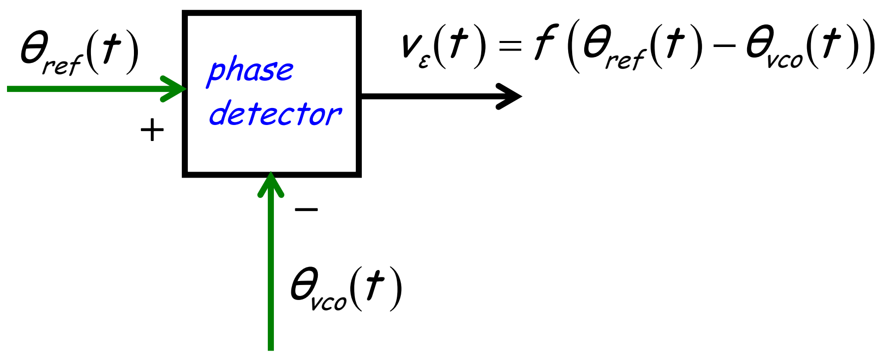
Phase Detectors

Recall that one of the three fundamental elements of a PLL is the phase detector:



The phase detector must create a voltage signal $v_\varepsilon(t)$ (the error voltage) that provides and indication of the difference between the two fundamental phase functions $\theta_{ref}(t)$ and $\theta_{vco}(t)$.

Recall that when the loop is locked, these two phase functions are ideally equal to each other, so that their difference is zero (i.e., $\theta_{ref}(t) - \theta_{vco}(t) = 0$ when locked).

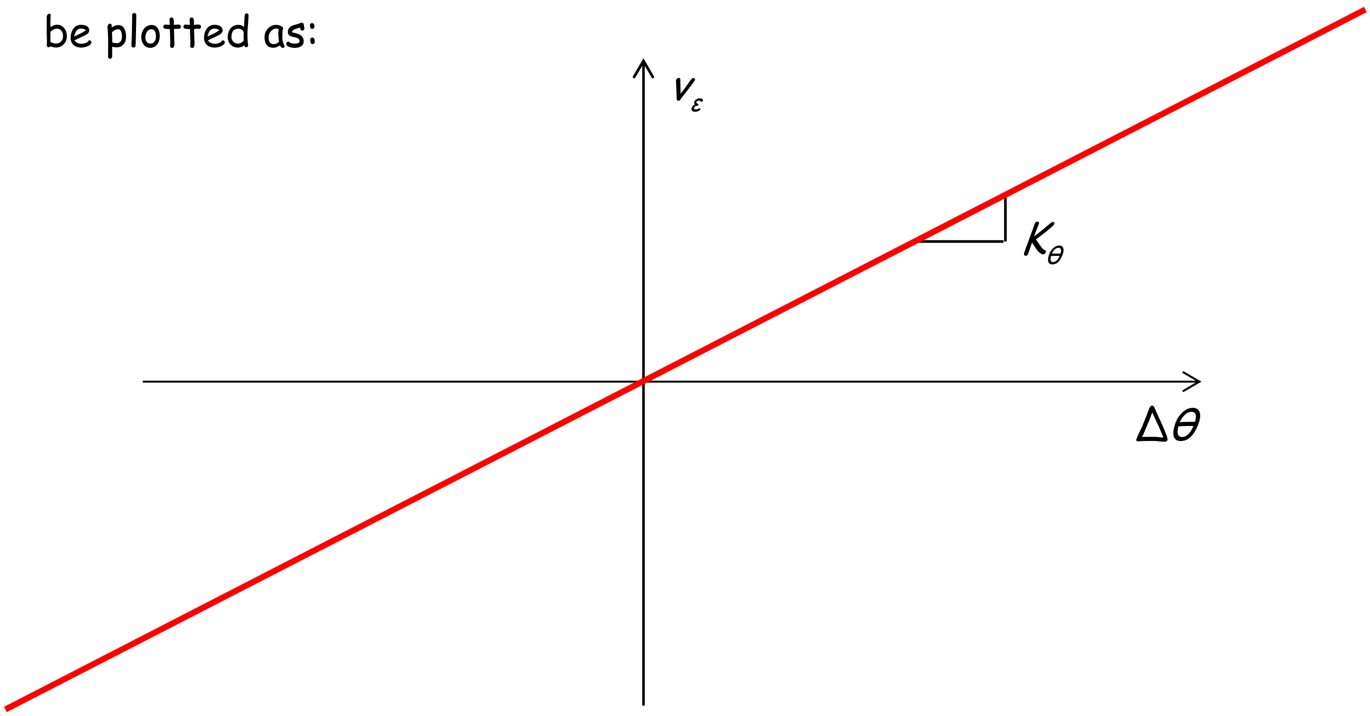
Ideally, the relationship between phase difference $\Delta\theta = \theta_{ref} - \theta_{vco}$ and the error voltage v_ε is simple and straight forward:

