Circuits 212 Lab

Lab #5
Steady State Power Analysis

Steady state power analysis refers to the power analysis of circuits that have one or more sinusoid stimuli. This lab covers the concepts of RMS voltage, maximum power transfer, and complex power.

Labs will be performed in groups of two or three students. Each person will turn in his or her own copy of the required work for the lab.

For this lab, one person from your group will need to check out the following from the EECS Shop.

- A probe kit with a breadboard (metal toolbox)

Parts List

- 1 – 250 [Ω] Resistor (use the Decade Resistance Box)
- 1 – 0.1 [μF] Capacitor
- 1 – 100 [mH] Inductor

Definitions

Common Ground – Some instruments use the common ground as their reference ground. Common ground is sometimes referred to as chassis ground and is the ground supplied by the power company (round pin on the three pin power plug). A DSO (Digital Storage Oscilloscope) uses a common ground. The potential of ground is fixed for such devices.

Floating Ground – Some instruments use a floating ground. The ground is only a reference to the output or input voltage. Its absolute value can float and really should be fixed at one potential by an external source. Sometimes this external source is common ground, but it can be any voltage source within the limitations specified by the equipment. The FG (function generator) and voltage supply have floating grounds.

Crest Factor – The 0-to-Peak value of the signal divided by the RMS value of a signal.

$$Crest\ Factor\ = \frac{V_{0-PK}}{V_{RMS}} = \frac{I_{0-PK}}{I_{RMS}} \qquad Crest\ Factor\ of\ Sinusoid\ = \sqrt{2}$$

1:1, 10:1, and 100:1 probes – These probes do not multiply the input signal, but rather they act as *a voltage divider and weaken* the signal by the amount specified. Therefore, a 10:1 probe actually attenuates the input into the oscilloscope by ten. The advantage is that they affect your circuit less and have higher bandwidth (can operate accurately at higher frequencies). Their disadvantage comes from the fact that the input signal is divided. Because of this voltage divider effect, you should only use the special probes with the oscilloscopes and never with any of the other equipment. Note that an ordinary wire does nothing to the signal and is considered a "1:1" probe. Make sure to set the oscilloscope channel settings to be matching the probe ratio connected to it; or otherwise your voltage readings will be multiplied by an incorrect probe factor.

Experiment 1: Root Mean Square or Effective Voltage

1. Connect the output port of your Function Generator (FG) to Channel 1 of the Oscilloscope (DSO). Use a **BNC-to-BNC cable** to connect the two instruments, as shown in Fig.1 below.

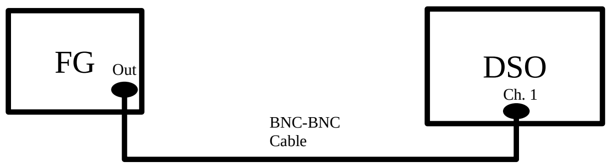

Figure 1

2. For each of the following waveforms:
   a) sinusoid with f = 100 [kHz], V_{PP} = 2V, zero DC offset
   b) sinusoid with f = 10 [kHz], V_{PP} = 2V, zero DC offset
   c) sinusoid with f = 10 [kHz], V_{PP} = 2V, 1V DC offset
   d) square wave with f = 10 [kHz], V_{PP} = 2V, zero DC offset, 50% duty cycle
   e) square wave with f = 10 [kHz], V_{PP} = 2V, zero DC offset, 20% duty cycle
   f) triangle wave with f = 10 [kHz], V_{PP} = 2V, zero DC offset
   g) ramp wave with f = 10 [kHz], V_{PP} = 2V, zero DC offset

   You will be measuring V_{RMS} and V_{PK-PK} using the oscilloscope. First, press the "Default Setup" button to return the oscilloscope to its default settings. Next, configure the oscilloscope inputs for BW Limiting to enable a 20 MHz low-pass filter on the input channels (This eliminates the high frequency noise and cleans up the waveforms). Press the "1" key and select "BW Limit" using the softkeys. To increase the accuracy of your results, use the oscilloscope's averaging function. You can turn on the averaging function by pressing the "Acquire" button and then choosing "Average" using the softkeys. Then choose "16" averaging points from the menus. Don't take any measurements until the measurements are holding completely still and the waveforms are all clearly in view. Measure the V_{RMS} (using the "DC RMS – N Cycles" measurement) and V_{PK-PK} (using the "Peak – Peak" measurement) with the oscilloscope.

