

Recursive definition for AVL tree node height:

$H[T] = -1$ if T is an empty tree

$H[T] = 0$ if T is a tree with a single node

$H[T] = \max (H[T \rightarrow \text{left}], H[T \rightarrow \text{right}]) + 1$

Notation:

$H[T]$: the height of a tree with root node T

$T \rightarrow \text{left}$: the left child of the node T

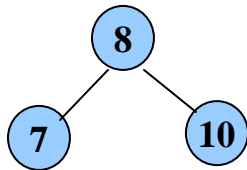
$T \rightarrow \text{right}$: the right child of the node T

Further Definitions:

The *height* of a node T is the height of the subtree rooted at the node T .

The *depth* of a node T is the height of the full tree minus the height of the subtree rooted at the node T

Example:



$$H[7] = 0$$

$$H[10] = 0$$

$$H[8] = \max (H[7], H[10]) + 1 = 1;$$

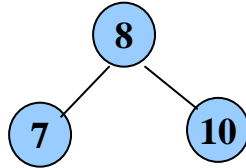
$$\text{Depth of node 7 is } H[8] - H[7] = 1 - 0 = 1;$$

$$\text{Depth of node 10 is } H[8] - H[10] = 1 - 0 = 1;$$

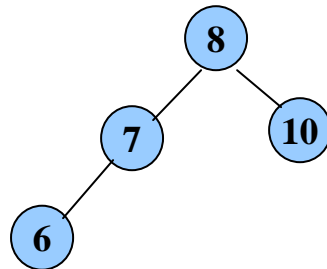
$$\text{Depth of node 8 is } H[8] - H[8] = 1 - 1 = 0;$$

AVL tree insertion single rotation.

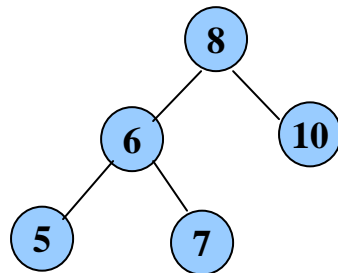
Insert 7:



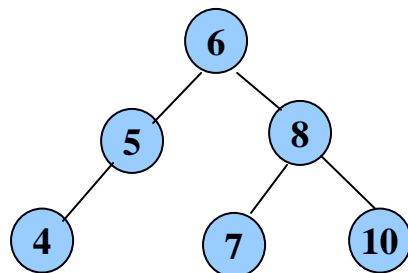
Insert 6



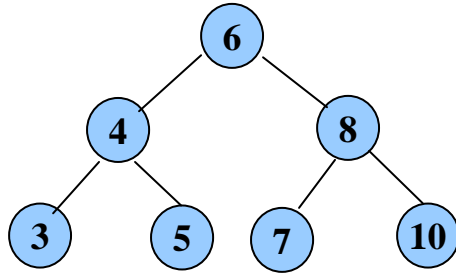
Insert 5:



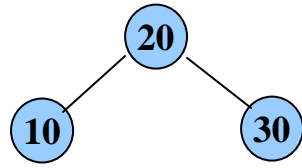
Insert 4:



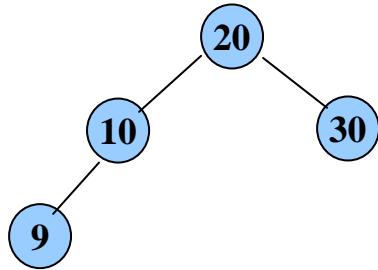
Insert 3:



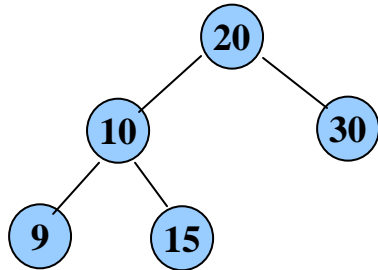
AVL tree insertion double rotation.



