EECS 865: Wireless Communication Systems
Spring 2015

Assignment 1 (Due on Jan. 29th 2015)

Reading Assignment:

1. Chapter 1 of T&V.
2. Chapter 2 of T&V.

Problems:

1. Let the random variable \( N \) be the number of stars in a region of space of volume \( V \). Assume that \( N \) is a Poisson random variable with probability mass function (pmf)

\[
p_N(n) = \exp\{-\rho V\} \frac{(\rho V)^n}{n!}, \quad n = 0, 1, 2, \ldots,
\]

where \( \rho \) is the “density” of the stars in space. We choose an arbitrary point in space and define the random variable \( X \) to be the distance from the chosen point to the nearest star. Find the probability density function (pdf) of \( X \) in terms of \( \rho \).

2. (Gallager) Consider the electric field in (2.4).
   
   (a) It has been derived under the assumption that the motion is in the direction of the line-of-sight from sending antenna to receive antenna. Find the electric field assuming that \( \phi \) is the angle between the line-of-sight and the direction of motion of the receiver. Assume that the range of time of interest is small enough so that changes in \( (\theta, \psi) \) can be ignored.

   (b) Explain why, and under what conditions, it is a reasonable approximation to ignore the change in \( (\theta, \psi) \) over small intervals of time.

3. (Gallager) Equation (2.13) was derived under the assumption that \( r(t) \approx d \). Derive an expression for the received waveform for general \( r(t) \). Break the first term in (2.11) into two terms, one with the same numerator but the denominator \( 2d - r_0 - vt \) and the other with the remainder. Interpret your results.

4. In the two-path example in Section 2.1.3 and 2.1.4, the wall is on the right side of the receiver so that the reflected wave and the directed wave travel in opposite directions. Suppose now that the reflecting wall is on the left side of transmitter. Redo the analysis. What is the nature of the multipath fading, both over time and over frequency? Explain any similarity or difference with the case considered in Sections 2.1.3 and 2.1.4.

5. Consider the propagation model in Section 2.1.5 where there is a reflected path from the ground plane.
(a) Let $r_1$ be the length of the direct path in Figure 2.6. Let $r_2$ be the length of the reflected path (summing the path length from the transmitter to the ground plane and the path length from the ground plane to the receiver). Show that $r_2 - r_1$ is asymptotically equal to $b/r$ and find the value of the constant $b$. Hint: Recall that for $x$ small, $\sqrt{1 + x} \approx 1 + x/2$ in the sense that $(\sqrt{1 + x} - 1)/x \to 1/2$ as $x \to 0$.

(b) Assume that the received waveform at the receive antenna is given by

$$E_r(f, t) = \frac{\alpha \cos 2\pi[ft - fr_1/c]}{r_1} - \frac{\alpha \cos 2\pi[ft - fr_2/c]}{r_2}.$$ 

Approximate the denominator $r_2$ by $r_1$ in the above equation and show that $E_r \approx \beta/r^2$ for $r^1$ much smaller than $c/f$. Find the value of $\beta$.

(c) Explain why this asymptotic expression remains valid without first approximating the denominator $r_2$ in the above equation by $r_1$.

6. Verify (2.39) and (2.40).