Figure 3.1 Transmission structures commonly used in microwave circuits.
$Z_o$ and $\gamma$ for coaxial lines [1]

For a coaxial line, the values of capacitance and inductance per unit length are obtained from static field analysis. We have

\[ C = \frac{2\pi \varepsilon}{\ln(b/a)} \]  \hspace{1cm} (3.11)

and

\[ L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \]  \hspace{1cm} (3.12)

where $\varepsilon$ and $\mu$ are the permittivity and the permeability of the dielectric medium, "b" denotes inner radius of the outer conductor and "a" the radius of the inner conductor. Using (3.11) the characteristic impedance for low loss lines may be written as

\[ Z_o = \frac{\eta}{2\pi} \ln\left(\frac{b}{a}\right) \]  \hspace{1cm} (3.13)

where $\eta$ is the intrinsic impedance (= $\sqrt{\mu/\varepsilon}$) of the dielectric medium. For air dielectric, we may write

\[ Z_o = 60 \varepsilon \ln\left(\frac{b}{a}\right) \text{ ohm} \]  \hspace{1cm} (3.14)

Series resistance $R$ per unit length is given by

\[ R = \frac{R_s}{2\pi} \left( \frac{1}{b} + \frac{1}{a} \right) \]  \hspace{1cm} (3.15)

where $R_s$ is the surface resistivity of the conductors. The conductance $G$ per unit length may be written as

\[ G = \frac{2\pi \sigma}{\ln\left(\frac{b}{a}\right)} = 2\pi \omega \varepsilon \sigma \tan\delta \]  \hspace{1cm} (3.16)

where $\sigma$ is the conductivity and $\tan\delta$ is the loss tangent of the dielectric. The attenuation coefficients due to $R$ and $G$ can be calculated as follows:

\text{Attenuation due to conductor,}
\[ \alpha_c = \frac{R}{(2 Z_o)} \text{ nepers/m} \] \hspace{1cm} (3.17)

\text{Attenuation due to dielectric,}
\[ \alpha_d = \frac{G Z_o}{2} = \frac{\sigma}{2} = \frac{\pi \sqrt{\varepsilon_r \tan\delta}}{\lambda_o} \text{ nepers/m} \] \hspace{1cm} (3.18)

Total attenuation, expressed in db/meter, is $8.686 (\alpha_c + \alpha_d)$.

From (3.15) it is seen that $R$, and hence the attenuation, decreases with an increase in $b$ and $a$. The upper limit on these dimensions is set by the existence of higher order modes. The lowest order TE wave with circumferential variation (i.e. TE$_{01}$ mode) is significant for this purpose. For this mode the cut-off wavelength is given by

\[ \lambda_c \approx \frac{2\pi}{b + a} \] \hspace{1cm} (3.19)

i.e., cut-off occurs for wavelength approximately equal to the average circumference.

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Characterization of Transmission Structure

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Figure 3.3(a) A coaxial line.
(b) Lumped circuit representation of a transmission line of length $\Delta z$. 

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3.3 STRIPLINES [4,5,6]

Stripline (shown in Figure 3.5(a)) is one of the most commonly used transmission lines at microwave frequencies. The dominant mode of propagation is TEM, and the design data can be obtained completely by electrostatic analysis.

The analysis of stripline is considerably simplified when the thickness “t” of the central strip is negligible. Cohn [4] has derived, using the conformal mapping technique, an expression for the capacitance of a stripline with \( t = 0 \). This results in the following equation for the characteristic impedance

\[
Z_o \sqrt{\varepsilon_r} = 30 \pi \frac{K'(k)}{K(k)} \text{ ohm}
\]  

(3.35)

where \( k = \tanh (\pi W/2b) \) and \( K \) represents a complete elliptic function of the first kind with \( K' \) its complementary function given by

\[
K'(k) = K(k') \quad ; \quad k' = \sqrt{1 - k^2}.
\]  

(3.36)

A simple expression for \( Z_o \) can be obtained by using an approximation for \( K/K' \) (accurate to 8 ppm) as given below:

\[
\frac{K(k)}{K'(k)} = \begin{cases} 
\frac{1}{\pi} \ln \left( 2 \frac{1 + \sqrt{k}}{1 - \sqrt{k}} \right) & \text{for } 0 \leq k \leq 0.7 \\
\frac{1}{\pi} \ln \left( 2 \frac{1 + \sqrt{k'}}{1 - \sqrt{k'}} \right) & \text{for } 0.7 \leq k \leq 1.
\end{cases}
\]

(3.37)

These formulas yield values for \( Z_o \) which are virtually exact.

