Fourier Series

Signals & Systems

Lab 5

The man with the plan

Any <u>periodic</u> function can be described by an infinite linear combination (a series) of Sines and Cosines.

$$x(t) = a_x[0] + \sum_{k=1}^{\infty} a_x[k] \cos\left(\frac{2\pi k t}{T}\right) + b_x[k] \sin\left(\frac{2\pi k t}{T}\right)$$

⇑ Fourier Series ⇑

Fourier Cosine Series

Any even (periodic) function can be described by an infinite linear combination (a series) of Cosines.

$$x(t) = x_e(t) = a_x[0] + \sum_{k=1}^{\infty} a_x[k] \cos\left(\frac{2\pi k t}{T}\right)$$

Fourier Sine Series

Any odd (periodic) function can be described by an infinite linear combination (a series) of Sines.

$$x(t) = x_o(t) = \sum_{k=1}^{\infty} b_x[k] \sin\left(\frac{2\pi k t}{T}\right)$$

Example

Look at these three first terms of the Cosine Series of some unknown function, $x(t)$:

$$x_1(t) = \frac{4}{\pi}\cos\left(\omega_0 t - \frac{\pi}{2}\right)$$

$$x_2(t) = \frac{4}{\pi}\cdot\frac{1}{3}\cos\left(3\omega_0 t - \frac{\pi}{2}\right)$$

$$x_3(t) = \frac{4}{\pi}\cdot\frac{1}{5}\cos\left(5\omega_0 t - \frac{\pi}{2}\right)$$

The nth term looks like:

$$x_n(t) = \frac{4}{\pi}\cdot\frac{1}{(2n-1)}\cos\left((2n-1)\omega_0 t - \frac{\pi}{2}\right) \qquad n \in \{1,2,\ldots\}$$

In series form:

$$x(t) = \sum_{n=0}^{\infty} \frac{4}{\pi} \cdot \frac{1}{(2n+1)} \cos\left((2n+1)\omega_0 t - \frac{\pi}{2}\right)$$

Lets plot the first fifty terms only ->

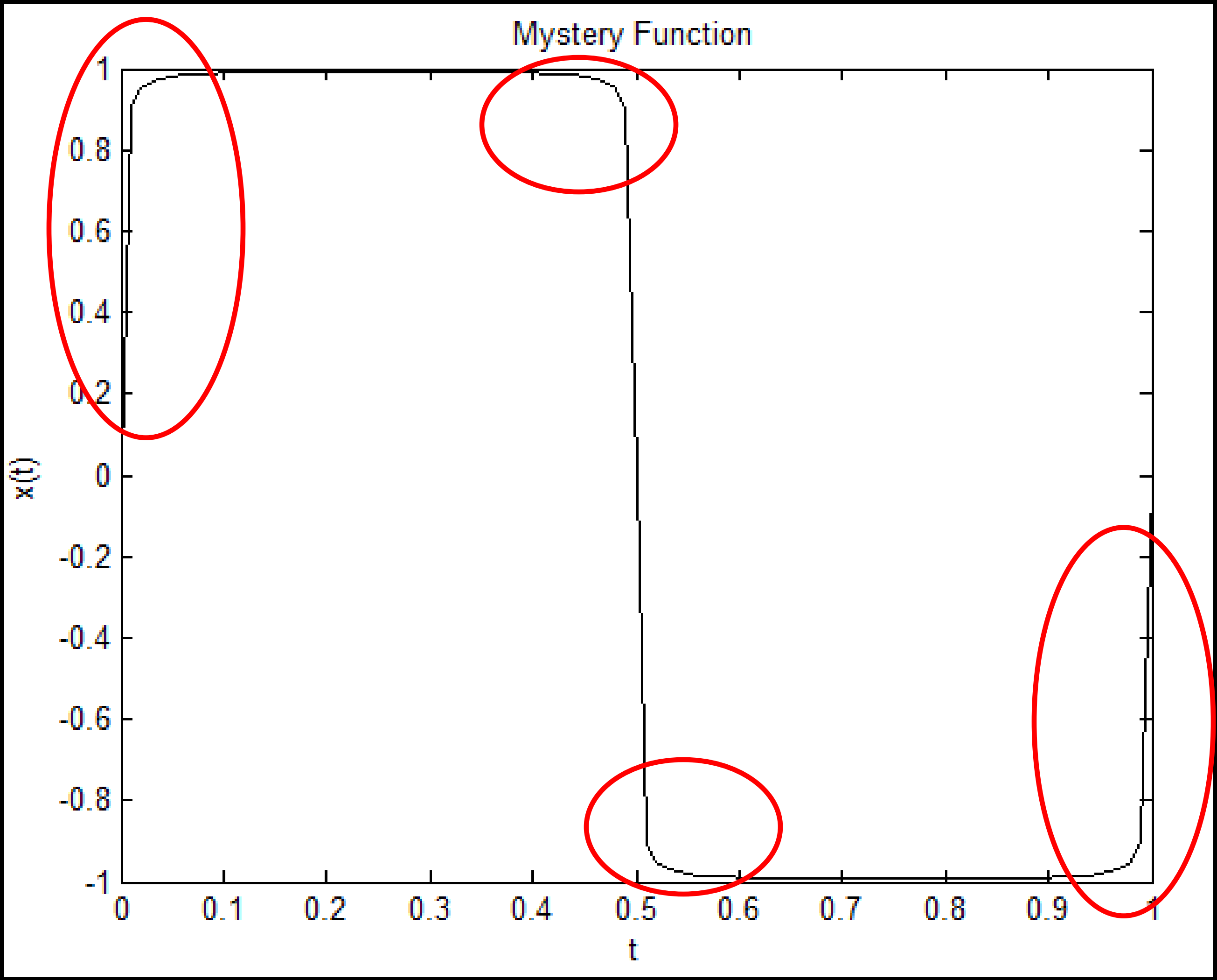
Hint let $\omega_0 = 2\pi$ for convenience (this makes $T_0 = 1$). Plot one complete period, $t \in [0,1]$.

