EECS 360 Short Quiz #2

Signal and System Analysis

September 25, 2012

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (35 %) Determine which of the following pairs of impulse responses correspond to inverse systems (justify your answer):

   (a) $h_1[n] = u[-n-1]$, and $h_2[n] = \delta[n-1] - \delta[n]$

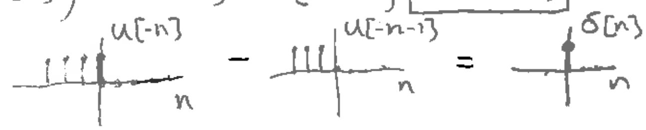
   (b) $h_1[n] = 0.5^n u[n]$, and $h_2[n] = \delta[n] - 0.5\delta[n-1]$

   (c) $h_1[n] = nu[n]$, and $h_2[n] = \delta[n+1] - 2\delta[n] + \delta[n-1]$

To be inverses of each other, $h_1[n]$ convolved with $h_2[n]$ must be the delta function $\Rightarrow$ $\boxed{h_1[n] * h_2[n] \overset{?}{=} \delta[n]}$

Recall that $x[n] * A\delta[n-1] = Ax[n-1]$

(a) $h_1[n] * h_2[n] = u[-n-1] * (\delta[n-1] - \delta[n]) = u[-n] - u[-n-1] \boxed{= \delta[n]}$ ✓



They are inverse systems

(b) $h_1[n] * h_2[n] = (0.5)^n u[n] * (\delta[n] - 0.5\delta[n-1]) = (0.5)^n u[n] - 0.5(0.5)^{n-1} u[n-1]$

$= (0.5)^n (u[n] - u[n-1])$

$= (0.5)^n \delta[n] \boxed{= \delta[n]}$ ✓

They are inverse systems

(c) $h_1[n] * h_2[n] = nu[n] * (\delta[n+1] - 2\delta[n] + \delta[n-1])$

$= (n+1)u[n+1] - 2nu[n] + (n-1)u[n-1]$ ← when all three are "on" ($n \geq 1$) it is zero, so we just need to figure out $n=-1$ and $n=0$, because for $n \leq 2$ they are all off

$$= \begin{cases} 0 & n \leq -2 \\ 0 & n = -1 \\ 1 & n = 0 \\ n-1-2n+n-1 = 0 & n \geq 1 \end{cases}$$

✓ $\boxed{= \delta[n]}$

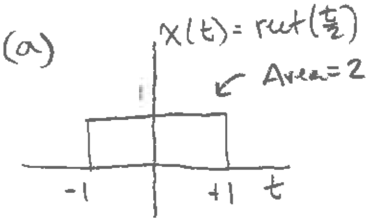
They are inverse systems

2. (35 %) Convolve the following pairs of signals. Provide a sketch of $x(t)$ and $h(t)$ in each case. Then compute $y(t) = x(t) * h(t)$ and provide an answer for $y(t)$ in equation form and in sketch form.

(a) $x(t) = \text{rect}\left(\frac{t}{2}\right)$, and $h(t) = \text{rect}\left(\frac{t-1.5}{3}\right)$

(b) $x(t) = e^{-2t}u(t)$, and $h(t) = u(t)$.

(a)



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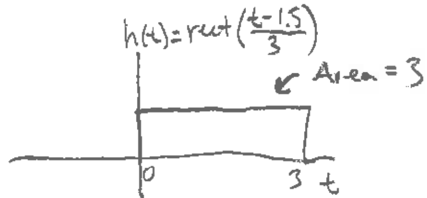

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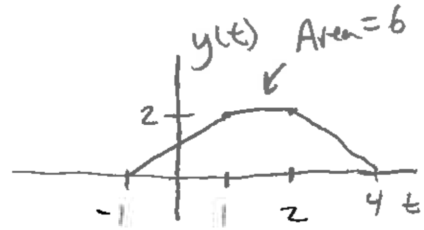
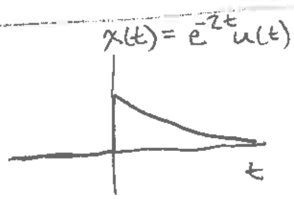


We know that:

- $y(t)$ starts at $t_{y_0} = t_{x_0} + t_{h_0} = -1 + 0 = -1$
- $y(t)$ ends at $t_{y_1} = t_{x_1} + t_{h_1} = +1 + 3 = +4$
- The maximum value of $y(t)$ occurs when the little rectangle is inside the big one, This will last for 1 second, and the area of the overlapping portions is 2
- $(\text{Area } y) = (\text{Area } x) \cdot (\text{Area } h) = 2 \cdot 3 = 6$

$$y(t) = \begin{cases} 0 & -\infty < t \leq -1 \\ t+1 & -1 < t \leq 1 \\ 2 & 1 < t \leq 2 \\ -t+4 & 2 < t \leq 4 \\ 0 & 4 < t < -\infty \end{cases}$$