Analysis for thick striplines has been reported by many authors [2, 5,6,7]. The most accurate of these approximate formulas is given by Wheeler [7]. According to this,

\[
Z_o \sqrt{\varepsilon_r} = 30 \ln \left[ 1 + \frac{4}{\pi} \frac{b - t}{W} \left[ \frac{8}{\pi} \frac{b - t}{W} + \sqrt{\left( \frac{8}{\pi} \frac{b - t}{W} \right)^2 + 6.27} \right] \right]
\]

(3.39)
where
\[
\frac{W'}{b - t} = \frac{W}{b - t} + \frac{\Delta W}{b - t}
\]  
(3.40)

with
\[
\frac{\Delta W}{b - t} = \frac{x}{\pi (1 - x)} \left\{ 1 - \frac{1}{2} \ln \left[ \left( \frac{x}{2 - x} \right)^2 + \left( \frac{0.0796 x}{W/b + 1.1x} \right)^m \right] \right\}
\]  
(3.41a)

in which \(m = 2 \left[ 1 + \frac{2}{3} \frac{x}{1 - x} \right]^{-1}\) and \(x = t/b\).  
(3.41b)

For \(W' / (b - t) < 10\), (3.39) to (3.41) are stated to yield data which are accurate to within 0.5 percent.

**Synthesis Equations for Striplines**

For computer-aided design and optimization of stripline circuits, synthesis equations are needed; i.e., strip width \(W\) for a given impedance line should be expressed in terms of parameters \(b\), \(\varepsilon_{r}\) and \(t\). For zero-thickness strips, \(W/b\) is obtained as a function of \(Z_{o}\) and \(\varepsilon_{r}\) from (3.35) and (3.36) as
\[
\frac{W}{b} = \frac{2}{\pi} \tanh^{-1} \sqrt{x} \quad k
\]  
(3.42)

where
\[
k = \begin{cases} 
\sqrt{1 - \left[ \frac{e^{\pi x} - 2}{e^{\pi x} + 2} \right]^4} & \text{for } x \geq 1 \\
\left[ \frac{e^{\pi x} - 2}{e^{\pi x} + 2} \right]^2 & \text{for } 0 \leq x \leq 1
\end{cases}
\]

with \(x = Z_{o} \sqrt{\varepsilon_{r}} / (30 \pi)\).

The synthesis equation for \(W/b\) (with \(t \neq 0\)) is obtained from (3.39)

and is given below [6]
\[
\frac{W}{b} = \frac{W_{o}}{b} - \frac{\Delta W}{b}
\]

with
\[
\frac{W_{o}}{b} = \frac{8(1 - x)}{\pi} \sqrt{\frac{e^{A} + 0.568}{e^{A} - 1}}, \quad A = \frac{Z_{o} \sqrt{\varepsilon_{r}}}{30}
\]  
(3.43)

and
\[
\frac{\Delta W}{b} = x \left\{ 1 - \frac{1}{2} \ln \left[ \left( \frac{x}{2 - x} \right)^2 + \left( \frac{0.0796 x}{W_{o}/b - 0.26 x} \right)^m \right] \right\}
\]  
(3.44)

The quantities \(x\) and \(m\) have been defined earlier in (3.41b).

**Stripline Losses**

As in the case of other transmission lines, the total loss \(\alpha_{T}\) in stripline can be divided in two parts: the conductor loss and the dielectric loss; i.e.,
\[
\alpha_{T} = \alpha_{c} + \alpha_{d}
\]  
(3.45)

where subscripts \(c\) and \(d\) stand for conductor and dielectric, respectively.