$$\begin{aligned} v_\varepsilon &= K_\theta \left(\theta_{ref} - \theta_{vco}\right) \\ &= K_\theta \, \Delta\theta \end{aligned}$$

In other words, the error voltage is directly proportional to the phase difference, with a proportionality constant of K_θ.

It should be evident to **you** that K_θ has units of **volts/radian** !

From the expression above, it appears that the "transfer function" between the phase error and the error voltage can be plotted as:



The reality however, is much different.

Remember, the phase detector only indirectly "knows" the phase functions $\theta_{ref}(t)$ and $\theta_{vco}(t)$. The inputs to the phase detector will of course be a voltage signal of some type (e.g., $v_{vco}(t) = \cos\left[\theta_{vco}(t)\right]$.

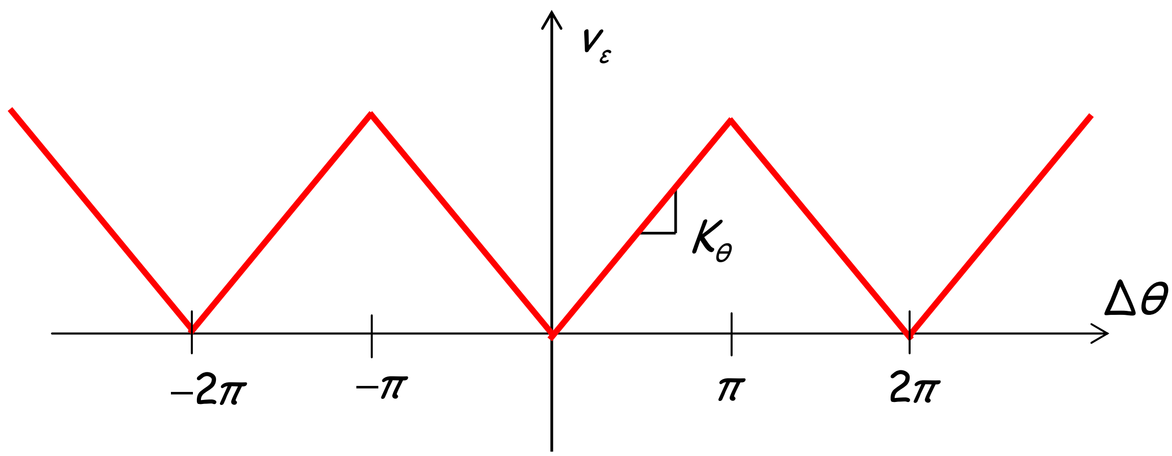
The problem here of course is the phase ambiguity :

$$\cos\left[\theta_{vco}(t)\right] = \cos\left[\theta_{vco}(t) + 2\pi\right] = \cos\left[\theta_{vco}(t) + 4\pi\right] = \cos\left[\theta_{vco}(t) + n2\pi\right]$$

