For the given decision table, determine a set of rules by

(a) the LEM1 algorithm

Solution:

First we will work with $D_+$. We start with

$$\{\text{size, color, feel, temp}\}^* = \{(1), (2), (3), (4), (5), (6), (7), (8)\} \subseteq \{D_+\}$$

We will now start from the left and drop any possible attributes. We can't drop size because

$$\{\text{color, feel, temp}\}^* = \{(1), (2), (3), (4, 5, 8), (6), (7)\} \not\subseteq \{D_+\}.$$

We can drop color, though, since

$$\{\text{size, feel, temp}\}^* = \{(1), (2, 8), (3), (4, 5), (6), (7)\} \subseteq \{D_+\}^*.$$

Now, with $\{\text{size, feel, temp}\}$, we can't drop feel since

$$\{\text{size, temp}\}^* = \{(1, 7), (2, 8), (3, 4, 5), (6)\} \not\subseteq \{D_+\}^*.$$

We also can't drop temp since

$$\{\text{size, feel}\}^* = \{(1), (2, 7, 8), (3, 6), (4, 5)\} \not\subseteq \{D_+\}^*.$$
That means our global covering for $D_+$ is $\{\text{size, feel, temp}\}$.

Moving on to $D_-$. We again start with

$$\{\text{size, color, feel, temp}\}^* = \{(1), (2), (3), (4), (5), (6), (7), (8)\} \leq \{D_-\}.$$  

Starting from the left, we try to drop size. We can drop size since

$$\{\text{color, feel, temp}\}^* = \{(1), (2), (3), (4, 5, 8), (6), (7)\} \leq \{D_-\}^*.$$  

With $\{\text{color, feel, temp}\}$, we can't drop color since

$$\{\text{feel, temp}\}^* = \{(1, 6), (2, 4, 5, 7), (3), (8)\} \not\leq \{D_-\}^*.$$  

Again with $\{\text{color, feel, temp}\}$, we can drop temp since

$$\{\text{color, feel}\}^* = \{(1, 3), (2), (4, 5, 7, 8), (6)\} \leq \{D_-\}^*.$$  

That gives us the global covering $\{\text{color, feel}\}$ for $D_-.$

Moving on to $D_{seo}$. We again start with

$$\{\text{size, color, feel, temp}\}^* = \{(1), (2), (3), (4), (5), (6), (7), (8)\} \leq \{D_-\}.$$  

Starting from the left, we try to drop size. We can't drop size since

$$\{\text{color, feel, temp}\}^* = \{(1), (2), (3), (4, 5, 8), (6), (7)\} \not\leq \{D_{seo}\}^*.$$  

Next we see if we can drop color. We can't drop color because

$$\{\text{size, feel, temp}\}^* = \{(1), (2, 3), (4), (5), (6), (7)\} \not\leq \{D_{seo}\}^*.$$  

Next we see if we can drop feel. We can drop feel since

$$\{\text{size, color, temp}\}^* = \{(1), (2), (3), (4, 5), (6), (7), (8)\} \leq \{D_{seo}\}^*.$$  

Next, with $\{\text{size, color, temp}\}$, we see if we can drop temp. We can drop temp since

$$\{\text{size, color}\}^* = \{(1, 2), (3), (4, 5, 6), (7, 8)\} \leq \{D_+\}^*.$$  

This gives us the global covering $\{\text{size, color}\}$ for $D_{seo}$.

Using $\{\text{size, feel, temp}\}$ for $D_+$, we get the rules

- $(\text{size, big}) \&(\text{feel, soft}) \&(\text{temp, low}) \rightarrow (\text{Attitude, +})$ (can drop temp)
- $(\text{size, med}) \&(\text{feel, soft}) \&(\text{temp, high}) \rightarrow (\text{Attitude, +})$ (can drop size)
- $(\text{size, med}) \&(\text{feel, hard}) \&(\text{temp, high}) \rightarrow (\text{Attitude, +})$ (can drop feel)
After dropping the unnecessary attributes, we have
- \( (\text{size}, \text{big}) \land (\text{feel}, \text{soft}) \rightarrow (\text{Attitude}, +) \)
- \( (\text{feel}, \text{soft}) \land (\text{temp}, \text{high}) \rightarrow (\text{Attitude}, +) \)
- \( (\text{size}, \text{med}) \land (\text{temp}, \text{high}) \rightarrow (\text{Attitude}, +) \)