The conductor loss is determined by considering the incremental inductance associated with the penetration of magnetic flux into each of the conducting surfaces [17]. For stripline, it may be evaluated from the following relation [3],
\[
\alpha_{c} = \frac{0.0231}{Z_{o} \sqrt{\varepsilon_{r}}} \left\{ \frac{\partial Z_{o}}{\partial b} - \frac{\partial Z_{o}}{\partial W} - \frac{\partial Z_{o}}{\partial t} \right\} \quad \text{db/m}
\]  
(3.46)

where \(R_{s}\) is the sheet resistivity (ohm/square) for the conductor and is given by \(\sqrt{\pi / \rho_{o}}\rho\) with \(\rho\) being the resistivity of the conductor. Using relations (3.39) to (3.42) for \(Z_{o}\), \(\alpha_{c}\) becomes
\[
\alpha_{c} = \frac{0.0231}{Z_{o} \sqrt{\varepsilon_{r}}} \frac{\partial Z_{o}}{\partial W} \left\{ 1 + \frac{2W}{b - t} - \frac{1}{\pi} \left[ \frac{3x}{2 - x} + 2 \ln \frac{x}{2 - x} \right] \right\} \quad \text{db/m}
\]  
(3.47)
\[
\frac{\partial Z_o}{\partial W} = \frac{30 e^{-A}}{W \sqrt{\varepsilon_r}} \left[ \frac{3.135}{Q} - \left( \frac{8 b - t}{\pi W} \right)^2 (1 + Q) \right]
\]

(3.48)

with \(Q = \sqrt{1 + 6.27 \left( \frac{\pi}{8} \frac{W}{b - t} \right)^2} \).

(3.49)

It may be observed from (3.47) that for a given value of \(Z_o\), the conductor loss increases with the square root of frequency (because of the frequency dependence of \(R_c\)).

The dielectric loss for stripline (or any other TEM mode line) is given by [3]

\[
\alpha_d = 27.3 \sqrt{\varepsilon_r} \tan \delta / \lambda_o \text{ db/m}
\]

(3.50)

where \(\tan \delta\) is the loss tangent of the dielectric. Equation (3.50) shows that the dielectric loss is directly proportional to the frequency and the loss tangent.

The dielectric loss is, in general, very small compared to the conductor loss at microwave frequencies. But at millimeter waves it becomes comparable to the conductor loss because dielectric loss increases linearly with the frequency, whereas conductor loss is proportional to the square root of frequency.

Maximum frequency of operation of a stripline is limited by the excitation of TE modes [8]. For wide lines, the cut-off for the lowest order TE mode is given by

\[
f_T (\text{GHz}) = \frac{15}{b \sqrt{\varepsilon_r}} \frac{1}{(W/b + \pi/4)}
\]

(3.51)

where \(W\) and \(b\) are in cm. It may be observed that the cut-off frequency, \(f_T\), decreases when either the spacing between the ground planes or the dielectric constant is increased.

3.4 MICROSTRIP LINES [9, 18]

Unlike the stripline, the microstrip line (configuration shown in Figure 3.5(b)) is an inhomogeneous transmission line since the field lines between the strip and the ground plane are not contained entirely in the substrate. Therefore, the mode propagating along the microstrip is not purely TEM but quasi-TEM. For this mode of propagation, the phase velocity in the microstrip is given by (3.5). The effective dielectric constant \(\varepsilon_{re}\) is lower than the dielectric constant of the substrate and takes into account the fields external to the substrate.

The available numerical methods for the characterization of microstrip lines involve extensive computations. Closed form expressions are necessary for optimization and computer-aided design of microstrip circuits. A complete set of design equations for microstrip are presented in this section. These include closed form expressions for the characteristic impedance and effective dielectric constant, and their variation with metal strip thickness and frequency. Expressions for losses are also included.