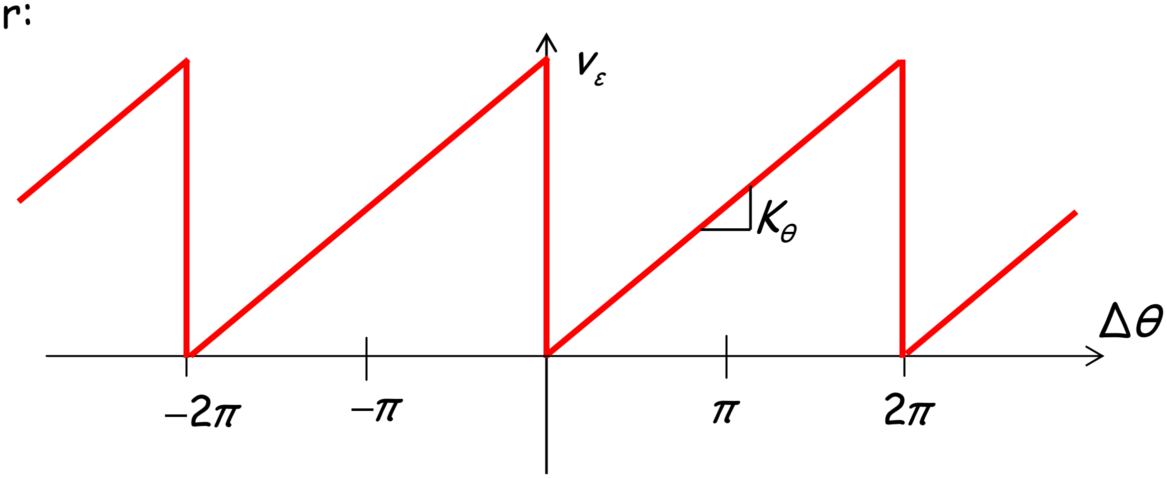
Even though we **know** that:

$$\theta_{vco}(t) \neq \theta_{vco}(t) + 2\pi \neq \theta_{vco}(t) + 4\pi \neq \theta_{vco}(t) + n2\pi$$

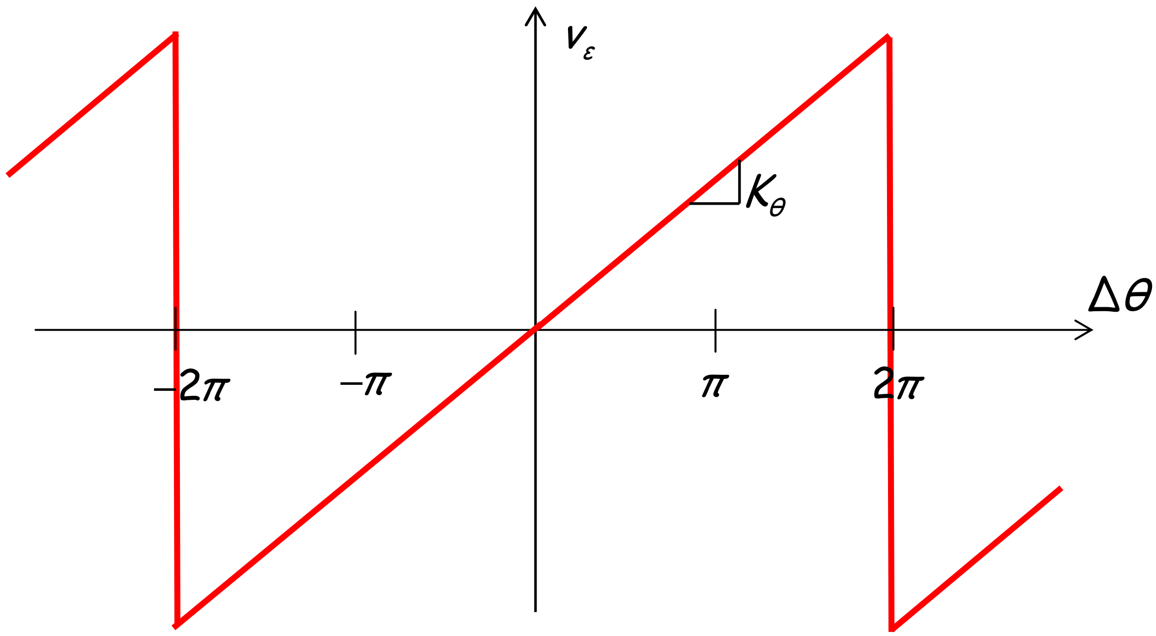
As a result, we find that real phase detectors exhibit transfer functions more like:



or:



or:



The first of these is known as a π-phase detector, the second a 2π-phase detector, and the last a 4π-phase detector.

Q: *But how are phase detectors made? How do they operate?*

A: The performance and operation of a phase detector depends completely on how the phase function manifests itself in the form of voltage signal.

