1-25. A political prisoner is to be exiled to either Siberia or the Urals. The probabilities of being sent to these places are 0.6 and 0.4, respectively. It is also known that if a resident of Siberia is selected at random the probability is 0.5 that he will be wearing a fur coat, whereas the probability is 0.7 that a resident of the Urals will be wearing one. Upon arriving in exile, the first person the prisoner sees is not wearing a fur coat. What is the probability he is in Siberia?

We are being asked a question of the form:

"What is the probability of ____ given ____ ?"

And we are also given a bunch of probabilities of this given that.

This sounds like a Bayes' Rule problem.

Bayes' Rule helps us find $P(A_i \mid B)$, so in this case

B = the event that the person is <u>not</u> wearing a fur coat.

A_1 = the event that the person is exiled to Siberia

A_2 = the event that the person is exiled to the Urals

→ remember that $\{A_i\}$ must partition the entire probability space into disjoint (mutually exclusive) events.

$$P(A_1 \mid B) = \frac{P(B \mid A_1)P(A_1)}{P(B \mid A_1)P(A_1) + P(B \mid A_2)P(A_2)}$$

We are given the data in terms of $\overline{B}$, but it is easy for us to see that $P(B \mid A_1) = 1 - P(\overline{B} \mid A_1)$ and $P(B \mid A_2) = 1 - P(\overline{B} \mid A_2)$

$$= \frac{(1-0.5)(0.6)}{(1-0.5)(0.6) + (1-0.7)(0.4)}$$

$$\boxed{= 0.7143}$$