Characteristic Impedance and Effective Dielectric Constant

The closed form expressions for \(Z_o\) and \(\varepsilon_{re}\) have been reported by Wheeler [19, 20], Schneider [10], and Hammerstad [21]. Wheeler and Hammerstad have also given a synthesis expression for \(Z_o\). The closed form expressions based on [10, 19] are given below:

\[
Z_o = \begin{cases} 
\frac{\eta}{2\pi \sqrt{\varepsilon_{re}}} \ln \left( \frac{8h}{W} + 0.25 \frac{W}{h} \right) & \text{for } (W/h < 1) \\
\frac{\eta}{\sqrt{\varepsilon_{re}}} \left[ \frac{W}{h} + 1.393 + 0.667 \ln \left( \frac{W}{h} + 1.444 \right) \right]^{-1} & \text{for } (W/h \geq 1)
\end{cases}
\]

(3.52a)

(3.52b)

where \(\eta = 120 \pi \text{ ohm}\), and

\[
\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} (1 + 10 h/W)^{-1/2}.
\]

(3.52c)

The maximum relative error in \(\varepsilon_{re}\) and \(Z_o\) is less than two percent. The expressions for \(W/h\) in terms of \(Z_o\) and \(\varepsilon_r\) are as follows:

For \(A > 1.52\)

\[
W/h = \frac{8 \exp(A)}{\exp(2A) - 2}
\]

(3.53a)
For $A \leq 1.52$
\[
W/h = \frac{2}{\pi} \left[ B - 1 - \frac{\ln (2B - 1) + \frac{e_r - 1}{2e_r} \left( \ln (B - 1) + 0.39 - \frac{0.61}{e_r} \right)}{B^2} \right]
\]
\[\text{where} \quad A = \frac{Z_o}{60} \left( \frac{e_r + 1}{2} \right)^{1/2} + \frac{e_r - 1}{e_r + 1} \left( 0.23 + \frac{0.11}{e_r} \right) \]
\[\text{and} \quad B = \frac{60 \pi^2}{Z_o \sqrt{e_r}} \]

These expressions also provide an accuracy better than two percent. The recent closed form expressions by Wheeler [20] provide lower accuracy. However, a single expression holds for the entire range of values of $W/h$.

The results discussed above assume the strip thickness to be negligible. But, in practice, the strip thickness "t" affects the characteristics. However, when $t/h \leq 0.005$, the agreement between experimental and theoretical results obtained by assuming $t/h = 0$ is excellent.

**Effect of Strip Thickness**

A number of expressions have been reported for incorporating the effect of strip thickness in the calculations of $Z_o$ and $\varepsilon_{re}$ of microstrip. Simple and accurate formulas for $Z_o$ and $\varepsilon_{re}$ with finite strip thickness are [22],

\[
Z_o = \begin{cases} 
\frac{60}{\sqrt{\varepsilon_{re}}} \ln \left( \frac{8h}{W_e} + 0.25 \frac{W_e}{h} \right) & (W/h \leq 1) \\
\frac{376.7}{\sqrt{\varepsilon_{re}}} \left[ \frac{W_e}{h} + 1.393 + 0.667 \ln \left( \frac{W_e}{h} + 1.444 \right) \right]^{-1} & (W/h \geq 1)
\end{cases}
\]

Characterization of Transmission Structure

Where \[
\frac{W_e}{h} = \frac{W}{h} + \frac{\Delta W}{h}, \quad \text{with}
\]

\[\Delta W = \begin{cases} 
\frac{1.25}{\pi} \frac{t}{h} \left( 1 + \frac{\ln \frac{4\pi W}{t}}{2} \right) & (W/h \leq 1/2\pi) \quad (3.55a) \\
\frac{1.25}{\pi} \frac{t}{h} \left( 1 + \frac{\ln \frac{2h}{t}}{2} \right) & (W/h \geq 1/2\pi) \quad (3.55b)
\end{cases}
\]

\[\varepsilon_{re} = \frac{e_r + 1}{2} + \frac{e_r - 1}{2} F(W/h) - Q \quad (3.56)
\]

In which

\[Q = \frac{e_r - 1}{4.6} \frac{t/h}{\sqrt{(W/h)}} \quad \text{and} \quad F(W/h) = (1 + 10 h/W)^{-1/2} \quad (3.57)
\]

**Effect of Dispersion**

The effect of frequency (dispersion) on $\varepsilon_{re}$ is described accurately through the dispersion model given by Gelsinger [23] and modified by Edwards and Owens [24]. The effect of frequency on $Z_o$ has been described by several investigators, and it is found that the results of Bianco et al. [25] are closer to numerical values.

