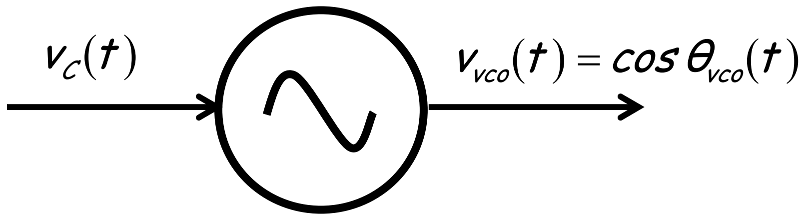
Voltage Controlled Oscillators

A voltage controlled oscillator is a rather simple device in theory—it's simply an oscillator whose **frequency** is related to a control voltage.



In other "words":

$$\frac{d\,\theta_{vco}(t)}{dt} = \omega_{vco} = f\left(v_C\right)$$

Thus, **if** control voltage v_C is a **constant** with respect to time, the oscillator frequency will likewise be a constant—the oscillator will produce a **"pure" tone** of the form:

$$v_{vco}(t) = cos(\omega_{vco}t + \theta_0)$$

Conversely, **if** the control voltage is **time-varying**, the oscillator frequency will also **change** with respect to time.

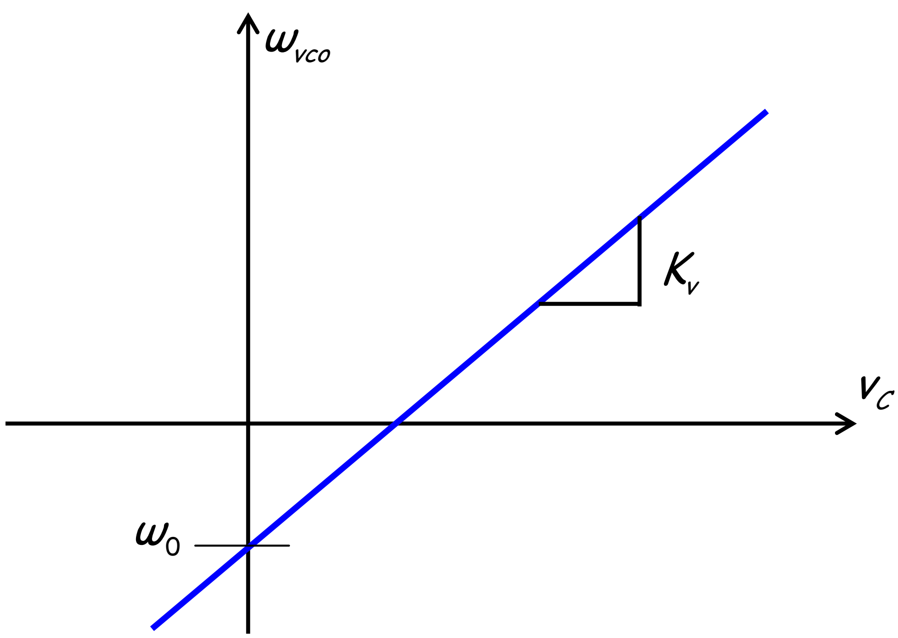
→ The result is a **frequency modulated** output signal!

Ideally, the relationship between control voltage v_C and oscillator frequency ω_{vco} is very simple; easily expressed as a **first-order** polynomial:

$$\begin{aligned}\omega_{vco} &= f(v_C)\\ &= K_v\, v_C + \omega_0\end{aligned}$$

where constant K_v **obviously** has units of $radians/sec\cdot volt$, sometime expressed as $2\pi\ Hz/volt$.

The result is thus an equation of a **line**, with **slope** K_v and ***y*-intercept** ω_0:



Now, consider the case where we **frequency modulate** this oscillator signal. The control voltage will change as a function of **time** (i.e., $v_C(t)$), and so the VCO output frequency will likewise:

$$\omega_{vco}(t) = K_v\, v_C(t) + \omega_0$$

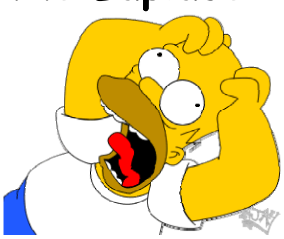
Q: *What then is the VCO output signal $v_{vco}(t) = \cos\theta_{vco}(t)$???*

A: Remember, the frequency function $\omega_{vco}(t)$ is the time derivative of the phase function $\theta_{vco}(t)$. So to determine $\theta_{vco}(t)$, we must **integrate** $\omega_{vco}(t)$!

$$\begin{aligned}\theta_{vco}(t) &= \int_0^t \omega_{vco}(t')\,dt' \\ &= K_v \int_0^t v_C(t)\,dt' + \omega_0 t + \theta_0\end{aligned}$$

Thus, the relationship between **two** of the **four** important PLL parameters has been established. The phase function $\theta_{vco}(t)$ is determined by integrating control voltage $v_C(t)$.

Typically, this relationship is mathematically described using the **Laplace Transform**!



$$\theta_{vco}(s) = \mathbf{L}\left[\theta_{vco}(t)\right] = \int_0^\infty \theta_{vco}(t)\,e^{-st}\,dt$$

Of course, it is now obvious to **you** that:

$$\theta_{vco}(s) = \frac{K_v}{s} v_C(s) + \frac{\omega_0}{s^2} + \frac{\theta_0}{s}$$

Now, to **simplify** the math a bit, most PLL mathematics **ignore** the last two terms of the expression above. Essentially, it is assumed that $\omega_0 = 0$ and $\theta_0 = 0$.

Although this is not **explicitly** true, this simplifying assumption will **not** ultimately affect our final conclusions, but will make the math a whole lot **easier**.

So, we can state that the VCO is mathematically described as:

$$\theta_{vco}(s) = \frac{K_v}{s} v_C(s)$$