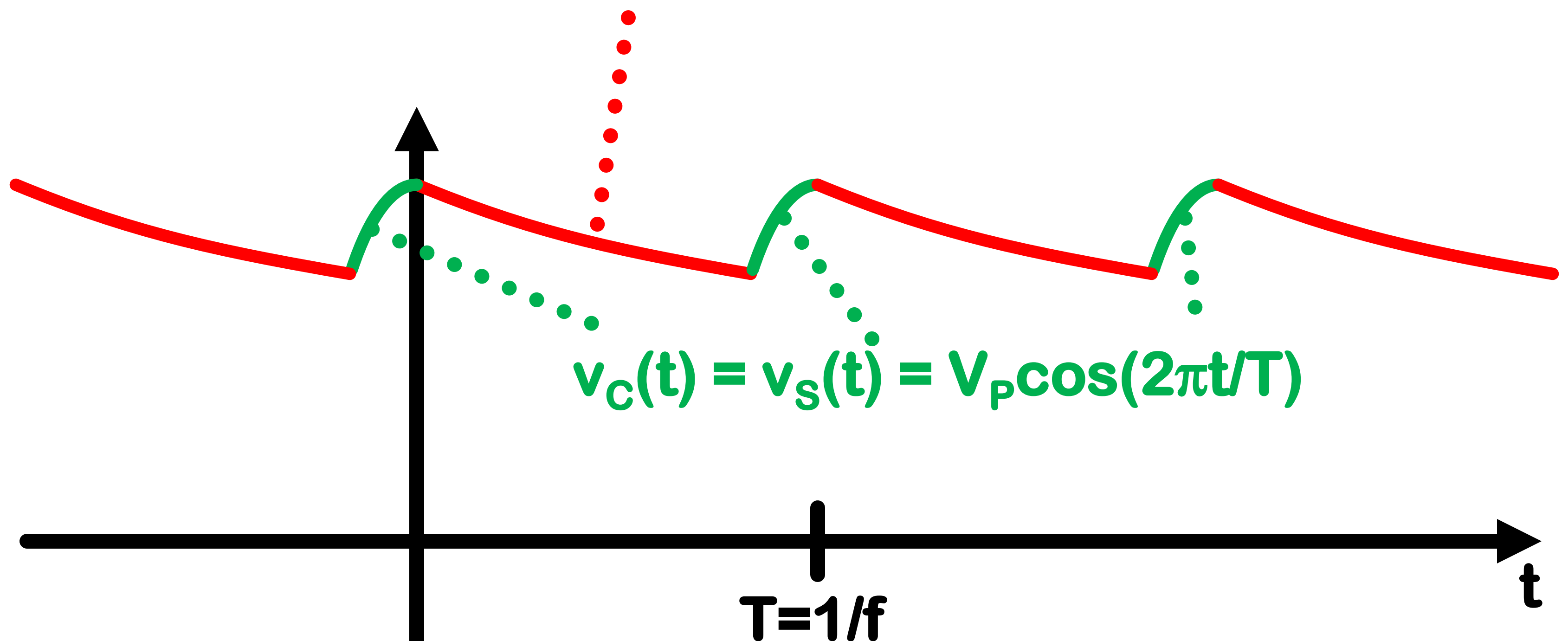


# Half Wave Rectifier with Capacitor And Resistor (CVD)



# Half Wave Rectifier with Capacitor And Resistor (Ideal)

$$v_C(t) = V_p \exp(-t/R_C C)$$



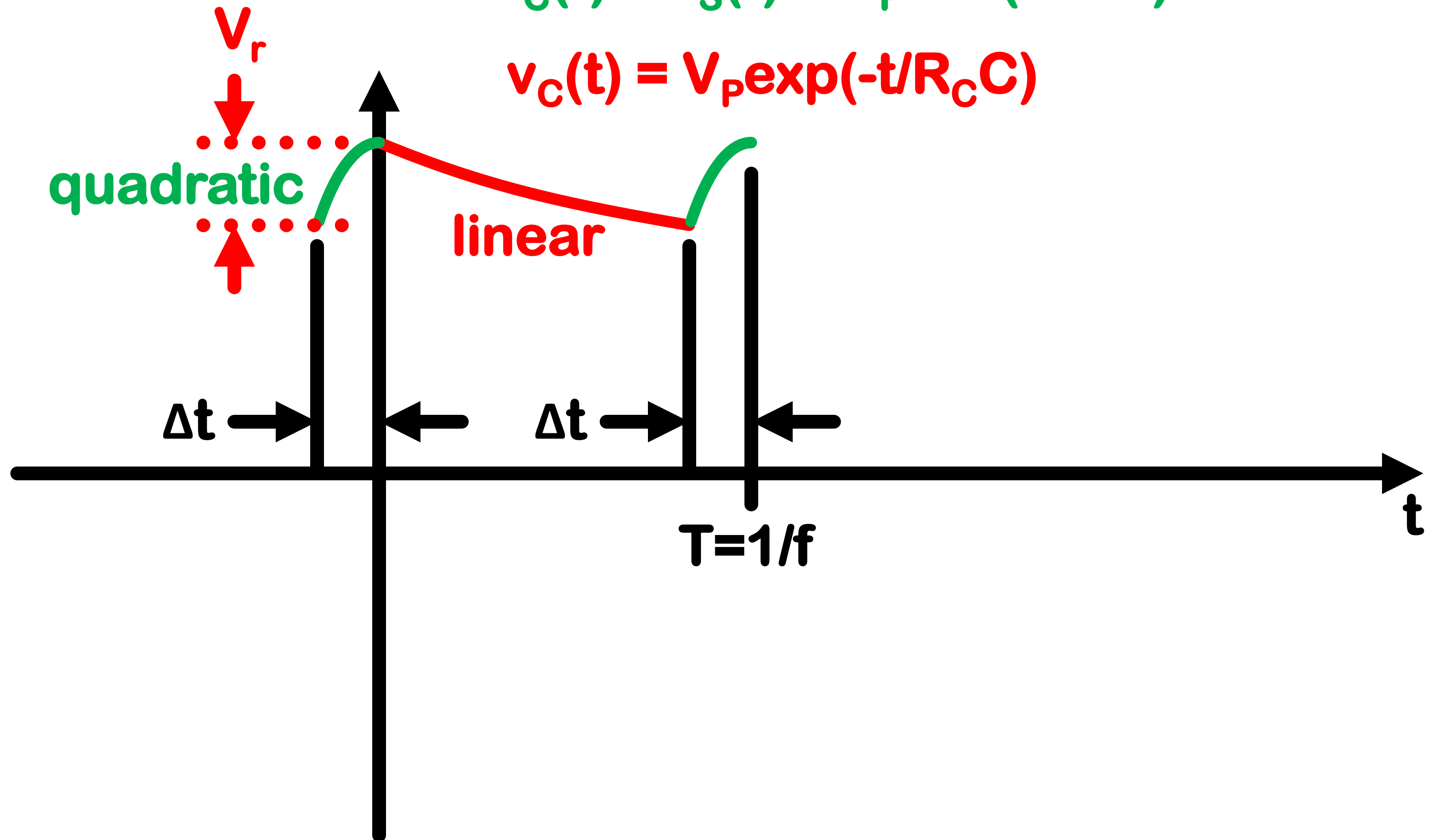