(b)



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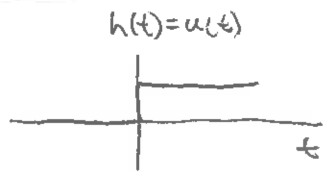
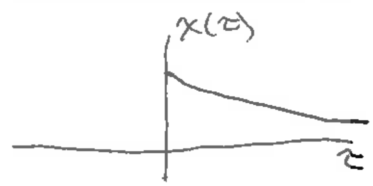
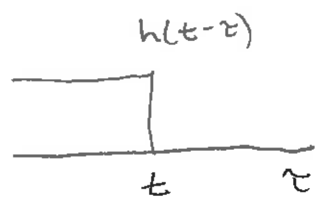


$\Rightarrow$





$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-2\tau} \underbrace{u(\tau)}_{\substack{\text{"on" for} \\ 0 \leq \tau}} \underbrace{u(t-\tau)}_{\substack{\text{"on" for} \\ \tau \leq t}} d\tau$$

$\hookrightarrow 0 \leq \tau \leq t$

$$= \int_0^t e^{-2\tau} d\tau = \left[-\frac{1}{2} e^{-2\tau}\right]_{\tau=0}^{t} \boxed{= \frac{1}{2}\left(1 - e^{-2t}\right) u(t)}$$

$u(t)$ is necessary because when $t < 0$ then the constraint $\tau < t < 0$ means there is nothing to integrate

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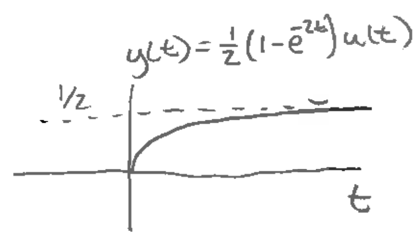


Stable means $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$

$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

$\boxed{\text{Causal means } \begin{array}{ll} h(t)=0 & t<0 \\ h(n)=0 & n<0 \end{array}}$

3. (30 %) For each of the following impulse responses, determine if the system is (i) memoryless, (ii) causal, and (iii) stable (justify your answer):

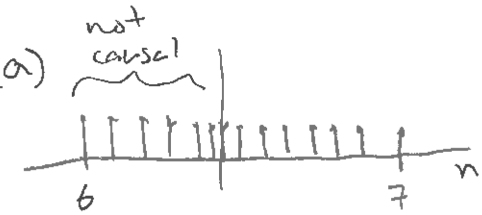
(a) $h[n] = u[n+6] - u[n-7]$

(b) $h(t) = \delta(t) + e^{-4t}u(t)$

(c) $h[n] = 0.8^{|n|}$

$\boxed{\text{The only memoryless system is } h(t) = A\delta(t) \text{ or } h[n] = A\delta[n]}$

(a)



(i) $h(n) \neq \delta(n) \Rightarrow$ has memory

(ii) $h(n) \neq 0 \quad n<0 \Rightarrow$ not causal

(iii) $\sum_{n=-\infty}^{\infty} |h(n)| = 14 < \infty \Rightarrow$ stable

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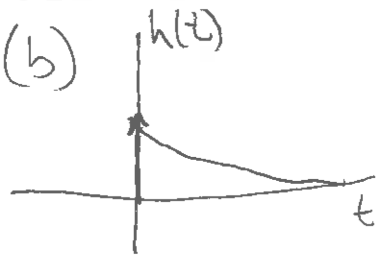
(b)



(i) $h(t) \neq \delta(t) \Rightarrow$ has memory

(ii) $h(t) = 0 \quad t<0 \Rightarrow$ causal

(iii) $\int_{-\infty}^{\infty} |h(t)|\,dt = 1 + \frac{1}{4} < \infty \Rightarrow$ stable

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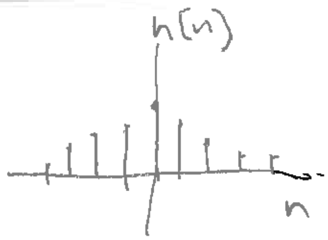
(c)



(i) $h[n] \neq \delta[n] \Rightarrow$ has memory

(ii) $h[n] \neq 0 \quad n<0 \Rightarrow$ non-causal

(iii) $\sum_{n=-\infty}^{\infty} |h(n)| = \left(\frac{1}{1-.8}\right)\cdot 2 - 1 < \infty \Rightarrow$ stable