EECS 360 Short Quiz #3–B

Signal and System Analysis
November 10, 2016

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (45 %) Let $x(t)$ be any signal with Fourier transform $X(f)$ [or $X(j\omega)$ if you prefer]. The time-shift property of the Fourier transform may be stated as

$$x(t - t_0) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j2\pi t_0 f} X(f).$$

   (a) Prove the time-shift property by applying the time shift to the synthesis equation

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi f t}\, df.$$

   (b) Prove the time-shift property by utilizing the inverse Fourier tranform of $e^{-j2\pi t_0 f}$ in conjunction with the multiplication–convolution duality of the Fourier transform.

(a)

$$x(t-t_0) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f(t-t_0)}\, df = \int_{-\infty}^{\infty} X(f) e^{-j2\pi f t_0} e^{j2\pi f t}\, df$$

$$= \mathcal{F}^{-1}\left\{ X(f) e^{-j2\pi t_0 f} \right\}$$

(b)

$$x(t) \longleftrightarrow X(f)$$

$$\delta(t-t_0) \longleftrightarrow e^{-j2\pi t_0 f}$$

$$x(t) * \delta(t-t_0) \longleftrightarrow X(f) \cdot e^{-j2\pi t_0 f}$$

$$\hookrightarrow x(t-t_0)$$

2. (40 %) Find the DTFT of the signal

$$x[n] = \begin{cases} (0.6)^n, & 0 \le n \le 7 \\ 0, & \text{otherwise,} \end{cases}$$

using the transform of $(0.6)^n u[n]$, and the linearity and time-shifting properties.

write $x[n]$ as

$$x[n] = (0.6)^n u[n] - (0.6)^n u[n-8]$$

$$= \underbrace{(0.6)^n u[n]}_{g[n]} - (0.6)^8 \underbrace{(0.6)^{n-8} u[n-8]}_{g[n-8]}$$

$$g[n] \xleftrightarrow{\mathcal{F}} \frac{1}{1 + 0.6 e^{-j\Omega}}$$

$$g[n-8] \xleftrightarrow{\mathcal{F}} \frac{1}{1 + 0.6 e^{-j\Omega}} e^{-j8\Omega}$$

$$\Rightarrow X(e^{j\Omega}) = \frac{1}{1 - 0.6 e^{j\Omega}} - \frac{(0.6)^8 e^{-j8\Omega}}{1 - 0.6 e^{-j\Omega}}$$

$$= \frac{1 - (0.6)^8 e^{-j8\Omega}}{1 - 0.6 e^{-j\Omega}}$$

3. (15 %) True/False Questions. Circle the True (T) or False (F) for each part below.

(T)(F) The time-domain variable for the DTFT is a discrete (integer-valued) variable whereas the time-domain variable for the CTFT is a continuous-valued variable.

(T)(F) The DTFT is always periodic with period 2π whereas the CTFT is generally not periodic except under special conditions.

(T)(F) The inverse DTFT is a summation over frequency whereas the inverse CTFT is an integration over frequency.

(T)(F) The DTFT is a summation over time whereas the CTFT is an integration over time.

(T)(F) The highest frequency for the DTFT is effectively π whereas the highest frequency for the CTFT is ∞.

(T) DTFT operates on $x[n]$, CTFT operates on $x(t)$

(T) Some CTFTs, like the CTFT of the impulse train, are periodic

(F) Both the inverse DTFT and CTFT are integrals

(T) True, because time is discrete (summation) vs. continuous (integral)

(T) Because it is periodic with period 2π, the "highest" frequency with the DTFT is π