EECS 360 Short Quiz #5

Signal and System Analysis

April 26, 2012

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

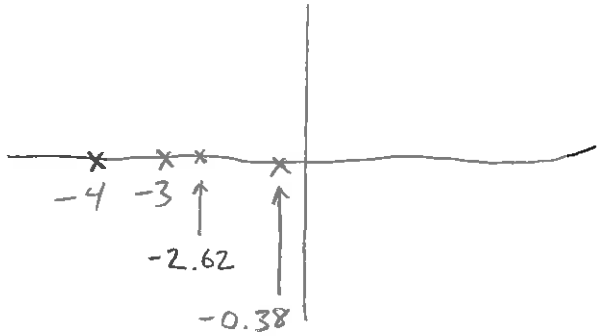
1. (30 %) Given the transfer function

$$H(s) = \frac{s^2 - s - 6}{(s^2 + 3s + 1)(s^2 + 7s + 12)}$$

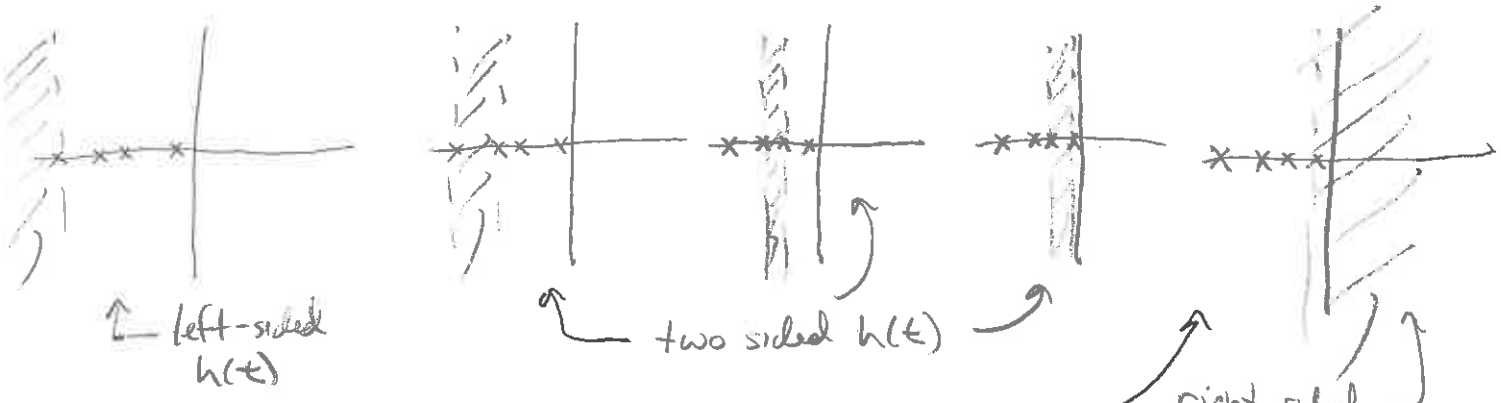
(a) Determine all possible choices for the ROC. For each answer, state whether the corresponding impulse response $h(t)$ is left sided, right sided, or two sided.

(b) Is it possible for this system to be causal and stable? Why or why not?

poles = roots of the denominator = $\left\{\frac{-3-\sqrt{5}}{2}, \frac{-3+\sqrt{5}}{2}, -3, -4\right\}$



(a) There are 5 possible ROC's



(b) Yes, the right sided ROC is causal (because it is right sided) and is stable (because the ROC includes the jω axis)

2. (30 %) A continuous-time system is given by the differential equation

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} - 4x(t).$$

(a) Determine the transfer function $H(s)$ of the system.

(b) Determine the impulse response $h(t)$ of the system.

(c) Is this system stable? Why or why not?

(d) Is this *inverse* to this system stable? Why or why not?

Taking the Laplace transform of the differential equation, we get

$$s^2Y(s) + 5sY(s) + 6Y(s) = sX(s) - 4X(s)$$

(a) $$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \boxed{\frac{s-4}{s^2+5s+6}}$$

(b) Using partial fractional expansion (after factoring the denominator)

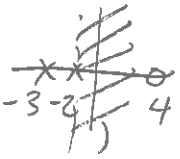
$$H(s) = \frac{A}{s+2} + \frac{B}{s+3} \quad \text{when } A = -6 \text{ and } B = 7$$

$$\updownarrow \mathcal{L}$$

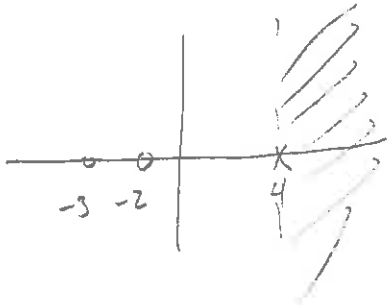
$$\boxed{h(t) = \left[-6e^{-2t} + 7e^{-3t}\right]u(t)}$$

If we assume it to be causal, which is reasonable because it is time-based

(c) Given our assumption that it is causal, then yes because the ROC is to the right of the right-most pole ($s=-2$) which includes the $j\omega$ axis