The results of Bianco et al. for $Z_o(f)$, and Edwards and Owen for $\varepsilon_{re}(f)$ may be stated as follows

\[Z_o(f) = Z_{oT} - \frac{Z_{oT} - Z_o}{1 + G(t/f_p)^2} \quad (3.58)
\]

And

\[\varepsilon_{re}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{re}}{1 + G(t/f_p)^2} \quad (3.59)
\]

Where

\[G = \left[ \frac{Z_o - 5}{60} \right]^{1/2} + 0.004 Z_o \quad (3.60)
\]
and \( f_p (\text{GHz}) = 15.66 Z_o / h \) .

(3.61)

In the above equations \( h \) is in mil, \( Z_o \) in ohm and \( Z_oT \) is twice the characteristic impedance of stripline of width \( W \) and height \( 2h \). The characteristics \( Z_o \) and \( \epsilon_{re} \) are quasi-static values obtained earlier.

Kuester and Chang have reviewed the various numerical methods for dispersion in microstrip [26]. They have also reported [27] an analytical method for dispersion in microstrip which leads to an expression for \( \epsilon_{re} \) in terms of frequency dependent capacitance and inductance parameters for the line.

**Losses**

The closed form expressions for the total loss have been reported in the literature [28]. An expression for the conductor loss \( \alpha_c \) derived using (3.54) may be written as

\[
\alpha_c = \begin{cases} 
1.38 A \frac{R_s}{hZ_o} \frac{32 - (W_e/h)^2}{32 + (W_e/h)^2} & \text{db/m} \quad (W/h \leq 1) \\
6.1 \times 10^{-5} A \frac{R_s Z_o \epsilon_{re}}{h} \left( \frac{W_e/h}{W_e/h + \frac{0.667 W_e}{W_e/h + 1.444}} \right) & \text{db/m} \quad (W/h \geq 1)
\end{cases} 
\]

(3.62)

(3.62a)

(3.62b)

where \( (W_e/h) \) is given by (3.55),

\[
A = 1 + \frac{h}{W_e} \left( 1 + \frac{1}{\pi} \ln \frac{2B}{t} \right) 
\]

(3.63)

\[
R_s = \sqrt{\pi f \mu_0 \rho} 
\]

(3.64)

\[
B = \begin{cases} 
h & \text{for} \quad \frac{W/h}{2} \geq 1 \\
2\pi W & \text{for} \quad \frac{W/h}{2} \leq 1
\end{cases}
\]

and \( \rho \) is the resistivity of the strip conductor.

**Characterization of Transmission Structure**

Dielectric loss \( \alpha_d \) is given by

\[
\alpha_d = 27.3 \frac{\epsilon_r}{\epsilon_r - 1} \frac{\epsilon_{re} - 1}{\sqrt{\epsilon_{re}}} \tan \delta \frac{\lambda_0}{\lambda_0} \text{ db/m} 
\]

(3.65)

where \( \tan \delta \) is the loss tangent of the dielectric. The dielectric loss is normally very small compared with the conductor loss.

**Fig. 3.2** The dispersive behavior of \( \epsilon_e (f) \) in a microstrip.

**3.6 COPLANAR LINES [9]**

Coplanar waveguides are finding extensive applications in microwave integrated circuits. Inclusion of coplanar waveguides in microwave circuits adds to the flexibility of circuit design and improves the performance for some circuit functions. The configuration of a coplanar waveguide (CPW) is shown in Figure 3.7(a). Another promising configuration which is complementary to CPW is known as coplanar strips (CPS) and is shown in Figure 3.7(b). Both of these configurations belong to the category of “coplanar lines” wherein all the conductors are in the same plane (i.e. on the top surface of the dielectric substrate).
Characterization of Transmission Structure

Frequency-dependent behavior of phase velocity and characteristic impedance of CPW and CPS have also been evaluated.