Using \( \{\text{color}, \text{feel}\} \) for \( D_- \), we get the rules
- \( (\text{color}, \text{yellow}) \land (\text{feel}, \text{hard}) \rightarrow (\text{Attitude}, -) \) (can't drop any attributes)
- \( (\text{color}, \text{blue}) \land (\text{feel}, \text{soft}) \rightarrow (\text{Attitude}, -) \) (can't drop any attributes)

Using \( \{\text{size}, \text{color}\} \) for \( D_{\text{soso}} \), we get the rules
- \( (\text{size}, \text{big}) \land (\text{color}, \text{blue}) \rightarrow (\text{Attitude}, \text{soso}) \) (can't drop any attributes)

(b) computing all global coverings for conceptual variables and then using linear dropping condition technique

Solution:

\[
\begin{align*}
\{\text{size}\}^* &= \{\{1,2,7,8\},\{3,4,5,6\}\} \not\subseteq D_+^* \\
\{\text{color}\}^* &= \{\{1,2,3\},\{4,5,6,7,8\}\} \not\subseteq D_+^* \\
\{\text{feel}\}^* &= \{\{1,3,6\},\{2,4,5,7,8\}\} \not\subseteq D_+^* \\
\{\text{temp}\}^* &= \{\{1,6,7\},\{2,3,4,5,8\}\} \not\subseteq D_+^* \\
\end{align*}
\]

So no single-attribute global coverings

\[
\begin{align*}
\{\text{size, color}\}^* &= \{\{1,2\},\{3\},\{4,5,6\},\{7,8\}\} \not\subseteq D_+^* \\
\{\text{size, feel}\}^* &= \{\{1\},\{2,7,8\},\{3,6\},\{4,5\}\} \not\subseteq D_+^* \\
\{\text{size, temp}\}^* &= \{\{1,7\},\{2,8\},\{3,4,5\},\{6\}\} \not\subseteq D_+^* \\
\{\text{color, feel}\}^* &= \{\{1,3\},\{2\},\{4,5,7,8\},\{6\}\} \not\subseteq D_+^* \\
\{\text{color, temp}\}^* &= \{\{1\},\{2,3\},\{4,5,8\},\{6,7\}\} \not\subseteq D_+^* \\
\{\text{feel, temp}\}^* &= \{\{1,6\},\{2,4,5,7\},\{3\},\{8\}\} \not\subseteq D_+^* \\
\end{align*}
\]

So no double-attribute global coverings

\[
\begin{align*}
\{\text{size, color, feel}\}^* &= \{\{1\},\{2\},\{3\},\{4,5\},\{6\},\{7,8\}\} \subseteq D_+^* \\
\{\text{size, color, temp}\}^* &= \{\{1\},\{2\},\{3\},\{4,5\},\{6\},\{7\},\{8\}\} \subseteq D_+^* \\
\{\text{size, feel, temp}\}^* &= \{\{1\},\{2,8\},\{3\},\{4,5\},\{6\},\{7\}\} \subseteq D_+^* \\
\{\text{color, feel, temp}\}^* &= \{\{1\},\{2\},\{3\},\{4,5,8\},\{6\},\{7\}\} \not\subseteq D_+^* \\
\end{align*}
\]

The first three give us triple-attribute global coverings.
For the other contexts, we have

\[
\{\text{size}\}^* \notin \{D_\rightarrow, D_{soso}\} \\
\{\text{color}\}^* \notin \{D_\rightarrow, D_{soso}\} \\
\{\text{feel}\}^* \notin \{D_\rightarrow, D_{soso}\} \\
\{\text{temp}\}^* \notin \{D_\rightarrow, D_{soso}\} \\
\{\text{size, color}\}^* \notin \{D_\rightarrow\} \\
\{\text{size, feel}\}^* \notin \{D_\rightarrow, D_{soso}\} \\
\{\text{size, temp}\}^* \notin \{D_\rightarrow, D_{soso}\} \\
\{\text{color, feel}\}^* \notin \{D_{soso}\} \\
\{\text{color, temp}\}^* \notin \{D_\rightarrow, D_{soso}\} \\
\{\text{feel, temp}\}^* \notin \{D_\rightarrow, D_{soso}\} \\
\{\text{size, feel, temp}\}^* \notin \{D_\rightarrow, D_{soso}\} \\
\{\text{color, feel, temp}\}^* \notin \{D_{soso}\}
\]

