EECS 210 Spring 2009

Section 1.7: Proof Methods and Strategy

Proof Methods:

- Proof by Cases
  - Exhaustive Proofs
  - Without Loss of Generality
- Existence Proof
  - Constructive
  - Non-Constructive
- Uniqueness Proof

Proof Strategies:

- Forward and Backward Reasoning
- Adapting Existing Proofs
- Looking for Counterexamples

1 Proof by Cases

Idea:
$[(p_1 \vee p_2 \vee \ldots \vee p_n) \rightarrow q] \leftrightarrow [(p_1 \rightarrow q) \wedge (p_2 \rightarrow q) \wedge \ldots \wedge (p_n \rightarrow q)]$

1.1 Exhaustive Proofs

Example 1: Prove that $(n+1)^2 \geq 3^n$ if n is a positive integer with $n \leq 2$.
Idea: Try for $n = 1, 2$.

Example 2: Prove that the only consecutive positive integers not exceeding 100 that are perfect powers are 8 and 9. Note that an integer is a **perfect power** if it equals n^a where a is an integer greater than 1.
Idea: List all powers til 100 and check. Note: Easier with computer.

1.2 Non-Exhaustive

Example 3: Prove that if n is an integer, then $n^2 \geq n$.
Idea: Separate cases for (i) $n = 0$, (ii) $n \leq -1$, and (iii) $n \geq 1$.

Example 4: Let n be an integer. Prove that $9n^2 + 3n - 2$ is even.
Idea: Factor the polynomial, and separate cases for parity of one factor.

1.2.1 Without Loss of Generality

Example 5 (Exercise 5): Prove the **triangle inequality** which states that if x and y are real numbers, then $|x| + |y| \geq |x + y|$. Note that absolute value $|r|$ is equal to r if $r \geq 0$ and is equal to $-r$ is $r < 0$.
Idea: Just consider cases for x and y to be negative or non-negative.

Example 6: PRove that $|xy| = |x||y|$.
Idea: Just consider cases for x and y to be negative or non-negative.

2 Existence Proofs

How to prove that some object with particular properties exists.

2.1 Constructive

Example 7: Show that there is a positive integer that can be written as the sum of cubes of positive integers in two different ways.
Idea: Search and find it, $1729 = 10^3 + 9^3 = 12^3 + 1^3$.

2.2 Non-Constructive

Example 8: Assuming that $\sqrt{2}$ is irrational, show that there exist irrational numbers x and y such that x^y is rational.

Idea: Consider cases for $x = y = \sqrt{2}$ and then for $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$.