

Problem 8.25 from Roberts

The CTFS harmonic function $X[k]$ for the signal
 $x(t) = \text{rect}(2(t-1)) * 3\delta_3(t) = \text{rect}(2t) * 3\delta_3(t-1)$
is of the form $X[k] = A \text{sinc}(ak) e^{-jb\pi k}$.

Find A , a , and b using a representation time $T_F = T_0$

Using Table 8.1 on p. 273 and $T_F = T_0$

$$\frac{1}{\omega} \text{rect}\left(\frac{t}{\omega}\right) * \delta_{T_0}(t) \xleftrightarrow{\text{FS}} f_0 \text{sinc}(\omega k f_0)$$

we have $\omega = 1/2$ and $f_0 = 1/T_0 = 1/3$, therefore

$$\text{rect}(2t) * 3\delta_3(t) \leftrightarrow \frac{1}{2} \text{sinc}(k/6)$$

Now we have to account for the delay, $t-1$, using
the time-shifting property ($t_0 = 1$)

$$\text{rect}(2(t-1)) * 3\delta_3(t) \xleftrightarrow{\text{FS}} \frac{1}{2} \text{sinc}(k/6) e^{-j2\pi k(\frac{1}{3})(1)}$$

Therefore, our final answer is

$$\text{rect}(2(t-1)) * 3\delta_3(t) \leftrightarrow \frac{1}{2} \text{sinc}(k/6) e^{-j2\pi k/3}$$