- What is the Ripple Level ( $v_{C:MAX} - v_{C:MIN}$ )?
- What percentage of time is it FB?
- What is the average and max diode current?

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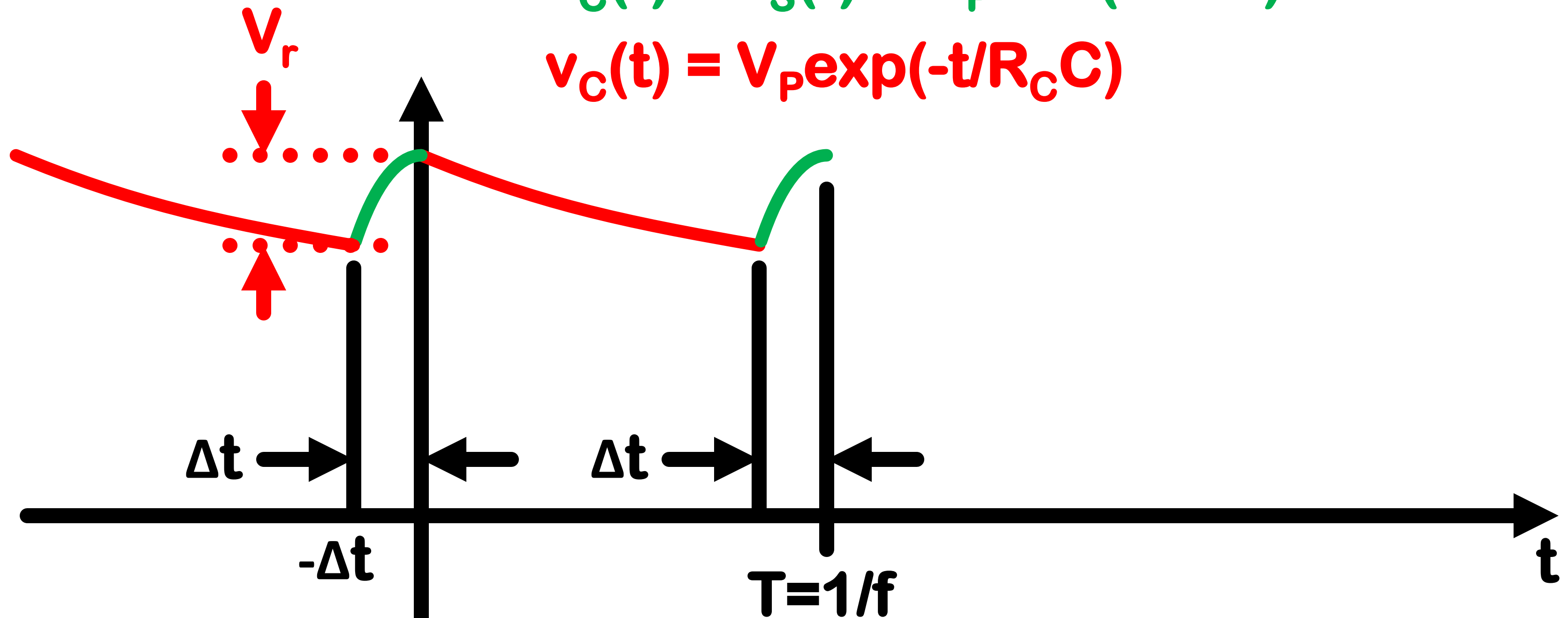


