An LTI System S with impulse response $h[n]$ and frequency response $H(e^{j\Omega})$ is Known to have the property that, when $-\pi \le \omega_0 \le \pi$

$$\cos(\omega_0 n) \longrightarrow \omega_0 \cos(\omega_0 n)$$

(recall that this notation means $\xrightarrow[\cos(\omega_0 n)]{} \boxed{S} \xrightarrow[\omega_0 \cos(\omega_0 n)]{}$)

(a) Determine $H(e^{j\Omega})$

(b) Determine $h[n]$

(a) It is given that when the excitation is $\cos(\omega_0 n)$ the response is $\omega_0 \cos(\omega_0 n) = \underbrace{\omega_0 e^{j\omega_0 n} + \omega_0 e^{-j\omega_0 n}}$

This means that $e^{j\omega_0 n} \longrightarrow |\omega_0| e^{j\omega_0 n}$, for any ω_0 in $-\pi \le \omega_0 \le \pi$

Therefore, the frequency response is $\boxed{H(e^{j\Omega}) = |\Omega|}$

(b) Taking the inverse DTFT we get

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\Omega}) e^{j\Omega n}\, d\Omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} -\Omega e^{j\Omega n}\, d\Omega + \frac{1}{2\pi}\int_{0}^{\pi} \Omega e^{j\Omega n}\, d\Omega$$

$$= \frac{1}{\pi}\int_{0}^{\pi} \Omega \cos(\Omega n)\, d\Omega$$

$$\boxed{= \frac{1}{\pi}\left[\frac{\cos(n\pi) - 1}{n^2}\right]}$$