EECS 360 Short Quiz #3

Signal and System Analysis

March 11, 2008

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (50 %) Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency f_1 and CTFS harmonic function $X_1[k]$. Given that

$$x_2(t) = x_1(1-t) + x_1(t-1),$$

find the answers to the following questions:

(a) How is the fundamental frequency f_2 of $x_2(t)$ related to f_1?

(b) Find a relationship between the CTFS harmonic functions for $x_1(t)$ and $x_2(t)$, i.e. find a relationship between $X_1[k]$ and $X_2[k]$.

(a) In the formulation of $x_2(t)$, $x_1(t)$ undergoes

- time shifting
- time reversal

Both of these operations have some effect on the CTFS, but they do not alter the fundamental frequency, therefore, $\boxed{f_2 = f_1}$

(as a counter example, the time scaling operation, which doesn't take place in this problem, would alter the CTFS)

(b)

$$x_1(-t) \xleftrightarrow{\mathcal{FS}} X_1[-k]$$

$$x_1(1-t) = x_1(-(t-1)) \xleftrightarrow{\mathcal{FS}} e^{j2\pi k f_F} X_1[-k]$$

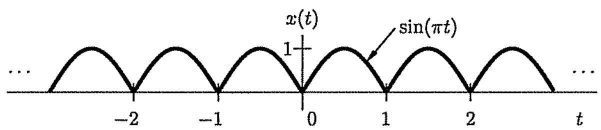
$$x_1(t-1) \xleftrightarrow{\mathcal{FS}} e^{-j2\pi k f_F} X_1[k]$$

Therefore

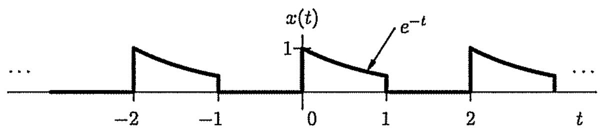
$$X_2[k] = e^{j2\pi k f_F} X_1[-k] + e^{-j2\pi k f_F} X_1[k]$$

2. (50 %) Find the CTFS harmonic function $X[k]$ for the following signals:

(a)



(b)



(a) The signal is periodic with fundamental period $T_0 = 1$.

Let $T_F = T_0 = 1$, $\Rightarrow f_F = 1$

$$X[k] = \frac{1}{T_F}\int_{T_F} x(t)e^{-j2\pi k f_F t}\,dt = \frac{1}{1}\int_0^1 \sin(\pi t)e^{-j2\pi k t}\,dt$$

$$= \int_0^1 \frac{1}{2j}\left[e^{j\pi t} - e^{-j\pi t}\right]e^{-j2\pi k t}\,dt = \frac{1}{2j}\left[\frac{1}{j\pi(1-2k)}e^{j\pi t(1-2k)} - \frac{1}{-j\pi(1+2k)}e^{-j\pi t(1+2k)}\right]_{t=0}^{1}$$

$$\boxed{= -\frac{1}{2\pi(1-2k)}\left(e^{j\pi(1-2k)} - 1\right) - \frac{1}{2\pi(1+2k)}\left(e^{-j\pi(1+2k)} - 1\right),\quad k \neq 0}$$

$$X[0] = \int_0^1 \sin(\pi t)\,dt = \frac{1}{\pi}\cos(\pi t)\Big|_{t=0}^{1} \boxed{= \frac{2}{\pi} \quad k=0}$$ (actually, this one works also for k=0)

(b) The signal is periodic with fundamental period $T_0 = 2$.

Let $T_F = T_0 = 2$, $\Rightarrow f_F = \frac{1}{2}$

$$X[k] = \frac{1}{2}\int_0^1 e^{-t}e^{-j2\pi k/2\,t}\,dt = \frac{1}{2}\int_0^1 e^{-(j\pi k+1)t}\,dt = \frac{1}{-2(j\pi k+1)}e^{-(j\pi k+1)t}\Big|_{t=0}^{1}$$

$$\boxed{= \frac{1}{-2(j\pi k+1)}\left(e^{-(j\pi k+1)} - 1\right),\ \forall k \text{ [this means "for all k"]}}$$