Consider a real, odd, and periodic signal $x(t)$ whose Fourier Series representation may be expressed as

$$x(t) = \sum_{k=0}^{5} \left(\tfrac{1}{2}\right)^k \sin(k\pi t).$$

Let $\hat{x}(t)$ represent the signal obtained by performing impulse-train sampling on $x(t)$ using a sampling period of $T = 0.2$.

(a) Does aliasing occur when this impulse-train sampling is performed on $x(t)$?

(b) If $\hat{x}(t)$ is passed through an ideal lowpass filter with cutoff frequency π/T and passband gain T, determine the Fourier series representation of the output signal $g(t)$.

(a) The sampling frequency is $\frac{2\pi}{1/5} = 10\pi$

The maximum frequency in $x(t)$ is the sinusoid with $k=5$

$\omega_m = 5\pi$

Aliasing <u>does</u> occur since this signal is <u>critically</u> sampled

$2\omega_m = 10\pi$

(b) The cutoff frequency is $\omega_c = \pi/(1/5) = 5\pi$

The $k=5$ term is already aliased to $\left(\tfrac{1}{2}\right)^5 \sin(0\pi t) = 0$, the remaining terms are:

$$g(t) = \sum_{k=0}^{4} \left(\tfrac{1}{2}\right)^k \sin(k\pi t)$$

$$= \sum_{k=-4}^{4} X[k]\, e^{jk\pi t} \quad \text{where} \quad X[k] = \begin{cases} 0, & k=0 \\ -j\left(\tfrac{1}{2}\right)^{k+1} & 1 \le k \le 4 \\ j\left(\tfrac{1}{2}\right)^{-k+1} & -4 \le k \le -1 \end{cases}$$