Instruction: You must show all your work clearly for credit. Partial credit will only be given to meaningful answers.

1. (12) When implementing an ADT for a set of records $S$, $|S| = 2^6$, it is determined that a find operation, find($x$, $S$), will requires $0.5$ ms ($10^{-3}$ s) to execute. If the complexity of the find operation is given by the following closed-form expressions $T(n)$, compute the time required to execute this operation when $|S| = 2^{16}$.
   (a) $T(n) = 560$.
   (b) $T(n) = n \log n$.
   (c) $T(n) = n^2 \log n$.
   (d) $T(n) = n^3$.

2. (6) If an algorithm requires $0.5$ ms to solve a problem with input size of 100, how large a problem it can solve in 1 min if the complexity of the algorithm is given by the following function $T(n)$ in closed-form?
   (a) $T(n) = n$.
   (b) $T(n) = n^2$.

3. (10) Given the following algorithm for finding the two largest integers in an array $A[1..n]$ of $n$ distinct positive integers. Base on the number of comparisons between elements in $A$, compute $T_B(n)$ and $T_n(n)$. You must justify your answer and show your work clearly for credit.

```plaintext
    then   largest = $A[1]$;               
            $s_{\text{largest}} = A[2]$
            $s_{\text{largest}} = A[1]$
endif;

    if $A[i] > s_{\text{largest}}$   // $A[i]$ is one of the two largest integers
        if $A[i] > $largest        // $A[i]$ is the current largest integer
            then  $s_{\text{largest}} = $largest;
                     largest = $A[i]$
            else    $s_{\text{largest}} = A[i]$
        endif
    endif
endfor;
```
4. (10) Assuming that all basic operations require the same constant cost $C$, by concentrating on the dominating step(s), compute the cost of the resource function $R(n)$ for the following program segment in closed-form.

```plaintext
x = 2;
y = 10;
for i = 1 to n do
  for j = i to n do
    y = x * y / 2;
  for k = 1 to j do
    x = x + y - 10;
endfor;
endfor;
endfor;
```

5. (7) Let $A_1$ and $A_2$ be two algorithms with closed-form complexity $T_1(n) = 10n^2$ and $T_2(n) = 499n + 50$. Find smallest integer $n_0$ such that for all $n > n_0$, algorithm $A_2$ will always be more efficient than algorithm $A_1$.

6. (7) Use the definition of big-O to prove or disprove that $2^{2^n} = O(3^n)$.

7. (7) Prove or disprove that if $T_1(n) = O(f(n))$ and $T_2(n) = O(f(n))$, the $T_1(n) + T_2(n) = O(f(n))$.

8. (7) Prove or disprove that if $T_1(n) = O(f(n))$ and $T_2(n) = O(f(n))$, then $\frac{T_1(n)}{T_2(n)} = O(1)$.

9. (10) Use the definition of big-$\Theta$ to prove that

$$\frac{2n^4 - n^3 - 5n^2 + 4}{n^2 - 6n + 7} = \Theta(n^2).$$


   (a) By using the hash function $h(x) = x \mod m$ and chaining with singly linked list in constructing an open hash table $H$ with $m = 11$ buckets, insert a set of 7 records with keys 35, 28, 43, 17, 39, 3, and 46, in the given order, into $H$. You must show your computations for locations and illustrate the final structure of your hash table $H$ clearly for credit. Remark: Insertion must be done at the beginning of the list.

   (b) By using the hash function $h(x) = x \mod m$ and quadratic probing in constructing a closed hash table $H$ with $m = 11$ buckets, insert a set of 7 records with keys 35, 28, 43, 17, 39, 3, and 46, in the given order, into $H$. You must show your computations for locations and illustrate the final structure of your hash table $H$ clearly for credit.

   (c) Given two hash functions $h(x) = x \mod m$ and $h^+(x) = p - x \mod p$. By using open addressing with $f_i = i \cdot h^+(x)$ and double hashing in constructing a closed hash table $H$ with $m = 11$ buckets and $p = 5$, insert a set of 7 records with keys 35, 28, 43, 17, 39, 3, and 46, in the given order, into $H$. You must show your computations for locations and illustrate the final structure of your hash table $H$ clearly for credit.