(a) Determine which, if any, of the real signals depicted in the figure below have CTFTs that satisfy each of the following conditions:

(1) $\mathrm{Re}\{X(j\omega)\} = 0$

(2) $\mathrm{Im}\{X(j\omega)\} = 0$

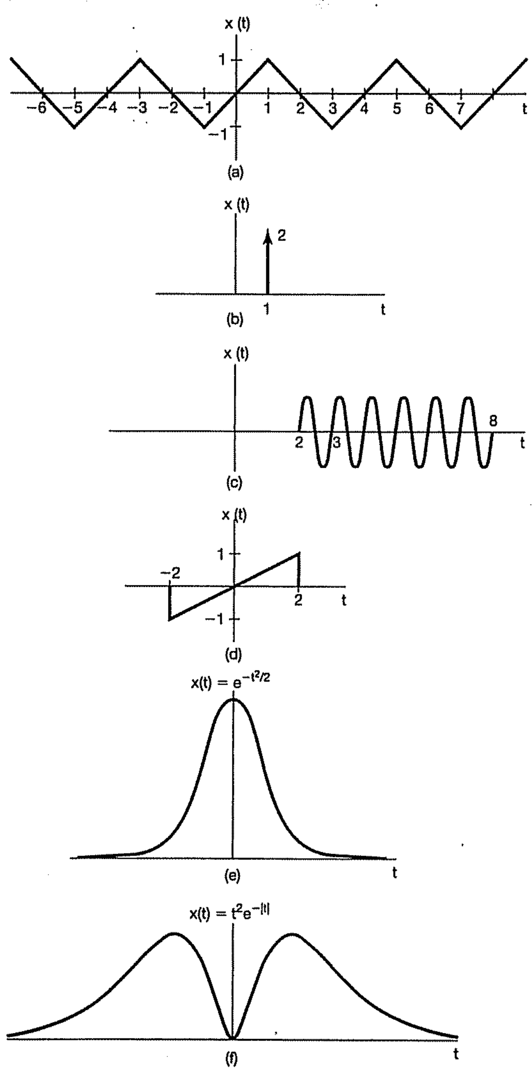
(3) There exists a real α such that $e^{j\alpha\omega} X(j\omega)$ is real

(4) $\int_{-\infty}^{\infty} X(j\omega)\, d\omega = 0$

(5) $\int_{-\infty}^{\infty} \omega X(j\omega)\, d\omega = 0$

(6) $X(j\omega)$ is periodic

(b) Construct a signal that has properties (1), (4), and (5) and does <u>not</u> have the others.



Solution,

(a) (1) for $Re\{X(j\omega)\} = 0$ the signal $X(t)$ must be real and odd.

The signals in figures (a) and (c) have this property

(2) for $Im\{X(j\omega)\} = 0$ the signal $x(t)$ must be real and even.

The signals in figures (e) and (f) have this property

(3) for there to exist a real α such that $e^{j\alpha\omega} X(j\omega)$ is real,

we require $x(t+\alpha)$ to be a real and even signal.

The signals in figures (a), (b), (e), and (f)

$\alpha = \pm 1$ $\quad \alpha = 1$ $\quad \alpha = 0$ $\quad \alpha = 0$

(4) For this condition to be true $x(0) = 0$

The signals in figures (a), (b), (c), (d), and (f) have this property.

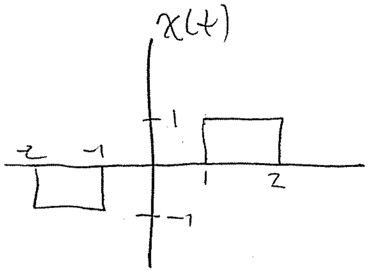
(5) For this condition to be true, the derivative of $x(t)$ has to be zero at $t = 0$

The signals in figures (b), (c), (e), and (f) have this property.

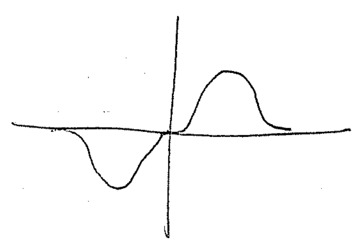
(6) For this to be true, the signal $x(t)$ has to be periodic

only the signal in figure (a) has this property

(b) the signal must be real and odd, have $x(0) = 0$ and $x'(0) = 0$



or



or . . .