Determine sets of certain and possible rules for all three concepts by the LEM2 algorithm.

	Attributes				Decision	Conceptual Variables		
	Size	Color	Feel	Temperature	Attitude	$Attitude_{positive}$	$Attitude_{negative}$	$Attitude_{so-so}$
1	big	yellow	soft	low	positive	SPECIAL	SPECIAL	SPECIAL
2	big	yellow	hard	high	negative	SPECIAL	negative	SPECIAL
3	medium	yellow	soft	high	positive	SPECIAL	SPECIAL	SPECIAL
4	medium	blue	hard	high	so-so	SPECIAL	SPECIAL	so-so
5	medium	blue	hard	high	negative	so-so	SPECIAL	SPECIAL
6	medium	blue	soft	low	negative	SPECIAL	negative	SPECIAL
7	big	blue	hard	low	so-so	SPECIAL	SPECIAL	so-so
8	big	blue	hard	low	positive	positive	SPECIAL	SPECIAL

$$A^* = \{\{1\}, \{2\}, \{3\}, \{4,5\}, \{6\}, \{7,8\}\}$$
$$\{d\}^* = \{\{1,3,8\}, \{2,5,6\}, \{4,7\}\}$$
$$A^* \nsubseteq \{d\}^*$$

$$Attitude_{positive} = \{1,3,8\} \qquad Attitude_{positive} = \{1,3\}$$
$$Attitude_{negative} = \{2,5,6\} \qquad Attitude_{negative} = \{2,6\}$$
$$Attitude_{so-so} = \{4,7\} \qquad Attitude_{so-so} = \{\}$$

$$\underline{Attitude_{positive}} = \{1,3\} \qquad \underline{Attitude_{positive}} = \{1,3,7,8\}$$
$$\underline{Attitude_{negative}} = \{2,6\} \qquad \underline{Attitude_{negative}} = \{2,4,5,6\}$$
$$\underline{Attitude_{so-so}} = \{\} \qquad \underline{Attitude_{so-so}} = \{4,5,7,8\}$$

(a,v)	{a,v}		{a,v}	
(Size, big)	{1,2,7,8}	{1}	{1,3}	{1,3}
(Size, medium)	{3,4,5,6}	{3}		{1}
(Color, yellow)	{1,2,3}			{3}
(Color, blue)	{4,5,6,7,8}	{1,3}		{1,3}
(Feel, soft)	{1,3,6}			
(Feel, hard)	{2,4,5,7,8}			{1}
(Temperature, low)	{1,6,7,8}	{1}		{3}
(Temperature, high)	{2,3,4,5}	{3}		
	$\{1,2,3\} \nsubseteq \{1,3\}$		$\{1,2,3\} \cap \{1,3,6\} = \{1,3\} \subseteq \{1,3\}$	

Table 1

(a,v)	{a,v}	{2,6}	{2}
(Size, big)	{1,2,7,8}	{2}	{2}
(Size, medium)	{3,4,5,6}	{6}	
(Color, yellow)	{1,2,3}	{2}	{2}
(Color, blue)	{4,5,6,7,8}	{6}	
(Feel, soft)	{1,3,6}	{6}	
(Feel, hard)	{2,4,5,7,8}	{2}	{2}
(Temperature, low)	{1,6,7,8}	{6}	
(Temperature, high)	{2,3,4,5}	{2}	{2}
	{1,2,3} ⊈ {2,6}	{1,2,3} ∩ {1,2,7,8} = {1,2} ⊈ {2,6}	{1,2,3} ∩ {2,3,4,5} = {2} ⊆ {2,6}

Table 2

(a,v)	{a,v}	{6}	{6}	{6}
(Size, big)	{1,2,7,8}			
(Size, medium)	{3,4,5,6}	{6}	{6}	{6}
(Color, yellow)	{1,2,3}			
(Color, blue)	{4,5,6,7,8}	{6}	{6}	{6}
(Feel, soft)	{1,3,6}	{6}	{6}	{6}
(Feel, hard)	{2,4,5,7,8}			
(Temperature, low)	{1,6,7,8}	{6}	{6}	{6}
(Temperature, high)	{2,3,4,5}			
	{1,3,6} ⊈ {2,6}	{1,3,6} ∩ {3,4,5,6} = {3,6} ⊈ {2,6}	{1,3,6} ∩ {1,6,7,8} = {6} ⊆ {2,6}	