So we have the global coverings

\[
\{\text{size, color, feel}\}^*, \{\text{size, color, temp}\}^*, \\
\{\text{size, feel, temp}\}^* \leq \{D_+\}
\]

\[
\{\text{color, feel}\}^*, \{\text{size, color, temp}\}^*, \leq \{D_\rightarrow\}
\]

\[
\{\text{size, color}\}^* \leq \{D_{soso}\}
\]

From here, we can induce rules for each of these coverings. Starting with \(D_+\)

\[
(size, \text{big}) \& (color, \text{yellow}) \& (feel, \text{soft}) \rightarrow (\text{Attitude}, +)
\]

We can drop size, which gives

\[
(color, \text{yellow}) \& (feel, \text{soft}) \rightarrow (\text{Attitude}, +)
\]

Next,

\[
(size, \text{med}) \& (color, \text{yellow}) \& (feel, \text{soft}) \rightarrow (\text{Attitude}, +)
\]

We can only drop size, which gives

\[
(color, \text{yellow}) \& (feel, \text{soft}) \rightarrow (\text{Attitude}, +)
\]

which collapses into the previous rule. Next,

\[
(size, \text{med}) \& (color, \text{blue}) \& (feel, \text{hard}) \rightarrow (\text{Attitude}, +)
\]

We can only drop color, which gives

\[
(size, \text{med}) \& (feel, \text{hard}) \rightarrow (\text{Attitude}, +)
\]
Next, for $D_-$

$$(\text{color, yellow}) \& (\text{feel, hard}) \rightarrow (\text{Attitude, } -)$$

which can’t drop any attributes. Next,

$$(\text{color, blue}) \& (\text{feel, soft}) \rightarrow (\text{Attitude, } -)$$

and again, neither attribute can be dropped. Finally, for $D_{soso}$,

$$(\text{size, big}) \& (\text{color, blue}) \rightarrow (\text{Attitude, soso})$$

Ultimately, we have the following rules:

- $(\text{color, yellow}) \& (\text{feel, soft}) \rightarrow (\text{Attitude, } +)$
- $(\text{size, med}) \& (\text{feel, hard}) \rightarrow (\text{Attitude, } +)$
- $(\text{color, yellow}) \& (\text{feel, hard}) \rightarrow (\text{Attitude, } -)$
- $(\text{color, blue}) \& (\text{feel, soft}) \rightarrow (\text{Attitude, } -)$
- $(\text{size, big}) \& (\text{color, blue}) \rightarrow (\text{Attitude, soso})$
\[ G = \{ (\text{Attitude}, +) \} = B = \{ 1, 3, 4, 5 \} \]

<table>
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<tr>
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<th>( [ (a, u) ] )</th>
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<th>( { 3, 4, 5 } )</th>
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<tr>
<td>(Color, blue)</td>
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<td>{ 4, 5 }</td>
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<tr>
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<tr>
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<tr>
<td>(Temperature, high)</td>
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<td>{ 3, 4, 5 }</td>
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<td>{ 1 }</td>
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</tbody>
</table>

\[ \{ (\text{Size, medium}) \} = \{ 3, 4, 5, 6 \} \nsubseteq \{ 1, 3, 4, 5 \} \]

\[ \{ (\text{Size, medium}) \} \cap \{ (\text{Temperature, high}) \} = \{ 3, 4, 5 \} \nsubseteq \{ 1, 3, 4, 5 \} \]

then we get our first rule: \[ (\text{Size, medium}) \& (\text{Temperature, high}) \rightarrow (\text{Attitude, +}) \]

new \( G = \{ 1 \} \)

\[ \{ (\text{Color, yellow}) \} = \{ 1, 2, 3 \} \nsubseteq \{ 1, 3, 4, 5 \} \]

\[ G = \{ 1, 3 \} \cap \{ 1 \} = \{ 1 \} \]

\[ \{ (\text{Color, yellow}) \} \cap \{ (\text{Feel, soft}) \} = \{ 1, 3 \} \nsubseteq \{ 1, 3, 4, 5 \} \]