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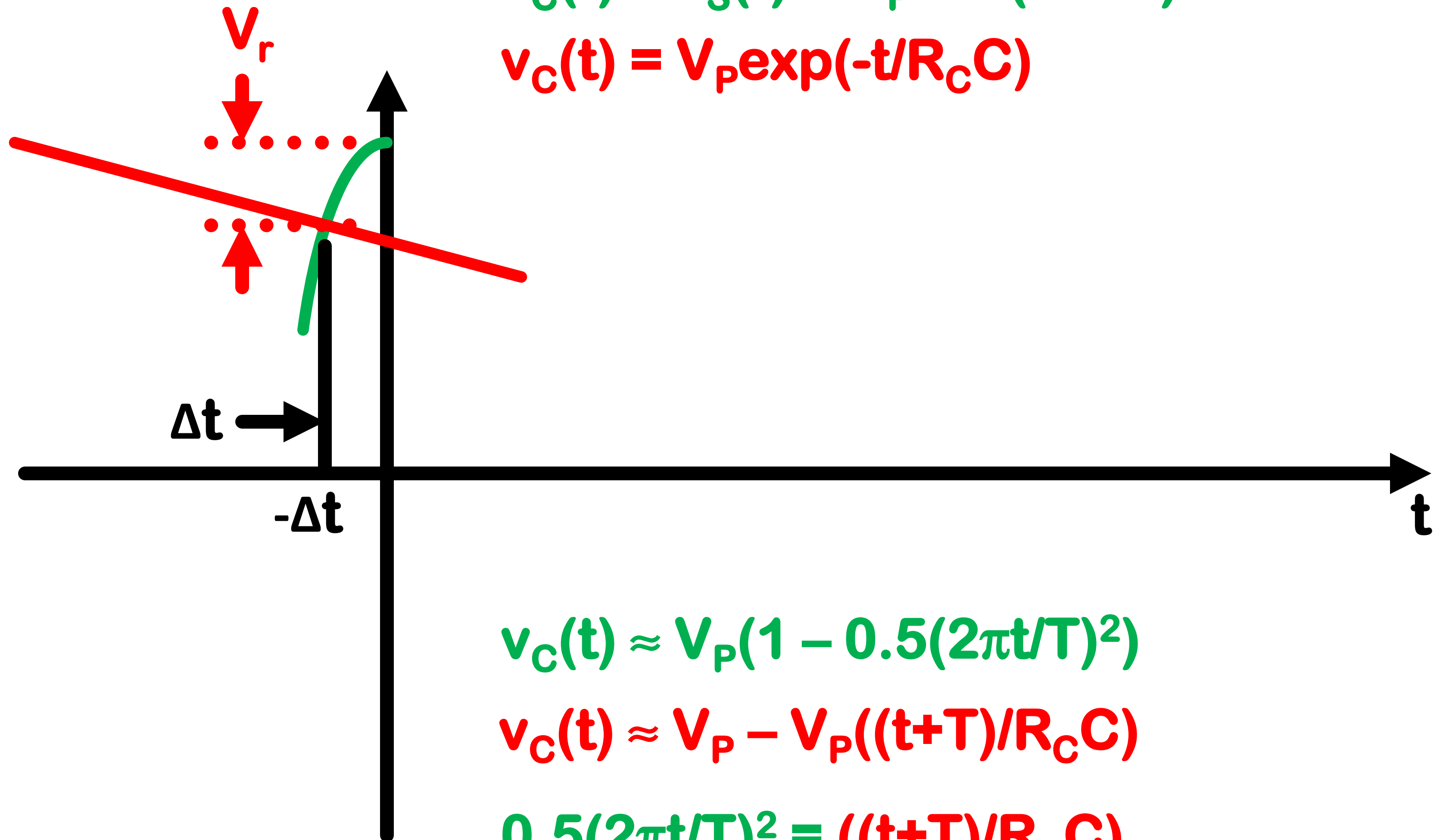
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$$0.5(2\pi t/T)^2 = ((t+T)/R_C C)$$

$$2\pi^2(t/T)^2 - (T/R_c C)(t/T) - (T/R_c C) = 0$$

$$\frac{t}{T} = \frac{\frac{T}{R_c C} - \sqrt{\left(\frac{T}{R_c C}\right)^2 + 4 \cdot 2\pi^2 \cdot \left(\frac{T}{R_c C}\right)}}{4\pi^2}$$