Experiment 1 Lab Report Deliverables

- For the report, use your measured values to calculate the Crest Factor for measurements a) – g) (except for measurement set c), and compare these calculated values to the theoretical values.
- Using Equation 1 explain why the frequency has no effect on the RMS value.
- Using Equation 1 explain why the duty cycle of the square wave has no effect on the RMS value.
- Explain how the DC offset affects the power of the signal.

  Hint: To determine how the DC offset affects the power of the signal keep in mind that a DC signal (frequency = 0) is orthogonal to the AC part of the signal (frequency = 10 [kHz]). This means that the power of each signal can be superimposed to find the total power. Look at Equation 1 which defines RMS along with Equations 2 and 3.

$$\boldsymbol{V_{rms}} = \sqrt{\frac{1}{T}\int_0^T [\boldsymbol{v(t)}]^2\, \boldsymbol{dt}} \qquad \text{Equation 1}$$

$$\boldsymbol{v(t)} = \boldsymbol{DC_{offset}} + (\boldsymbol{V_{0-pk}}) * \boldsymbol{cos}(\boldsymbol{w} * \boldsymbol{t} + \boldsymbol{\varphi}) \qquad \text{Equation 2}$$

$$\boldsymbol{P} = \frac{\boldsymbol{V_{rms}^2}}{\boldsymbol{R}} \qquad \text{Equation 3}$$

Experiment 2: Maximum Power Transfer Theorem

Maximum Power Transfer: When a source having a complex output impedance of Z**s** = **r**+j**x** is connected to a load with a complex impedance of $\mathbf{Z_L}$=**R**+j**X**, the maximum power transferred to (or dissipated in) the load is achieved when $\boldsymbol{Z_s} = \boldsymbol{Z_L^*}$, where * denotes the complex conjugate. Therefore, the condition of maximum power transfer in the circuit shown in Fig. 2 will be achieved if

$$\mathbf{r} = \mathbf{R}, \quad \text{and} \quad \mathbf{x} = -\mathbf{X},$$

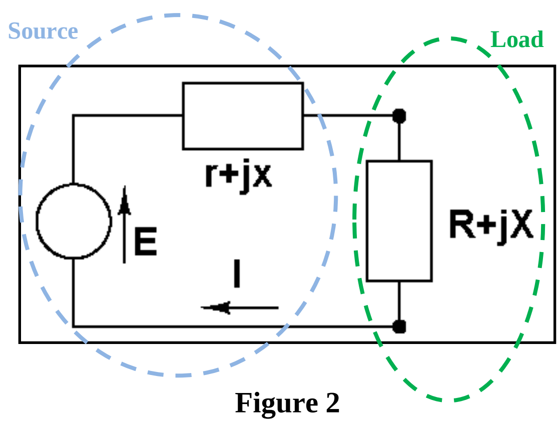

Figure 2

Differential Voltage Measurement: The voltages across the inductor, capacitor and resistor shown in Fig. 3 must be measured carefully. You may NOT simply measure the voltage at points A, B and C and just subtract them to get the corresponding values! This is because you are subtracting complex phasors which have a magnitude and phase angle associated with them and as a result the magnitudes may not simply be added (shown in Equations 4 and 5) and subtracted. For differential voltage measurements the Channel 1 probe is attached to one side of the component being measured across and the Channel 2 probe is attached to the other side. The oscilloscope (when it is configured properly) subtracts the two waveforms as shown in Equations 6 and 7.

$$|\bar{\boldsymbol{V}}_{\boldsymbol{Source}}| \neq |\bar{\boldsymbol{V}}_{\boldsymbol{L}}| + |\bar{\boldsymbol{V}}_{\boldsymbol{C}}| + |\bar{\boldsymbol{V}}_{\boldsymbol{R}}| \quad \text{Equation 4}$$

$$\bar{\boldsymbol{V}}_{\boldsymbol{Source}} = \bar{\boldsymbol{V}}_{\boldsymbol{L}} + \bar{\boldsymbol{V}}_{\boldsymbol{C}} + \bar{\boldsymbol{V}}_{\boldsymbol{R}} \quad \text{Equation 5}$$

$$\bar{V}_{Differential} = \bar{V}_{Channel1} - \bar{V}_{Channel2} \quad \text{Equation 6}$$

$$v(t)_{Differential} = v(t)_{Channel1} - v(t)_{Channel2} \quad \text{Equation 7}$$

