EECS 360 Short Quiz #3–A

Signal and System Analysis

November 8, 2016

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (30 %) Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency f_1 and CTFS harmonic function $c_{x_1}[k]$. Given that

$$x_2(t) = x_1(1-t) + x_1(t-1),$$

find the answers to the following questions:

(a) How is the fundamental frequency f_2 of $x_2(t)$ related to f_1?

(b) Find a relationship between the CTFS harmonic functions for $x_1(t)$ and $x_2(t)$, i.e. find a relationship between $c_{x_1}[k]$ and $c_{x_2}[k]$.

(a) In the formulation of $x_2(t)$, $x_1(t)$ undergoes:

- time shifting
- time reversal

Both of these have some effect on the CTFS, but neither effects the fundamental frequency, therefore $\boxed{f_2 = f_1}$

(as a counter example, the time scaling operation, which doesn't take place in this problem, would alter the fundamental freq.)

(b)

$$x_1(-t) \xleftrightarrow{\mathcal{FS}} c_{x_1}[-k]$$

$$x_1(1-t) = x_1(-(t-1)) \xleftrightarrow{\mathcal{FS}} e^{j2\pi k/T} c_{x_1}[-k]$$

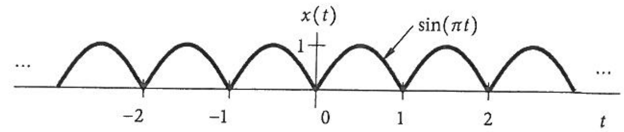
$$x_1(t-1) \xleftrightarrow{\mathcal{FS}} e^{-j2\pi k/T} c_{x_1}[k]$$

Therefore

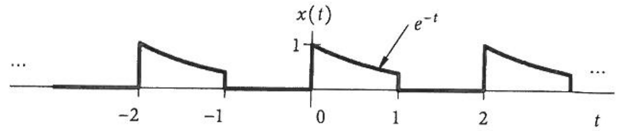
$$\boxed{c_{x_2}[k] = e^{j2\pi k/T} c_{x_1}[-k] + e^{-j2\pi k/T} c_{x_1}[k]}$$

2. (40 %) Find the CTFS harmonic function $c_x[k]$ for the following signals:

(a)



(b)



(a) The signal is periodic with period $T = 1$

$$C_x[k] = \frac{1}{T}\int_T x(t)\, e^{-j2\pi k t/T}\, dt = \frac{1}{1}\int_0^1 \sin(\pi t)\, e^{-j2\pi k t}\, dt$$

$$= \frac{1}{2j}\int_0^1 \left[e^{-j\pi t} - e^{-j\pi t}\right] e^{-j2\pi k t}\, dt = \frac{1}{2j}\left[\frac{1}{j\pi(1-2k)} e^{j\pi t(1-2k)} - \frac{1}{-j\pi(1+2k)} e^{-j\pi t(1+2k)}\right]_{t=0}^{1}$$

$$\boxed{= -\frac{1}{2\pi(1-2k)}\left(e^{j\pi(1-2k)} - 1\right) - \frac{1}{2\pi(1+2k)}\left(e^{-j\pi(1+2k)} - 1\right)}$$

We must account for the DC term. Because $\int e^{-j\pi(1 \pm 2k)t}\, dt$ behaves the same for $k = 0$, the answer still works for $k = 0$, and all k in fact. The DC term is $C_x[0] = \frac{2}{\pi}$

(b) The signal is periodic with period $T = 2$

$$C_x[k] = \frac{1}{2}\int_0^1 e^{-t} e^{-j2\pi k t/2}\, dt = \frac{1}{2}\int_0^1 e^{-(j\pi k + 1)t}\, dt = \frac{1}{-2(j\pi k + 1)} e^{-(j\pi k+1)t}\Big|_{t=0}^{1}$$

$$= \frac{1}{-2(j\pi k + 1)}\left(e^{-(j\pi k+1)} - 1\right)$$ for all k, for the same reason as above

3. (30 %) Consider a causal LTI system with frequency response

$$H(j\omega) = \frac{1}{j\omega + 3}.$$

For a particular input $x(t)$ this system is observed to produce the output

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t).$$

Determine $x(t)$.

$$Y(j\omega) = \frac{1}{j\omega + 3} - \frac{1}{j\omega + 4} = \frac{(j\omega+4) - (j\omega+3)}{(j\omega+4)(j\omega+3)} = \frac{1}{(j\omega+4)(j\omega+3)}$$

Because $Y(j\omega) = H(j\omega)X(j\omega)$

we have

$$\frac{1}{(j\omega+4)\cancel{(j\omega+3)}} = \frac{\cancel{1}}{\cancel{j\omega+3}} X(j\omega)$$

$$\Rightarrow X(j\omega) = \frac{1}{j\omega+4}$$

$$\Rightarrow \boxed{x(t) = e^{-4t}u(t)}$$