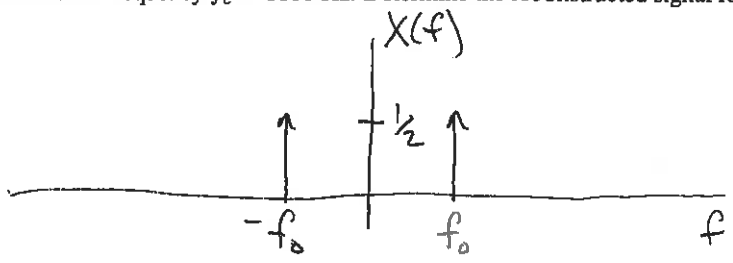
(d) The inverse of the system is $H^{-1}(s) = \frac{(s+2)(s+3)}{s-4}$



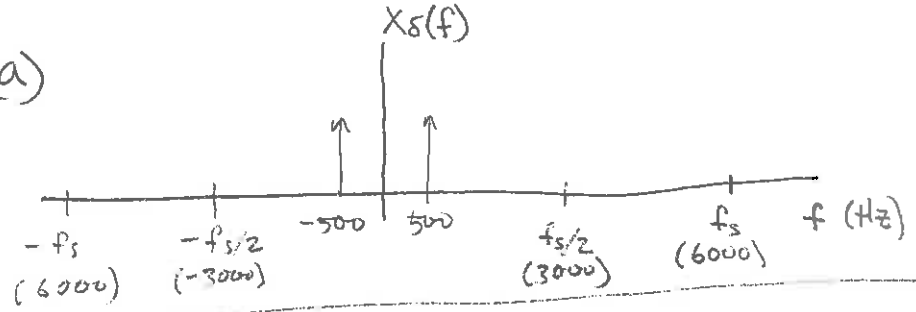
If we assume that it is causal then it cannot have an ROC that includes the $j\omega$ axis $\Rightarrow$ unstable

3. (40 %) A signal generator produces a sinusoidal tone $x(t) = \cos(2\pi f_0 t)$ with frequency f_0 between 1 Hz and 1 MHz. The signal is sampled with a sampling rate of $f_s = 6000$ samples/second and is reconstructed using an ideal low-pass filter (LPF) with cut-off frequency $f_c = 3000$ Hz. Determine the reconstructed signal for

(a) $f_0 = 500$ Hz
(b) $f_0 = 2.5$ kHz
(c) $f_0 = 3.5$ kHz
(d) $f_0 = 20$ kHz
(e) $f_0 = 1$ MHz



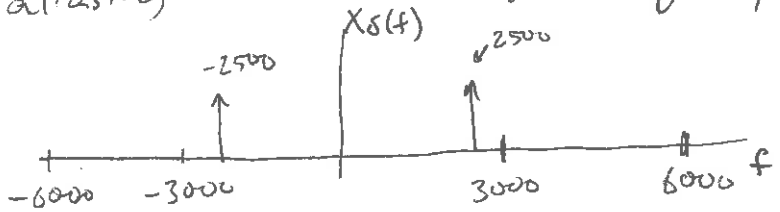
(a)



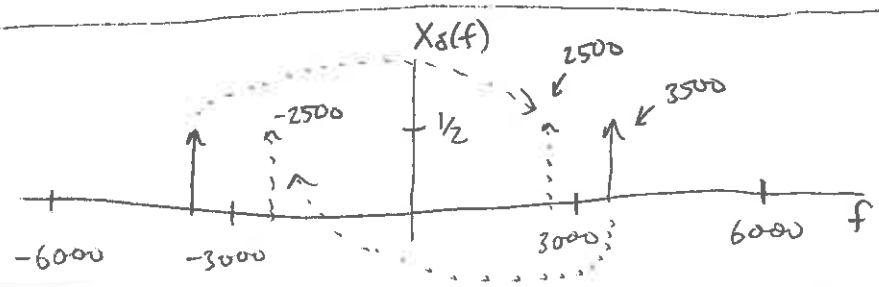
⇒ no aliasing

⇒ output frequency is **500 Hz**

(b) similar to the above, because $f_0 = 2500 < \frac{f_s}{2}$ there is no aliasing and the output frequency is **2.5 kHz**

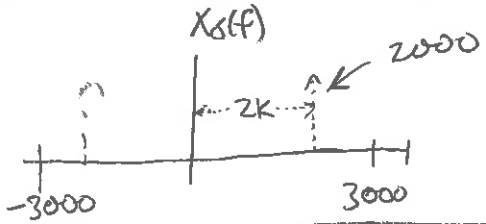


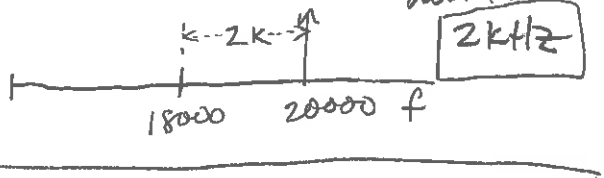
(c)



here we violate nyquist and 3500 Hz aliases down to **2500 Hz**

(d)





20 kHz aliases down to **2 kHz**

(e) 1 MHz aliases down to **2 kHz**