Experiment Procedure:

1. Connect the Function Generator directly to the Oscilloscope with a BNC-to-BNC cable. Set the Function Generator output voltage to **1** V_{RMS} **(NOT Peak-to-Peak!)**. Measure the RMS voltage at 1 kHz. This is the Function Generator's "Open Circuit Voltage" or "Thevenin's Equivalent Voltage". Check that the Oscilloscope measures the Function Generator output voltage to be 1 V_{RMS}.
2. Measure the actual value of each component in Fig. 3 (R_Load, C_Load, and L) including the parasitic resistance. The parasitic resistance of the inductor may be measured with the DMM attached to your bench.
3. Setup the circuit in Fig. 3 exactly as shown. Do not change the order of the components. You will need a 100 [mH] inductor, 250 [Ω] resistor (use the Decade Resistance Box), and a 0.1 [μF] capacitor. The inductor is associated with the *theoretical source* for this experiment, but you still must place a physical 100mH inductor in the circuit since the Inductor is not actually part of the Function Generator. Do not place a 200 Ω resistor in your circuit OR **whatever the parasitic resistance works out to be**, the parasitic resistance is there to remind you that the inductor has a parasitic resistance. Do not place a 50 Ω resistor in your circuit, this represents the Function Generator output impedance.

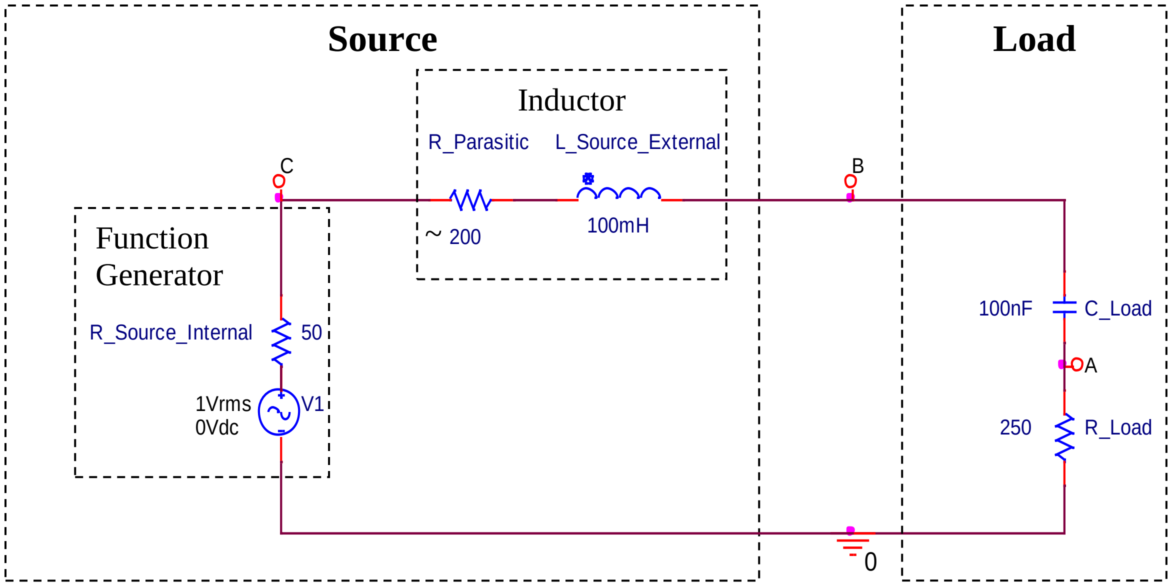

Figure 3

4. In this experiment you will be measuring the magnitude of the RMS voltages across the inductor, capacitor and resistor for each of the frequencies listed in the Table 1 below. You will measure these values using the "DC RMS-N Cycles" measurement on the oscilloscope. You will need to configure your oscilloscope probes to differentially measure the voltage across the inductor and the capacitor.
5. Configure the oscilloscope for differential voltage measurements.
   a. Connect a 10:1 (non-differential) probe to both "Channel 1" and "Channel 2" on the oscilloscope.
   b. Make certain each input channel is set to 10:1 input attenuation.
   c. Press the "Default Setup" key on the oscilloscope.
   d. Configure the inputs for Bandwidth (BW) Limiting. This puts a 20 MHz low-pass filter on the input signals to eliminate high frequency noise.
      i. Press the "1" key on the oscilloscope.
      ii. Select "BW Limit" using the softkeys.
      iii. Press the "2" key on the oscilloscope.
      iv. Select "BW Limit" using the softkeys.
   e. Configure the oscilloscope probes to measure the voltage differentially
      i. Press the "Math" key on the oscilloscope.
      ii. Select the "-" operator with the softkeys.
      iii. Press the "Meas" key on the oscilloscope.
      iv. Select the "Math: f(t)" as the source with the softkeys. A differenced waveform will appear on the oscilloscope screen. The differenced waveform's Volts/Div and vertical position may be adjusted using the knobs in the "Misc. Function Control Section" shown in Fig. 4.