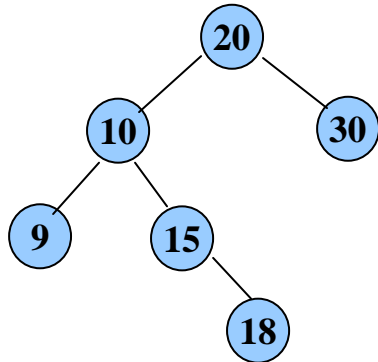
Insert 9:



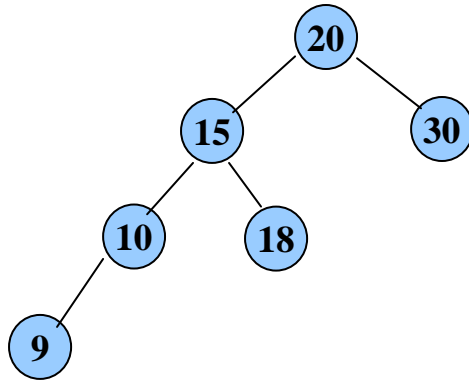
Insert 15:



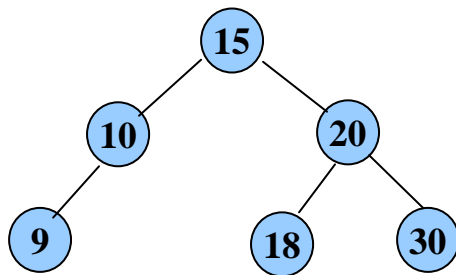
Insert 18:



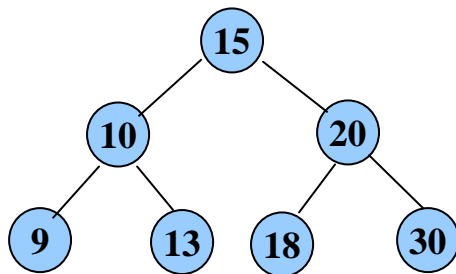
After left rotation at node 10:



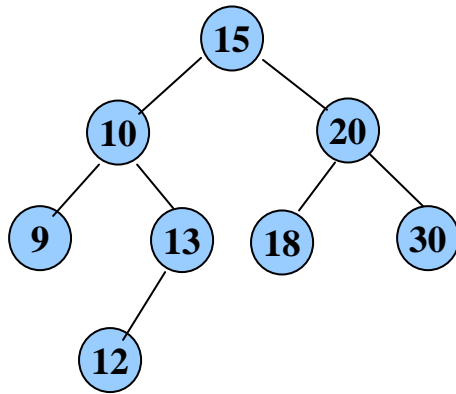
After right rotation at node 20 (root):



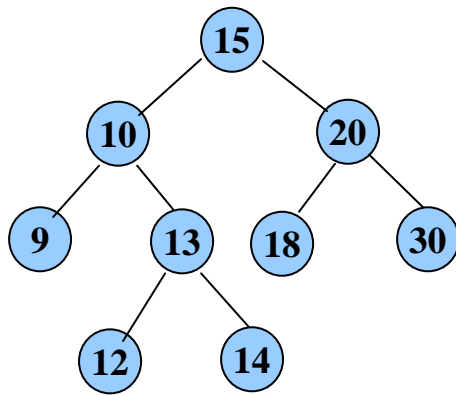
Insert 13



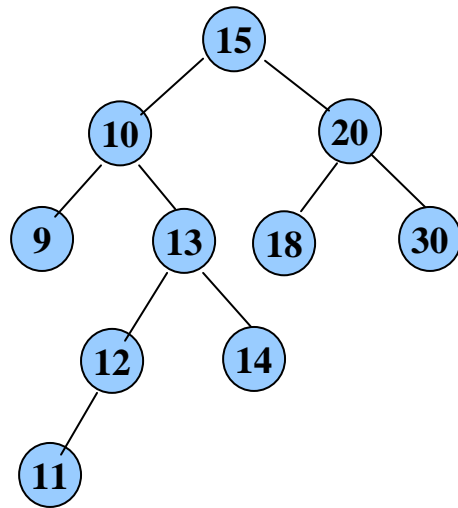
Insert 12



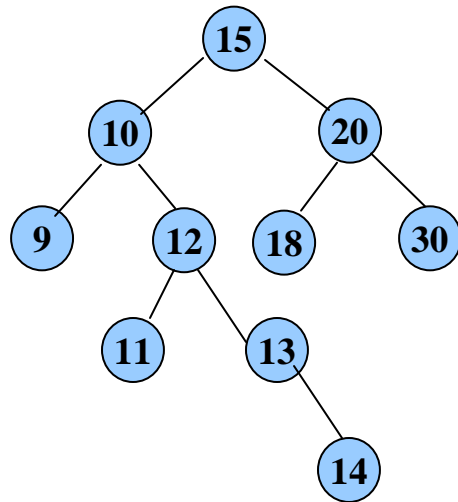
Insert 14



Insert 11:



After right rotation at node 13:



After left rotation at node 10(root):

