Consider a causal LTI system described by the difference equation $y[n] + \frac{1}{2} y[n-1] = x[n]$

(a) Determine the frequency response $H(e^{j\Omega})$ of the system

(b) Determine the impulse response $h[n]$ of the system

(a) Taking the DTFT of both sides of the equation we get

$$Y(e^{j\Omega}) + \frac{1}{2} e^{-j\Omega} Y(e^{j\Omega}) = X(e^{j\Omega})$$

$$\Rightarrow \quad \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} \triangleq H(e^{j\Omega}) = \boxed{\frac{1}{1 + \frac{1}{2} e^{-j\Omega}}}$$

(b) Using Table 11.1

$$\alpha^n u[n] \quad \longleftrightarrow \quad \frac{1}{1 - \alpha e^{-j\Omega}}, \quad |\alpha| < 1$$

In this case $\alpha = -1/2$

$$\boxed{h[n] = \left(-\frac{1}{2}\right)^n u[n]}$$