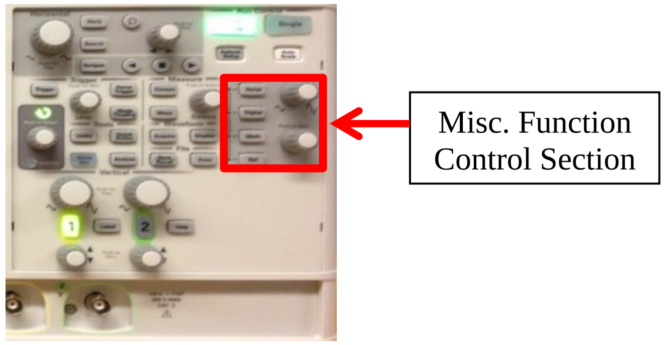

Figure 4

v. Select "DC RMS – N Cycles" as your measurement type and select "Add Measurement".

6. Measure the magnitude of the voltages across the inductor, capacitor and the resistor for the frequencies chosen in Table 1. For all of these measurements adjust the Volts/Div until there is no DC offset to the waveforms and until you can clearly see each sinusoidal waveform (if the sinusoidal waveforms appear distorted you must adjust the oscilloscope until the waveforms do not appear distorted). Note that the maximum power transfer (power of resistor) will happen at the frequency that sets $X_L = X_C$; and this is defined as the resonance frequency for this circuit. (when $X_L = X_C$ they will cancel each other because they have different signs, and, therefore, the current in the circuit will be maximized, and the power dissipated in the load resistor P_R is at its maximum).
   a. **Capacitor and Resistor voltages:** Attach the Channel 1 probe to point "B". Attach the Channel 2 probe to point "A". Use differential voltage measurement (the Math readings) and measure the magnitude of the rms voltage across the capacitor for every frequency chosen in Table 1. The rms readings of Channl 2 will represent the voltage of the resistor.
   b. **Inductor voltage:** Attach the Channel 1 probe to point "C". Attach the Channel 2 probe to point "B". Use differential voltage measurement and measure the magnitude of the rms voltage across the inductor for every frequency chosen in Table 1.

Table 1

Frequency (Hz)	$\lvert V_L(rms)\rvert$	$\lvert V_C(rms)\rvert$	$\lvert V_R(rms)\rvert$	$\lvert Q_L\rvert$	$\lvert Q_C\rvert$	P_R
Calculate the resonant frequency for the circuit. Vary the frequency between 250 Hz and 10 KHz. Choose your own step sizes and make certain to take more readings around the calculated resonant frequency for the circuit.						

Experiment 2 Lab Report Deliverables

- Calculate the load power P_R (also known as real power or average power) for each of the frequencies in Table 1.
- Calculate the magnitude of the reactive power (also known as the imaginary power or the quadrature power) for the inductor $|Q_L|$ and the capacitor $|Q_C|$ for each of the frequencies in Table 1 using Equations 8 and 9.
- Plot the load power P_R (on a linear scale) versus frequency (on a log scale) and determine the frequency at which maximum power transfer occurs.
- Plot the reactive power in the inductor (on a linear scale) versus frequency (on a log scale). On the same graph plot the reactive power in the capacitor (on a linear scale) versus frequency (on a log scale). Determine the frequency at which the two curves intersect. How close is this frequency to the resonant frequency?

$$|Q_L| \approx \left|\frac{V_L(rms)^2}{X_L}\right| \qquad \text{Equation 8}$$

$$|Q_C| = \left|\frac{V_C(rms)^2}{X_C}\right| \qquad \text{Equation 9}$$