$$\frac{t}{T} = \frac{-\sqrt{2\left(\frac{T}{R_c C}\right)}}{2\pi}$$

$$\Delta t = \frac{T}{2\pi} \sqrt{\left(\frac{2T}{R_c C}\right)}$$

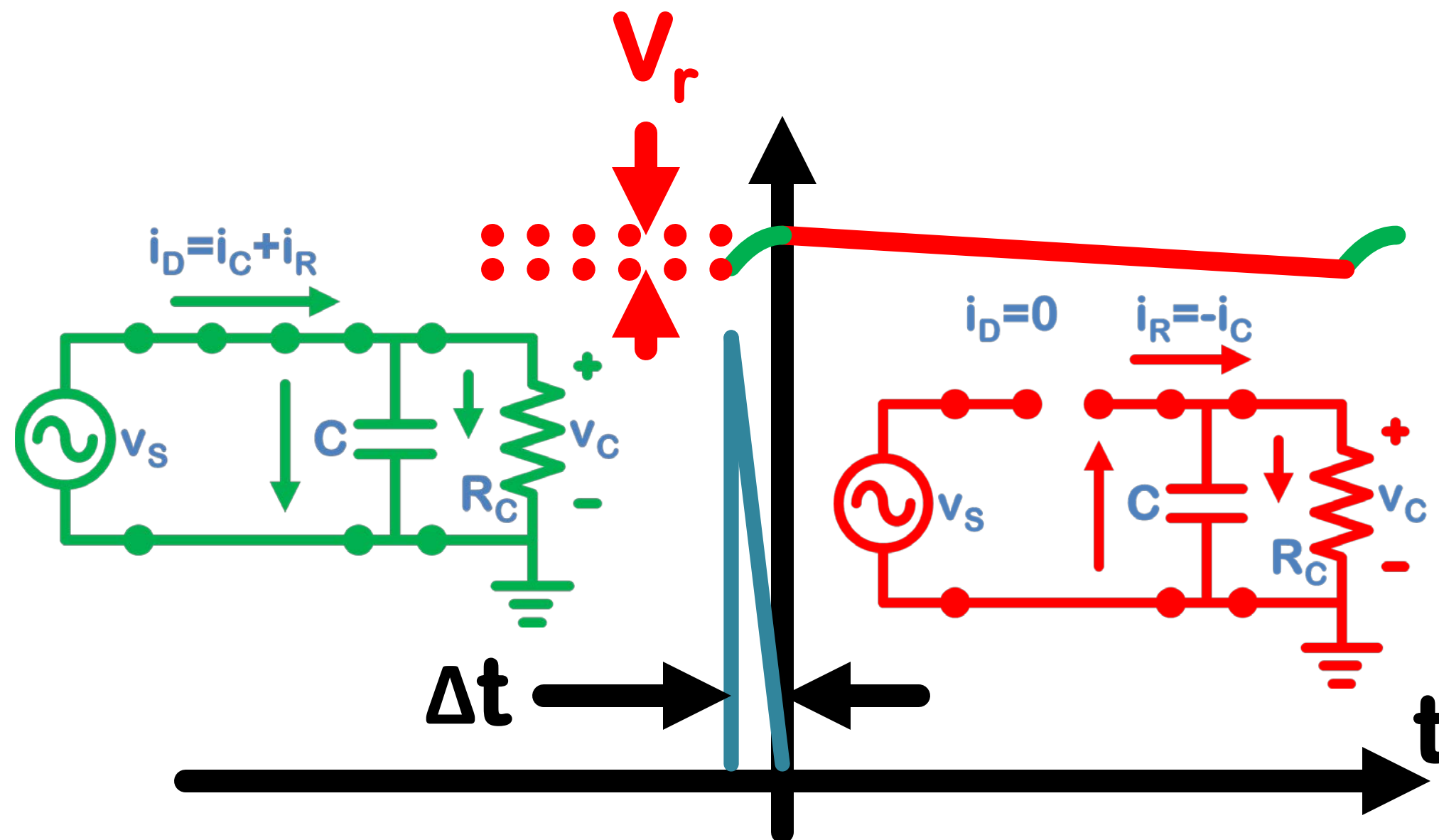
$$v_C(\Delta t) \approx V_P(1 - 2\pi^2 \frac{2(T/R_c C)}{4\pi^2})$$

$$v_C(\Delta t) \approx V_P(1 - T/(R_c C))$$

$$V_r = V_P - V_P + V_P T/(R_c C) = V_P/(fR_c C)$$

# Half Wave Rectifier with Capacitor And Resistor (Ideal)

What is the average and max diode current?



During FB  $i_D$  is initially large and then tapers to zero. We can model it as a triangle. If

$$i_{D_{av}} = I_R \cdot (1 + T/\Delta t)$$

Then

$$i_{D_{max}} = 2 \cdot I_R \cdot (1 + T/\Delta t)$$

The resistor is always conducting a constant:  
 $I_R = V_P/R_C$

During RB the capacitor is supplying this current.

During FB the capacitor needs to recover the lost charge from the source,  $v_s(t)$

$$Q_R = I_R T = V_P T/R_C$$

During FB, Cap recovers:

$$Q_C = i_{C_{av}} \Delta t = Q_R$$

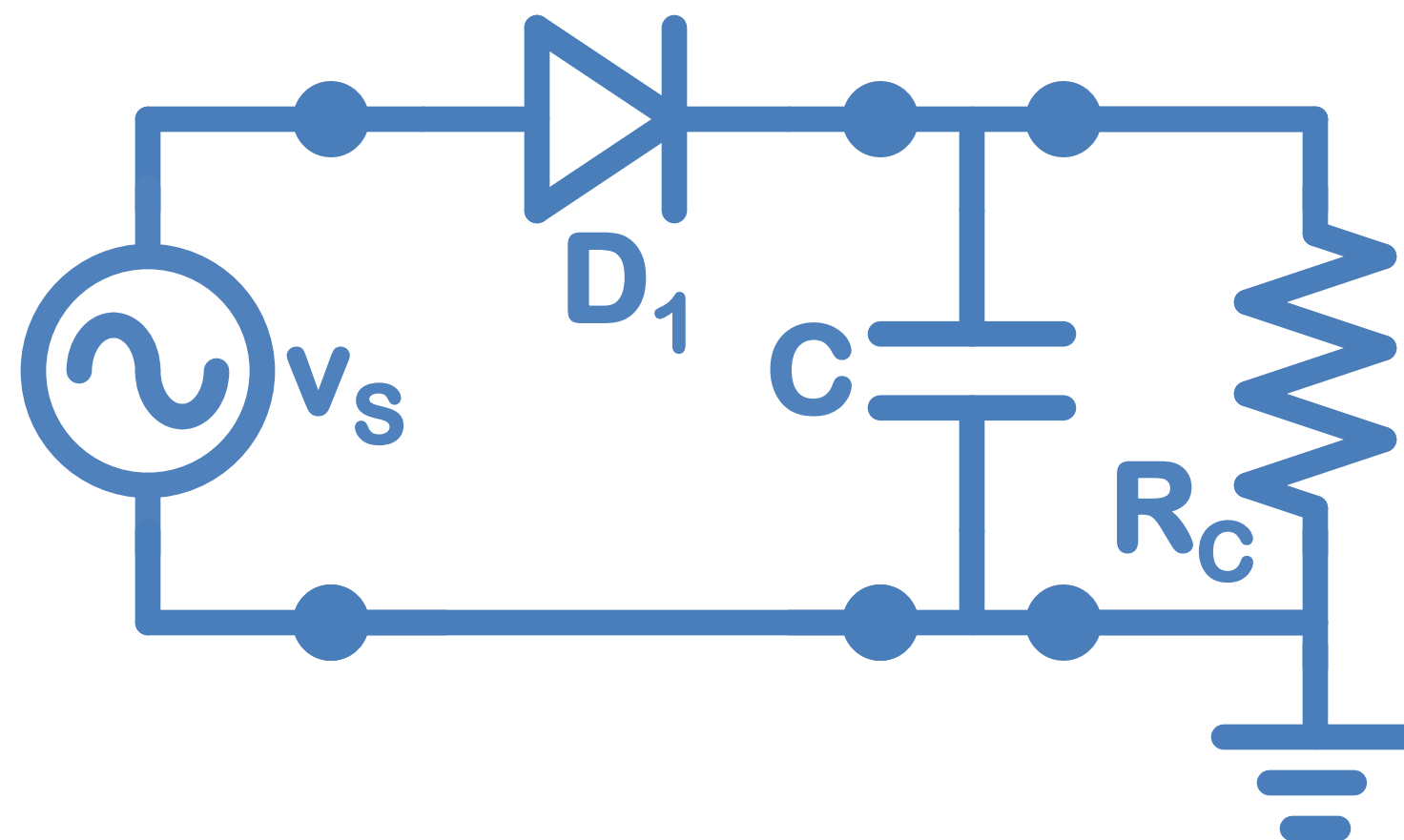
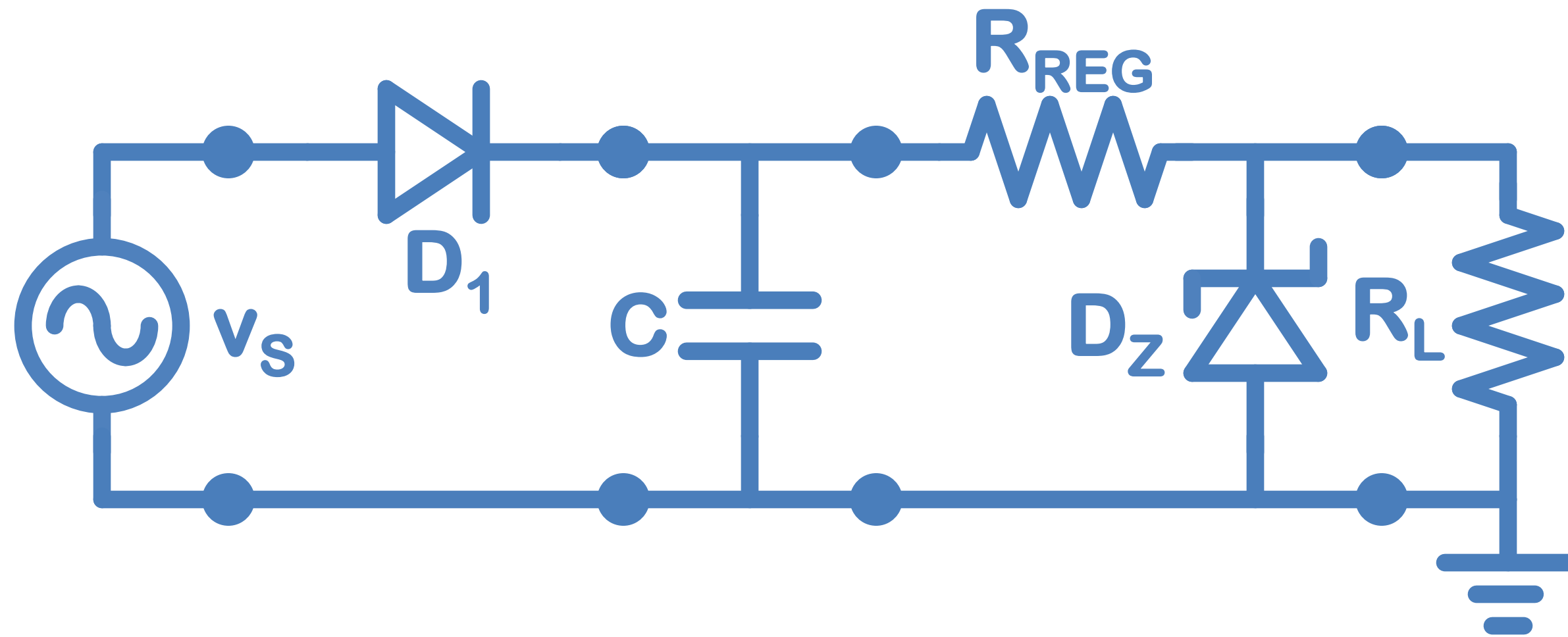
During FB,