The design equations and data presented in this section are based on the quasi-static analysis. Dispersion effects in coplanar lines have been studied by Yamashita and Atsuki [11] and found to be of the same order as for the microstrip lines. Also, the dispersion for low dielectric constant substrates is almost negligible [11]. These results indicate that the quasi-TEM analysis may be used below X-band frequencies.

**Characteristic Impedance and Effective Dielectric Constant**

The quasi-static results for CPW given by Wen [32] may be modified for finite dielectric thickness as

\[
Z_{ocp} = \frac{30\pi \sqrt{\varepsilon_{re}}}{\sqrt{\varepsilon_{re}}} K'(k) \quad K(k) \tag{3.73}
\]

where

\[
k = \frac{S}{S + 2W} \tag{3.74}
\]

A closed form expression for \( \varepsilon_{re} \) has been obtained [9] by curve fitting the numerical results of Davis et al. [33], and is given below:

\[
\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} \left\{ \tan^{-1} \left[ 0.775 \ln \left( h/W \right) + 1.75 \right] + \frac{kW}{h} \left[ 0.04 \right.ight.

\- 0.7 + 0.01 \left( 1 - 0.1 \varepsilon_r \right) \left( 0.25 + k \right) \left( 3.75 \right)
\]

The accuracy of this expression is better than 1.5 percent for \( \varepsilon_r \geq 9, h/W \geq 1 \) and \( 0 < k < 0.7 \), when compared with the results of Davis et al. [33].

For CPS, the characteristic impedance can be written as [9]

\[
Z_{ocs} = \frac{120 \pi}{\sqrt{\varepsilon_{re}}} \frac{K(k)}{K'(k)} \tag{3.76}
\]

where \( \varepsilon_{re} \) is again given by (3.75) in which \( W \) is now the strip width and \( S \) is the spacing between the strips. A good agreement is found between the values calculated using the above relations and the available results.
Effect of Strip Thickness

The results discussed above assume infinitesimally thin metallic strip conductor and ground planes. But in practice, metallization has a finite thickness "t" which affects the characteristics. The effect of strip thickness on the impedance of coplanar lines can be taken into account by defining effective values of the strip width and the spacing. This is similar to the concept of increase in microstrip width \( W \) due to thickness \( t \) as discussed in [22]. For CPW, we can write [9]

\[
S_e = S + \Delta \quad (3.77)
\]

and

\[
W_e = W - \Delta \quad (3.78)
\]

where \( \Delta \) can be evaluated (for all practical values of \( S/h \)) from the following relation,

\[
\Delta = (1.25 \frac{t}{\pi}) \left[ 1 + \ln \left( \frac{4 \pi S}{t} \right) \right] . \quad (3.79)
\]

The characteristic impedance is found as

\[
Z_{ocp} = \frac{30 \pi}{\sqrt{\varepsilon_{re}}} \frac{K'(k_e)}{K(k_e)} \quad (3.80)
\]

where \( k_e \) is the effective aspect ratio [9] given by

\[
k_e = \frac{S_e}{(S_e + 2W_e)} \approx k + (1 - k^2) \Delta/2W \quad (3.81)
\]

and \( \varepsilon_{re} \) is the effective dielectric constant for thick CPW. An expression for \( \varepsilon_{re} \) is derived by adding a term \( A \varepsilon_\infty \varepsilon_{re} \frac{t}{W} \), accounting for the increase in capacitance due to the metal thickness, to the expression for capacitance of CPW. The value of \( A \) is found empirically such that the results for \( \varepsilon_{re} \) agree with the numerically evaluated values given in [36] for \( \varepsilon_{re} = 20 \) and \( t/W \ll 0.1 \). It is observed that \( A = 2.8 \) gives reasonably accurate (better than 3 percent) results for \( \varepsilon_{re} \). This value of \( A \) is quite near to the value of \( A = 2 \) obtained by using the simple parallel plate capacitance formulation.

