

Derive the DTFT of the unit sequence (unit step)

$$X[n] = u[n]$$

by making use of the accumulation property and the knowledge that $g[n] = \delta[n] \xleftrightarrow{\mathcal{F}} G(e^{j\Omega}) = 1$

From chapter 2 we know that the unit sequence is the running sum of the unit impulse, that is

$$X[n] = \sum_{m=-\infty}^n g[m]$$

Taking the DTFT of both sides and using accumulation yields

$$X(e^{j\Omega}) = \frac{G(e^{j\Omega})}{1 - e^{-j\Omega}} + \pi G(e^{j0}) \delta_{2\pi}(\Omega)$$

$$= \frac{1}{1 - e^{-j\Omega}} + \pi \delta_{2\pi}(\Omega) \leftarrow \begin{array}{l} \text{verify in table} \\ 11.1 \end{array}$$
