1) Show that $(L^*)^* = L^*$ for all languages.

Star closure $L^* = L^0 \cup L^1 \cup L^2 \cup \ldots\ldots$

$$(L^*)^* = (L^*)^0 \cup (L^*)^1 \cup (L^*)^2 \cup \ldots\ldots$$

Substitute L^* from the definition of star closure.

$$(L^*)^* = (L^0 \cup L^1 \cup L^2 \cup \ldots)^0 \cup (L^0 \cup L^1 \cup L^2 \cup \ldots)^1 \cup$$

$$(L^0 \cup L^1 \cup L^2 \cup \ldots)^2 \cup \ldots\ldots$$

$\Rightarrow$ Since the union of same values is the value itself.

$$(L^*)^* = L^0 \cup L^1 \cup L^2 \cup \ldots\ldots$$

So after eliminating the repeated terms we found that

$$(L^*)^* = L^*$$

Hence proved.

2)

~~Show~~ Prove or disprove $(L^R)^* = (L^*)^R$ for all languages L.

$$L^R = \{w^R : w \in L\}$$

$$LHS \Rightarrow (L^R)^* = (L^R)^0 \cup (L^R)^1 \cup (L^R)^2 \ldots\ldots$$

$$RHS \Rightarrow (L^*)^R = (L^0 \cup L^1 \cup L^2 \ldots\ldots)^R$$

$$(L^*)^R = (L^0)^R \cup (L^1)^R \cup (L^2)^R \ldots\ldots$$

This is same as.

$$(L^R)^* = (L^R)^0 \cup (L^R)^1 \cup (L^R)^2 \ldots\ldots$$

Hence $(L^R)^* = (L^*)^R$ proved.

14c) $S \to bS \mid aA \mid \lambda$ no more than 3 a's

$A \to bA \mid aB \mid \lambda$

$B \to bB \mid aC \mid \lambda$

$C \to bC \mid \lambda$

17b) $L_2 = \{a^{3n} b^{2n} : n \ge 2\}$

when $n=2$ aaaaaabbbb

when $n=3$ aaaaaaaaabbbbbb

$S \to aaaaaaAbbbb$

$A \to aaaAbb \mid \lambda$

22) Show that $S \to SS \mid SSS \mid aSb \mid bSa \mid \lambda$

and $S \to SS \mid aSb \mid bSa \mid \lambda$

Let second grammar go to SS let this go to S

again SSS this was in first grammar, thus making them equal

23) $S \to aSb \mid ab \mid \lambda$ Show that they are equal

$S \to aaSbb \mid aSb \mid ab \mid \lambda$

take first grammar aSb

next take middle S and go back and insert "aSb"

aaSbb this state is represented in the second but not first, so must be equal.

Sect 1.3

12)