Table 3

(a,v)	{a,v}	{1,3,7,8}	{1,7,8}
(Size, big)	{1,2,7,8}	{1,7,8}	{1,7,8}
(Size, medium)	{3,4,5,6}	{3}	{3}
(Color, yellow)	{1,2,3}	{3}	{3}
(Color, blue)	{4,5,6,7,8}	{7,8}	{7,8}
(Feel, soft)	{1,3,6}	{1,3}	{1,3}
(Feel, hard)	{2,4,5,7,8}	{7,8}	{7,8}
(Temperature, low)	{1,6,7,8}	{1,7,8}	{1,7,8}
(Temperature, high)	{2,3,4,5}	{3}	{3}
	{1,2,7,8} ⊈ {1,3,7,8}	{1,2,7,8} ∩ {1,6,7,8} = {1,7,8} ⊆ {1,3,7,8}	

Certain Rules:

(Color, yellow) & (Feel, soft) → (Attitude, positive) **{1,3}**

(Color, yellow) & (Size, big) & (Temperature, high) → (Attitude, negative) **{2}**

(Feel, soft) & (Size, medium) & (Temperature, low) → (Attitude, negative) **{6}**

(a,v)	
(Size, big)	{1,2,7,8}
(Size, medium)	{3,4,5,6}
(Color, yellow)	{1,2,3}
(Color, blue)	{4,5,6,7,8}
(Feel, soft)	{1,3,6}
(Feel, hard)	{2,4,5,7,8}
(Temperature, low)	{1,6,7,8}
(Temperature, high)	{2,3,4,5}

Possible Rules:

(Size, big) & (Temperature, low) → (Attitude, positive)

(Color, yellow) & (Feel, soft) → (Attitude, positive)

{1,2,3} ⊈ {1,3,7,8}   {1,2,3} ∩ {1,3,6} = {1,3} ⊆ {1,3,7,8}

{3}   {3}   {3}₄   {3}₄
{3}₃   {3}₃
{1,7,8}
{1,3}

(a,v)	
(Size, big)	{1,2,7,8}
(Size, medium)	{3,4,5,6}
(Color, yellow)	{1,2,3}
(Color, blue)	{4,5,6,7,8}
(Feel, soft)	{1,3,6}
(Feel, hard)	{2,4,5,7,8}
(Temperature, low)	{1,6,7,8}
(Temperature, high)	{2,3,4,5}

(a,v)	
	{2,4,5,6}
	{2}
	{4,5,6} 5
	{6}
	{2,4,5} 5
	{6}
	{2,4,5}₄

{3,4,5,6} ⊈ {2,4,5,6}   {3,4,5,6} ∩ {4,5,6} {4,5,6,7,8} ⊆ {2,4,5,6}

{4,5,6}
{4,5}
{6}

We know the rules associated with {2} from figuring out the certain rules above

Possible Rules:

(Size, medium) & (Color, blue) → (Attitude, negative)

(Size, big) & (Temperature, high) → (Attitude, negative)

{4,5,6}
{2}

(a,v)	
(Size, big)	{1,2,7,8}
(Size, medium)	{3,4,5,6}
(Color, yellow)	{1,2,3}
(Color, blue)	{4,5,6,7,8}
(Feel, soft)	{1,3,6}
(Feel, hard)	{2,4,5,7,8}
(Temperature, low)	{1,6,7,8}
(Temperature, high)	{2,3,4,5}

(a,v)	
	{4,5,7,8}
	{7,8}
	{4,5}
	{4,5,7,8}
	{7,8}
	{4,5}

{4,5,6,7,8} ⊈ {4,5,7,8}   {4,5,6,7,8} ∩ {2,4,5,7,8} = {4,5,7,8} ⊆ {4,5,7,8}

{4,5,7,8}
{7,8}
{4,5}

Possible Rules:

(Color, blue) & (Feel, hard) → (Attitude, so-so)

2. In general, for subsets X and Y of the universe U

$$\bar{A}X - \bar{A}Y \subseteq \bar{A}(X - Y)$$

where **A is the set of all attributes.**
Show a decision table, and subsets X and Y of U with

$$\bar{A}X - \bar{A}Y \subset \bar{A}(X - Y)$$

	Price	Sale
1	High	Yes
2	High	No
3	Low	Yes
4	Low	Yes

$$A^* = \{\{1, 2\}, \{3, 4\}\}$$
$$X = (Sale, yes) = \{1, 3, 4\}$$
$$Y = (Sale, no) = \{2\}$$

$$\bar{A}X - \bar{A}Y = \{1, 3, 4\} - \{1, 2\} = \{3, 4\}$$
$$\bar{A}(X - Y) = \bar{A}(\{1, 3, 4\} - \{2\}) = \bar{A}(\{1, 3, 4\}) = \{1, 2, 3, 4\}$$
$$\{3, 4\} \subset \{1, 2, 3, 4\}$$