

We are given the following information about a signal $x(t)$:

- (1) $x(t)$ is real and odd
- (2) $x(t)$ is periodic with period $T_0 = 2$ and has Fourier harmonic function $X[k]$
- (3) $X[k] = 0$ for $|k| > 1$
- (4) $\frac{1}{2} \int_0^2 |x(t)|^2 dt = 1$

Specify two different signals that satisfy these conditions

- (1) Tells us that $X[k]$ is odd and purely imaginary
- (2) Mostly just establishes notation, and $f_0 = \frac{1}{2}$
- (3) Tells us that we are at most looking for $X[-1]$, $X[0]$ and $X[1]$, except that (1) tells us that $X[0] = 0$
- (4) Tells us that $|X[-1]|^2 + |X[1]|^2 = 1$
but (1) tells us that $X[1] = -X[-1]$ so
 $2|X[1]|^2 = 1 \Rightarrow |X[1]| = \frac{1}{\sqrt{2}}$

So... we know that $X[1]$ has a magnitude of $\frac{1}{\sqrt{2}}$ and is purely imaginary, so $X[1] = \pm j \frac{1}{\sqrt{2}}$
 $X[-1] = \mp j \frac{1}{\sqrt{2}}$

$$\begin{aligned} \text{and } x(t) &= \sum_{k=-\infty}^{\infty} X[k] e^{-2\pi k f_0 t} \\ &= \mp j \frac{1}{\sqrt{2}} e^{-j\pi t} \pm j \frac{1}{\sqrt{2}} e^{+j\pi t} \\ &= \pm \sqrt{2} \sin(\pi t) \end{aligned}$$