EECS 739: Homework 4

Due: Thursday, April 21, 2016 (At 4pm)
Submit to: EECS Office (2001 Eaton Hall)

Reminder: Please review the homework policies on the syllabus before working on the assignment.

Questions:

1. (20 points) The Hilbert matrix is a matrix whose elements are given by $H_{ij} = 1/(i+j-1)$. Consider solving linear systems of the form $Hx = b$, where $b = (1, 1, \cdots, 1)^T$.

   (a) For $n = 5$, show that H of size $n \times n$ is symmetric positive definite. (Note that H is symmetric positive definite for any positive integer value of n.)

   (b) Compute the condition number of H for $n = 5, 8, 12, 20$. Use the matrix infinity-norm for your calculations. Summarize your answers in a table.

   (c) Comment on how you would expect the conjugate gradient method to perform when used to solve the above linear system based on your answer to (b).

   Hint: The inverse of H is given by

$$(H^{-1})ij = (-1)^{i+j}(i+j-1)\left(\begin{array}{c} n+i-1 \\ n-j \end{array}\right)\left(\begin{array}{c} n+j-1 \\ n-i \end{array}\right)\left(\begin{array}{c} i+j-2 \\ i-1 \end{array}\right)^2.$$

   Note: Do NOT implement the conjugate gradient method to address (c).

2. (20 points) Develop and implement in MPI and C/C++ the parallel red-black Gauss-Seidel algorithm as described in Section 7.2.4 (starting on p. 383) in the textbook by Karniadakis and Kirby.

   Apply your algorithm to numerically solve the following partial differential equation in parallel using a finite-difference scheme:

$$-\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = \left(x^2 + y^2\right) e^{xy}, 0 < x < 1, 0 < y < 1;$$

   with $u(0, y) = u(1, y) = u(x, 0) = u(x, 1) = 0$ on the boundary. Use centered finite differences.

   (a) Solve the above PDE with $m, n = 10, 50$, and 100 mesh points. Demonstrate the convergence of your algorithm by plotting the norm of the residual vector vs. iteration number for each mesh; use the two-norm for your calculations. Use a single figure for your plots.

   (b) Perform a strong scaling experiment on up to 16 nodes on the finest mesh. Generate a bar plot which illustrates your strong scaling results.

   You must run your code on the Slurm cluster.

   Submit the following items electronically via e-mail to shontz@ku.edu: (1) your C/C++ code and (2) your PBS script.

   In addition, submit the following items in hard copy: (1) your MPI and C/C++ code, (2) your PBS script, (3) output demonstrating your code gives correct results

(as indicated above), (4) your convergence plot from (a), and (5) your bar plot from (b).

Both the electronic and hard copy submissions are required in order to receive credit for this question.