$$i_{C_{av}} = I_R \cdot (T/\Delta t)$$

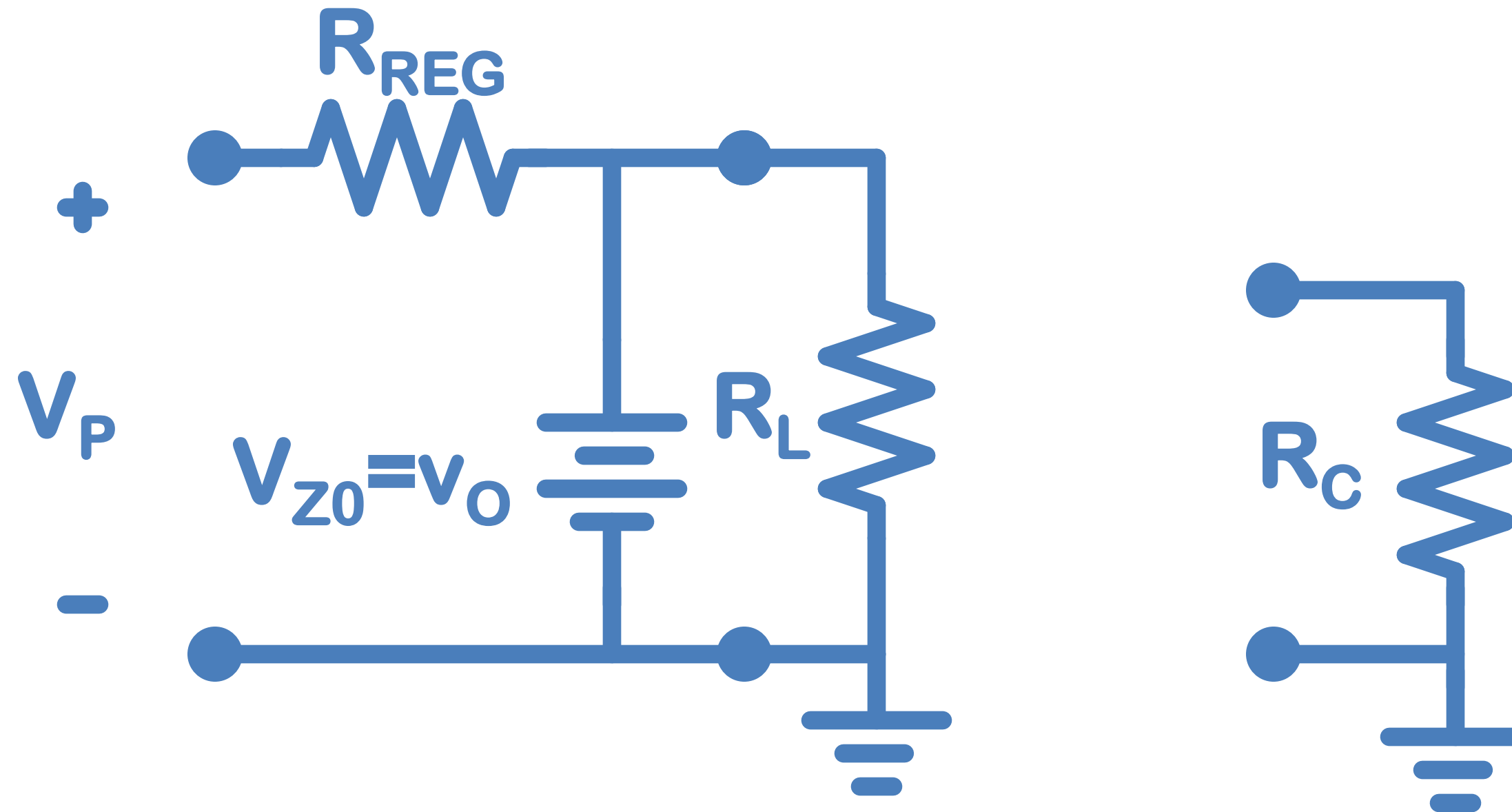
$$i_{D_{av}} = i_{C_{av}} + I_R$$

$$i_{D_{av}} = I_R \cdot (1 + T/\Delta t)$$

## How can we model the Regulator and Load as a single Resistor, $R_C$ ?



## How can we model the Regulator and Load as a single Resistor?



$V_P$  is the peak output of the Rectifier.

