Consider a system $\mathcal{H}$ with excitation $x[n]$ and response $y[n]$ related by $y[n] = x[n]\left(g[n] + g[n-1]\right)$

(a) If $g[n] = 1$ for all $n$, show that $H$ is time invariant

In this case $y[n] = 2x[n]$

Let $x_1[n] = g[n] \longrightarrow y_1[n] = 2g[n]$

Let $x_2[n] = g[n-n_0] \longrightarrow y_2[n] = 2g[n-n_0] \boxed{= y_1[n-n_0]}$

$\Rightarrow$ time invariant

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(b) If $g[n] = n$, show that $H$ is not time invariant

In this case $y[n] = x[n]\left(n + n-1\right) = (2n-1)\,x[n]$

Let $x_1[n] = g[n] \longrightarrow y_1[n] = (2n-1)\,g[n]$

Let $x_2[n] = g[n-n_0] \longrightarrow y_2[n] = (2n-1)\,g[n-n_0]$

$\Rightarrow$ not time invariant $\boxed{\neq y_1[n-n_0]}$

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(c) If $g[n] = 1 + (-1)^n$, show that $\mathcal{H}$ is time invariant

In this case $y[n] = x[n]\left(1 + (-1)^n + 1 + (-1)^{n-1}\right)$

This is +1 for n-even and −1 for n-odd

This is −1 for n-even and +1 for n-odd

they cancel!

$$y[n] = \underbrace{2x[n]}_{\text{Time invariant, see part (a)}}$$