In other words, a phase detector for the signals:

$$v_{ref}(t) = \cos\left[\theta_{ref}(t)\right] \qquad \text{and} \qquad v_{vco}(t) = \cos\left[\theta_{vco}(t)\right]$$

must be very different from the phase detector where:

$$v_{ref}(t) = rect\left[\theta_{ref}(t)\right] \qquad \text{and} \qquad v_{vco}(t) = rect\left[\theta_{vco}(t)\right]$$

and likewise different still from the phase detector where:

$$v_{ref}(t) = pulse\left[\theta_{ref}(t)\right] \qquad \text{and} \qquad v_{vco}(t) = pulse\left[\theta_{vco}(t)\right].$$

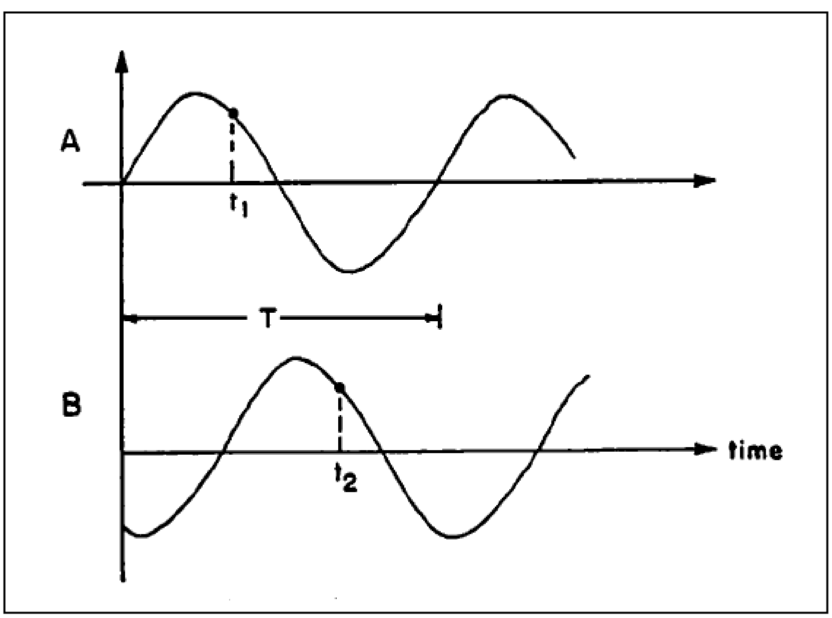
We will consider phase detectors for the two digital signals of the form $v(t) = rect\left[\theta(t)\right]$ and $v(t) = pulse\left[\theta(t)\right]$.

The first thing to know about phase detectors for digital signals is that they are actually **delay** detectors!

To see why, consider two sinusoidal signals whose phase difference is a simple **constant** θ_c. In other words:

$$\Delta\theta \equiv \theta_A(t) - \theta_B(t) = \theta_C$$

Note that the two sinusoidal signals must have identical frequency (do you see why?), and are dissimilar only because of a "phase shift" θ_c:



Note that this "phase shift" is really just a time shift of one signal with respect to the other. Note also that the value of voltage $v_A(t)$ at time $t = t_1$ is the same value as voltage $v_B(t)$ at time $t = t_2$.

The signal $v_B(t)$ is thus a delayed version of signal $v_A(t)$:

$$v_B(t) = v_A(t - \tau)$$

Where delay τ is clearly:

$$\tau = t_2 - t_1$$

Now, the two signals have the same period T (after all, they have the same frequency), and so the ratio τ/T provides a measure of the delay in terms of one period (e.g. if $\tau/T = 0.25$, then the signal is delayed a quarter of one period).

Remember though, that one period likewise represents the time required for the phase function to change by a value of 2π radians! I.E.,:

$$\theta(t + T) - \theta(t) = 2\pi$$

Thus, a delay of $0.25\,T$ corresponds to a phase shift of $0.25(2\pi) = \pi/2$!

More generally, we can conclude that if:

$$v_B(t) = v_A(t - \tau)$$

we can conclude that:

$$\Delta\theta \equiv \theta_A(t) - \theta_B(t) = 2\pi\left(\frac{\tau}{T}\right)$$

The ratio τ/T provides a direct measure of the phase difference! It is this value (τ/T) that most digital signal phase detectors attempt to provide us.