

Graphs:

A graph $G=(V,E)$ is two tuples with V and E where

V is a set called vertices; $E \subseteq V \times V$ called edges

if E is symmetric ($(u,v) \in E$ implies $(v,u) \in E$), G is *undirected*
otherwise it is *directed*

An edge is also called an arc; a vertex is also called a node

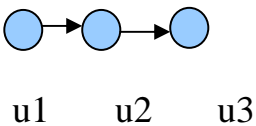
A path is a sequence of nodes $V_1, V_2, V_3, \dots, V_n$ such that
 $(V_i, V_{i+1}) \in E$ for $i \in [1, n-1]$. The *length* of the path is $n-1$

if $n=2$ and $V_2=V_1$, we call it a loop; a *simple path* is a path with no duplicated nodes; a *simple graph* is a graph with no loop

A cycle is a path with an additional edge from V_n to V_1

A directed graph is acyclic if it contains no cycles

A directed acyclic graph is called a DAG



Is $u_1 u_2 u_3$ a path?

Yes

Is it a simple path?

Yes

Is the graph a directed graph?

Yes

Is this an undirected graph?

No

Is this a DAG?

Yes

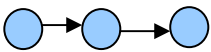
A graph is **connected** if for all $(u,v) \in V \times V$ there exists a path $V_1=u$ and $V_n=v$.

A directed graph is **strongly connected** if it is connected

A underlying graph G' of a directed graph $G=(V,E)$ is $G'=(V',E')$ such that
 $V'=V$

for all $(u,v) \in E \Rightarrow (u,v) \in E', (v,u) \in E'$

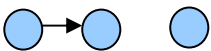
A directed graph is **weakly connected** if the underlying graph is connected



u1 u2 u3

Is the above graph connected? No

Is the above graph weakly connected? Yes



u1 u2 u3

Is the above graph connected? No

Clique: Undirected graph where $E= V \times V - \{(u,v) | u \neq v\}$