EECS 801: Homework 2

Due: Tuesday, February 16, 2016 (At 6pm)

Questions:

1. (20 points) Consider solving the two-dimensional Poisson problem given by

$$-(u_{xx} + u_{yy}) = 1 \in \Omega = (0,1)^2,$$
$$u|_{\partial\Omega} = 0.$$

   Derive the variational problem for the above PDE problem. Write down the corresponding linear system obtained from discretization of the variational problem.

   Then implement the finite element method with the standard basis functions and solve the linear system on a structured triangular mesh with $h = k = 0.1$.

2. (20 points) Suppose now you wish to solve the following two-dimensional Poisson PDE problem given by:

$$-(u_{xx} + u_{yy}) = 1 \in \Omega = (0,1)^2,$$

$$\frac{\partial u(x)}{\partial \mathbf{n}} = 0, \forall x \in \partial\Omega$$

   on the same structured triangular mesh as in Question 1.

   Compared to Question 1, what changes in the mathematical derivation of the variational problem and the corresponding linear system? What changes are there in the linear solution procedure? **Note: You do NOT need to implement anything for this question.**

3. (20 points) Suppose now you wish to solve the same PDE problem as in Question 1, i.e.,

$$-(u_{xx} + u_{yy}) = 1 \in \Omega = (0,1)^2,$$
$$u|_{\partial\Omega} = 0.$$

   However, this time, you wish to solve it on an unstructured triangular mesh.

   Compared to Question 1, what changes in the mathematical derivation of the variational problem and the corresponding linear system? What changes are there in the linear solution procedure? **Note: You do NOT need to implement anything for this question.**

4. (20 points)

   Consider solving the two-dimensional PDE problem given by:

$$-\Delta u + \beta_1 \frac{\partial u}{\partial x_1} + \beta_2 \frac{\partial u}{\partial x_2} + u = f \in \Omega = (0,1)^2,$$
$$u|_{\partial\Omega} = 0,$$

   where the β_i are constants.

   Derive the corresponding variational problem. Is the matrix in the corresponding linear system symmetric or nonsymmetric? Why? **Note: You do NOT need to obtain the corresponding linear system or implement anything for this question.**