Use a simple CVD model for the Zener Diode.

$$I_{REG} = (V_P - V_{Z0})/R_{REG}$$

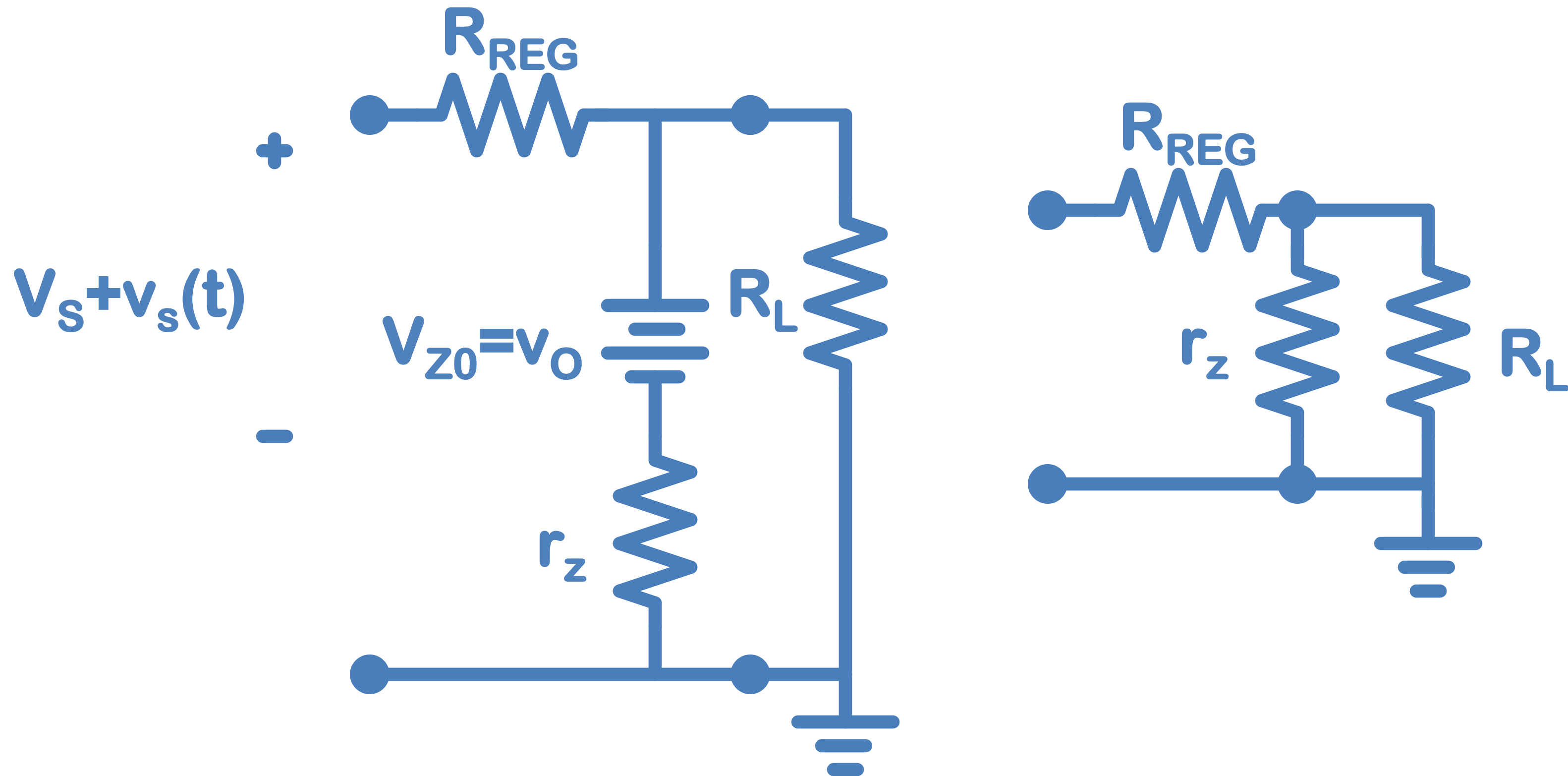
The effective resistance that is in parallel with the capacitor filter can be approximated as.