Then we get our second rule:

\[ (\text{Color, yellow}) \& (\text{Feel, soft}) \rightarrow (\text{Attitude, +}) \]

new \( G = \emptyset \)

Thus the first concept is completely alone.
Then the next concept, "so-so" 

$$G = \{ (\text{Attitude, so-so}) \} = B = \{7, 8\}$$

<table>
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<tr>
<td>(Color, yellow)</td>
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<tr>
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<tr>
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<tr>
<td>(Temperature, high)</td>
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</tbody>
</table>

$$\{1, 2, 7, 8\} \not\subseteq \{7, 8\}$$

$$G = \{7, 8\} \cap \{1, 2, 7, 8\} = \{7, 8\}$$

\((\text{Size, big}) \cap (\text{Color, blue})) = \{7, 8\} \subseteq \{7, 8\}

We get the first rule:

\((\text{Size, big}) \cap (\text{Color, blue}) \rightarrow (\text{Attitude, so-so})

\{7, 8\} - \{2, 6\} = \emptyset$$

so we can go to the next concept

\(G = \{ (\text{Attitude, +}) \} = B = \{7, 8\}\)

\{1, 2, 3\} \not\subseteq \{2, 6\}$$

$$G = \{1, 2, 3\} \cap \{2, 6\} = \{2\}$$

\((\text{Color, yellow}) \cap (\text{Size, big}) = \{1, 2\} \not\subseteq \{2, 6\}\)

$$G' = \{2\}$$

\((\text{Color, yellow}) \cap (\text{Size, big}) \cap (\text{Feel, hard}) = \{2\} \subseteq \{2, 6\}\)

Here remove "Size", we can also say

\((\text{Color, yellow}) \cap (\text{Feel, hard}) = \{2\} \subseteq \{2, 6\}\)

So we get first rule:

\((\text{Color, yellow}) \cap (\text{Feel, hard}) \rightarrow (\text{Attitude, +})\)

New \(G = \{2, 6\} - \{2\} = \{6\}\)

\{1, 3, 6\} \not\subseteq \{6\}\)

\((\text{Feel, soft}) \cap (\text{Temperature, low}) = \{1, 3\} \not\subseteq \{2, 6\}\)

\((\text{Feel, soft}) \cap (\text{Temperature, low}) \cap (\text{Size, medium}) = \{1\} \subseteq \{2, 6\}\)

We can remove "Feel" and get

\((\text{Temperature, low}) \cap (\text{Size, medium}) \rightarrow (\text{Attitude, -})\)
2. (a) R:  
   1. (Size, medium) & (Temp, high) \rightarrow (Attitude, positive)  
   2. (Color, yellow) & (Feel, soft) \rightarrow (Attitude, positive)  
   3. (Color, yellow) & (Feel, hard) \rightarrow (Attitude, negative)  
   4. (Temp, low) & (Size, medium) \rightarrow (Attitude, negative)  
   5. (Size, big) & (Color, blue) \rightarrow (Attitude, so-so) 

(b) R:  
   1. (Size, medium) \rightarrow (Attitude, positive)  
   2. (Feel, soft) \rightarrow (Attitude, positive)  
   3. (Color, yellow) & (Feel, hard) \rightarrow (Attitude, negative)  
   4. (Temp, low) \rightarrow (Attitude, negative)  
   5. (Size, big) \rightarrow (Attitude, so-so) 

(c) R:  
   1. (Size, medium) & (Temp, high) \rightarrow (Attitude, positive)  
   2. (Color, yellow) & (Feel, hard) \rightarrow (Attitude, negative)  
   3. (Size, big) & (Color, blue) \rightarrow (Attitude, so-so) 

(d) R:  
   1. (Size, medium) \rightarrow (Attitude, positive)  
   2. (Color, yellow) \rightarrow (Attitude, negative)  
   3. (Size, big) \rightarrow (Attitude, so-so)