Characterization of Transmission Structure

The final expression for \( \varepsilon_{re} \) may be written as

\[
\varepsilon_{re}^t = \varepsilon_{re} - \frac{0.7 \left( \varepsilon_{re} - 1 \right) t/W}{K(k) / K'(k)} + 0.7 t/W \quad (3.82)
\]

For coplanar strips, the effect of strip thickness on \( Z_{ocs} \) and \( \varepsilon_{re} \) is similar to that in CPW and the closed form expressions are obtained as

\[
Z_{ocs} = \frac{120 \pi}{\sqrt{\varepsilon_{re}}} \frac{K(k_e)}{K'(k_e)} \quad (3.83)
\]

where \( k_e = S_e/(S_e + 2W_e) \approx k - (1 - k^2) \Delta/2W \quad (3.84) \)

with \( \Delta = (1.25t/\pi) \left[ 1 + \ln \left( \frac{4\pi W}{t} \right) \right] \) . \quad (3.85)

The effective dielectric constant for CPS may similarly be written as

\[
\varepsilon_{re}^t = \varepsilon_{re} - \frac{1.4 \left( \varepsilon_{re} - 1 \right) t/S}{K'(k) / K(k) + 1.4 t/S} \quad (3.86)
\]

Losses

When quasi-static approximation is valid, one can use Wheeler's incremental inductance formula [17] for evaluating ohmic losses, and the expression for attenuation in CPW becomes

\[
\alpha_{ocp} = 0.023 \frac{R_s}{Z_{ocp}} \left[ \frac{\partial Z_{ocp}^a}{\partial W} - \frac{\partial Z_{ocp}^a}{\partial S} - \frac{\partial Z_{ocp}^a}{\partial t} \right] \text{ db/m} \quad (3.87)
\]

where subscript \( cp \) denotes coplanar waveguide. After substituting the expressions for various partial derivatives in (3.87), the final expression for conductor loss becomes

\[
\alpha_c = 4.88 \times 10^{-4} \frac{R_s}{Z_{ocp}} \frac{P'}{W \pi} \left( \frac{1 + S}{W} \right) \left[ \frac{1.25}{\pi} \ln \left( \frac{4\pi S}{t} \right) + 1 + \frac{1.25t}{\pi S} \right] \left[ 2 + \frac{S}{W} - \frac{1.25t}{\pi W} \left( 1 + \ln \left( \frac{4\pi S}{t} \right) \right) \right]^2 \text{ db/m} \quad (3.88)
\]
where \( P' \) is given by

\[
P' = \begin{cases} 
\frac{k}{(1 - k')(k')^{3/2}} \left( \frac{K(k)}{K'(k)} \right)^2 & \text{for } 0 \leq k \leq 0.707 \\
\frac{1}{(1 - k)\sqrt{k}} & \text{for } 0.707 \leq k \leq 1.0 
\end{cases}
\] (3.89)

The expression for the attenuation constant due to dielectric loss in CPW is the same as that for microstrip lines and can be written as

\[
\alpha_d = 27.3 \frac{e_r}{\sqrt{\varepsilon_{re}}} \times \frac{e_{re} - 1}{e_r - 1} \times \frac{\tan\delta}{\lambda_o} \text{ db/m}.
\] (3.90)

In the present case, \( e_{re} \) is given by (3.75).

For coplanar strips, the expression for the conductor loss becomes

\[
\alpha_c^{cs} = 17.34 \frac{R_s}{Z_{oCS}} \frac{P'}{\pi S} \left( 1 + \frac{W}{S} \right)
\]

\[
\times \left[ \frac{1.25}{\pi} \left( \frac{\ln \frac{4\pi W}{t}}{t} + 1 + \frac{1.25 t}{\pi W} \right) \right]^{2} \text{ db/m} \] (3.91)

where \( P' \) is again given by (3.89). The expression for dielectric loss is the same as that given by (3.90).