$$R_C = V_P/I_{REG} = V_P R_{REG}/(V_P - V_{Z0})$$

To ensure FB  $I_{REG} > I_L$ , or  $(V_P - V_{Z0})/R_{REG} > V_{Z0}/R_L$

$$R_{REG} < R_L (V_P - V_{Z0})/V_{Z0}$$

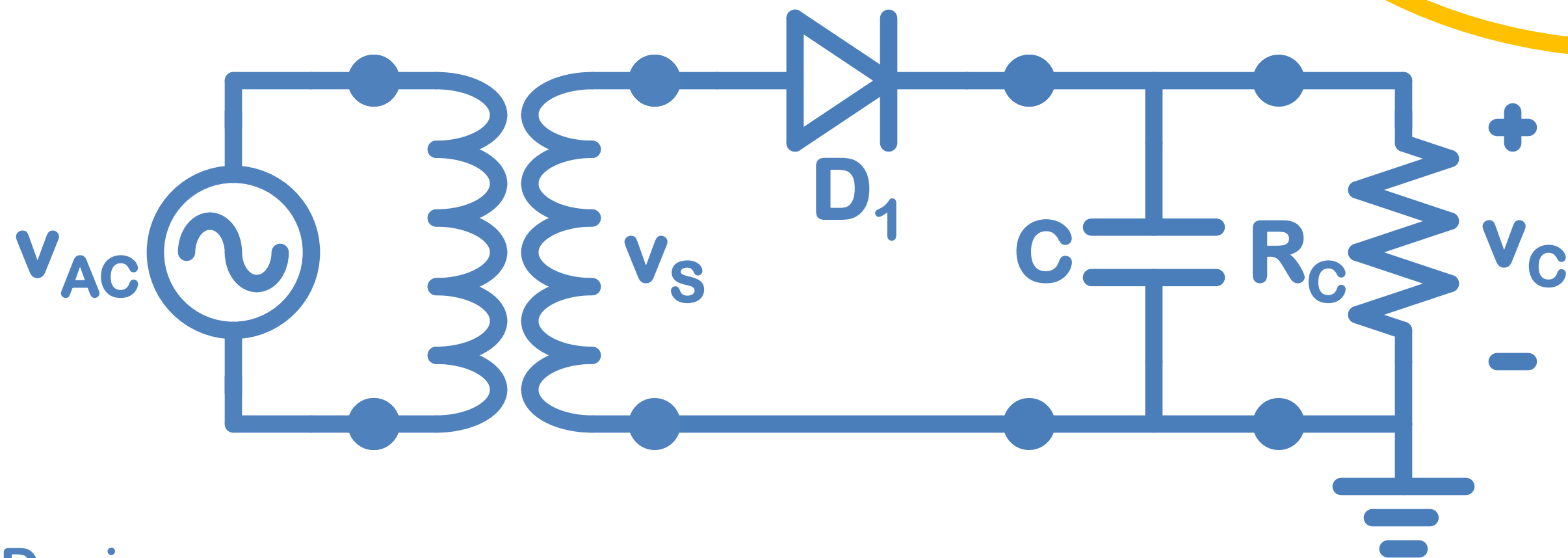
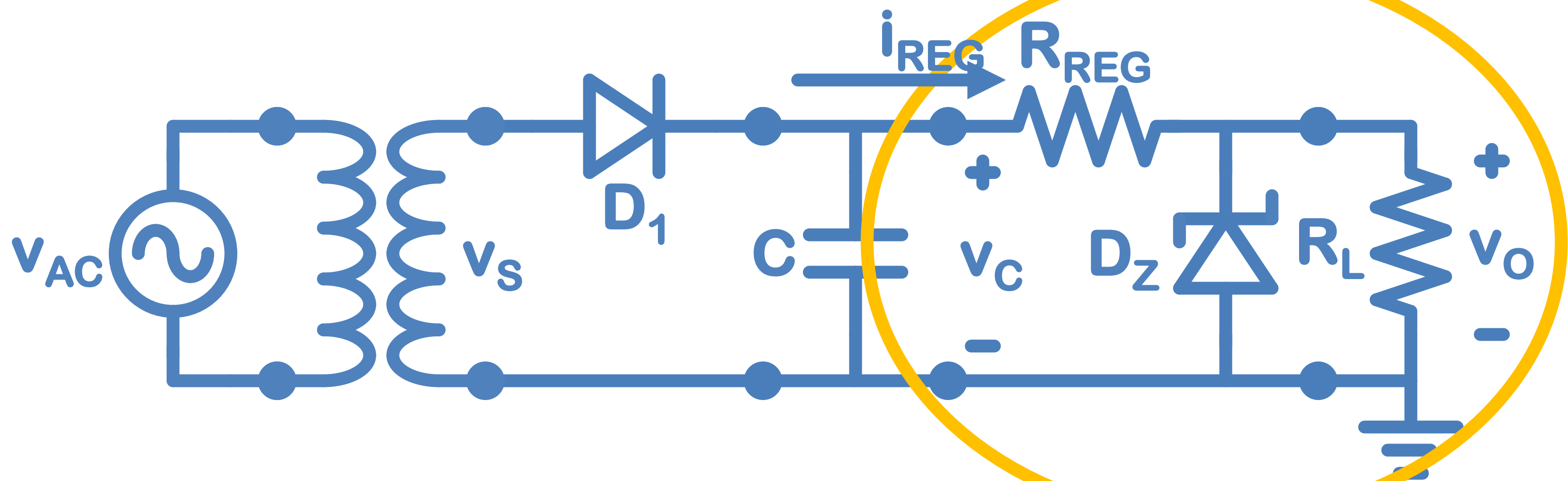
## Remember line regulation



$V_S \approx V_P$  is the peak output of the Rectifier (DC component).

$v_s(t)$  is the time varying component which has a peak to peak value corresponding to the ripple voltage (discussed later).

$$\text{line reg} = v_{\text{out}}(t)/v_s(t) = (r_z \parallel r_L)/(r_z \parallel R_L + R_{REG})$$



**Model Regulator  
as a resistor.**

$$R_C = v_C / i_{REG}$$

## Design

$R_L$  is fixed by the device we supply power to.

$R_{REG}$  is determined by peak output of the rectifier and  $R_L$ .

The amount of ripple at the output,  $\Delta v_{out}$ , of the regulator is a design requirement.

The amount of ripple at the output of the rectifier is  $V_r = \Delta v_{out} / (\text{line regulation})$ .

The  $R_C$  is determined from  $i_{REG}$  and  $R_{REG}$ .

The value of the filter capacitor is determined by  $V_r$ ,  $V_p$ ,  $R_C$ , and frequency.

# Waveforms