$$x(t) = \sum_{n=1}^{50} \frac{4}{\pi} \cdot \frac{1}{(2n+1)} \cos\left((2n+1)\omega_0 t - \frac{\pi}{2}\right)$$

Code

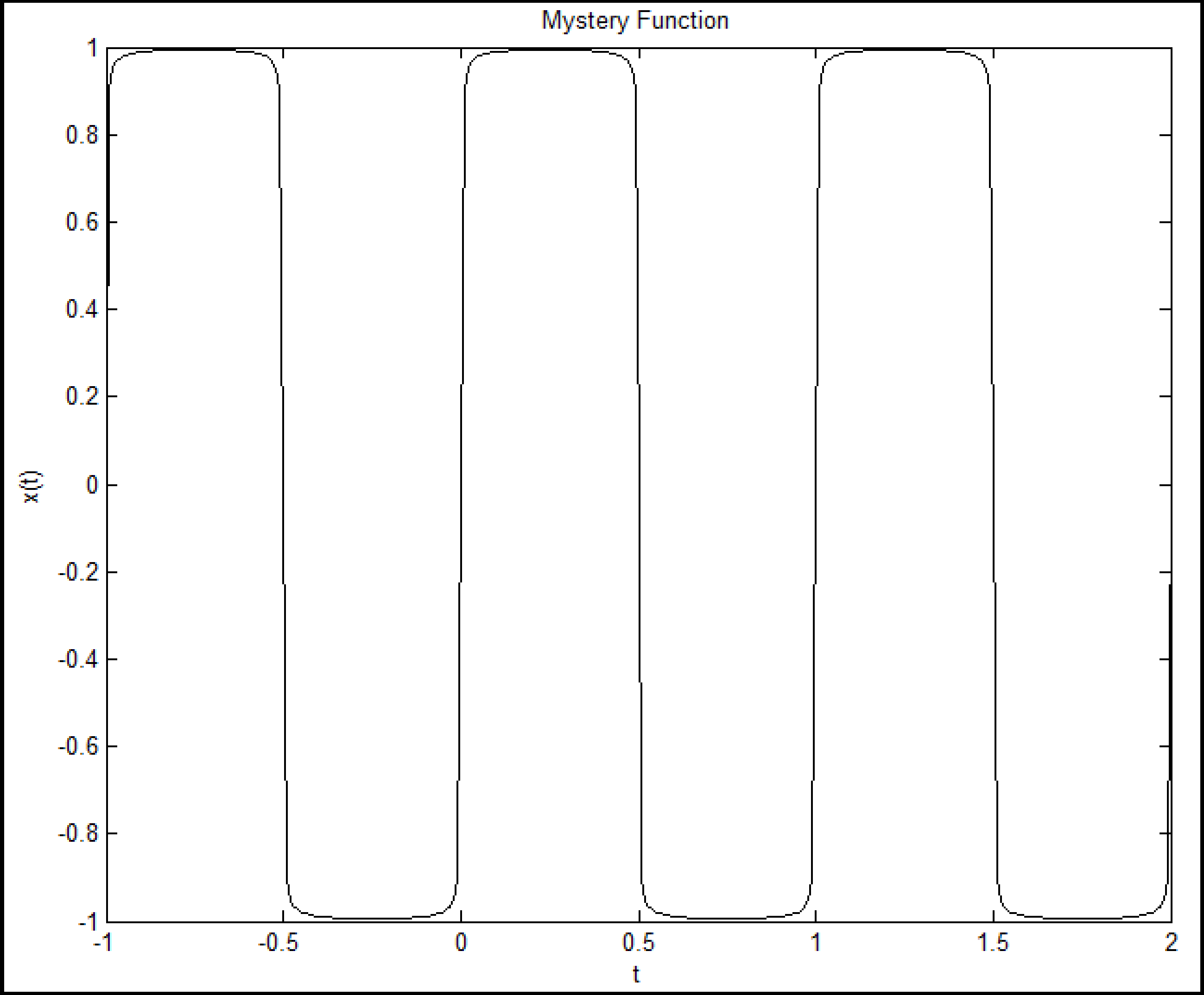
```
%set up time interval
t=0:0.01:1;
%preallocate a matrix where each row is one term of the
%series
x=zeros(50,length(t));
for n=1:50
    x(n,:)=(4/(pi*(2*n-1)))*cos((2*n-1)*2*pi.*t-pi/2);
end
%sum all the columns of the term matrix
%this gives the "summed value" at each point in time
sum5=sum(x(1:50,:));
%plot
plot(t,sum5,'k')
title('Mystery Function')
xlabel('t')
ylabel('x(t)')
```

Remember its periodic! So this repeats every $T = \frac{2\pi}{\omega_0}$.



These smooth corners are not present in the function we are trying to represent but are due to the inability of expressing sharp edges with smooth cosines.

Graphed with 3 periods



Have you guessed what the graph is yet?

Looks like $$x(t) = -sgn\left(t - \frac{1}{2}\right)$$ for one period

Actually it is:

$$x(t) = sgn\left(\cos\left(2\pi t - \frac{\pi}{2}\right)\right)$$

This is know as a square wave!

Sources

Roberts, M.J. (2012). *Signals and Systems: Analysis Using Transform Methods and MATLAB®* (2nd ed.). New York, NY: McGraw-Hill.