z Transform

Lab Notes 13

Review

Generalized CTFT -> Laplace Transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{+j\omega t}d\omega$$

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

$$x(t) = \frac{1}{j2\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{+st}ds$$

Fourier Family

Generalizes to Laplace Transform

	Continuous Frequency, $X(f)$	Discrete Frequency, $X[k]$
Continuous Time, $x(t)$	Continuous Time Fourier Transform (CTFT)	Continuous Time Fourier Series (CTFS)
Discrete Time, $x[n]$	Discrete Time Fourier Transform (DTFT)	Discrete Time Fourier Series (DTFS) -OR- Discrete Fourier Transform (DFT)

Generalizes to z Transform

New Idea
Generalized DTFT -> z Transform

$$X\left(e^{j\Omega}\right) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega\mathrm{n}}$$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X\left(e^{j\Omega}\right)e^{+j\Omega\mathrm{n}}d\Omega$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$x[n] = \frac{1}{j2\pi}\oint_{C} X(z)z^{n-1}d\mathrm{z}$$

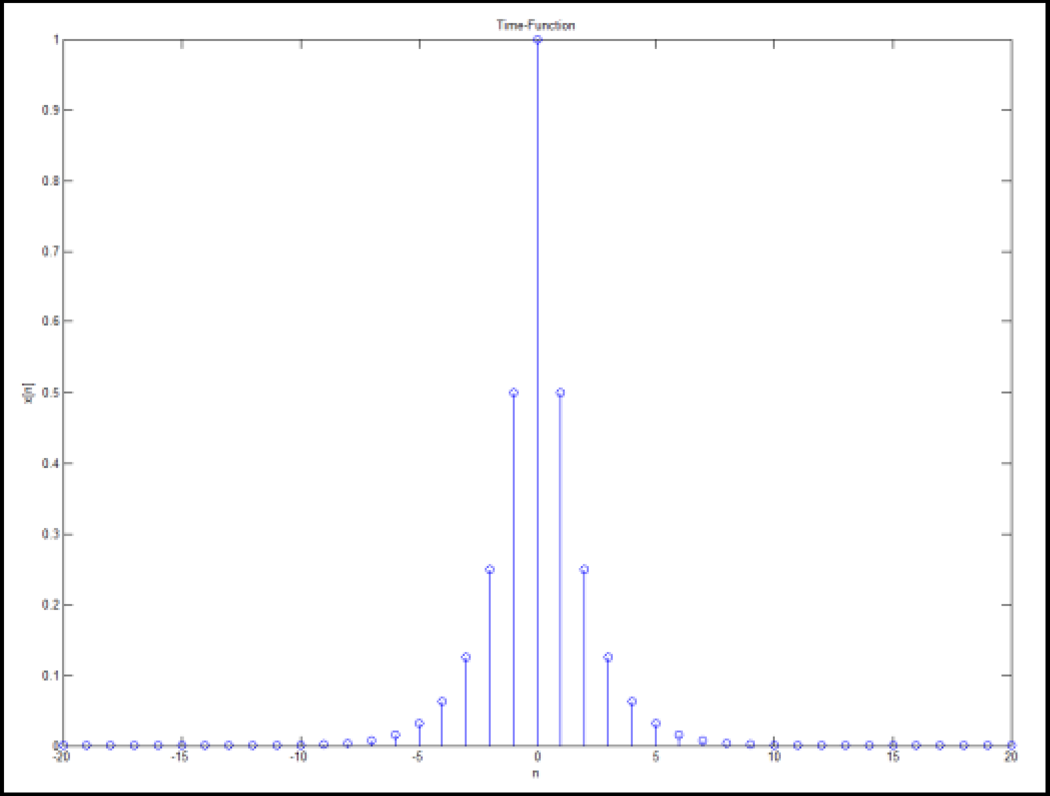
Bilateral z Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Region of Convergence for the *bilateral z-Transform* is dependent upon the nature of the discrete-time signal, $x[n]$.

Example

$$x[n] = 0.5^{|n|}\, u[|n|]$$



$$x[n] = \{\dots, 0.5^3, 0.5^2, 0.5^1, 1\,, 0.5^1, 0.5^2, 0.5^3, \dots\}$$

Find z Transform of x[n]

$$X(z) = \sum_{n=-\infty}^{\infty} 0.5^{|n|}\, u[|n|] z^{-n}$$

rewriting the sum since the amplitude values are the same,

$$X(z) = 2\left\{\sum_{n=0}^{\infty} 0.5^{n}\, z^{-n}\right\} - 1$$

ROC

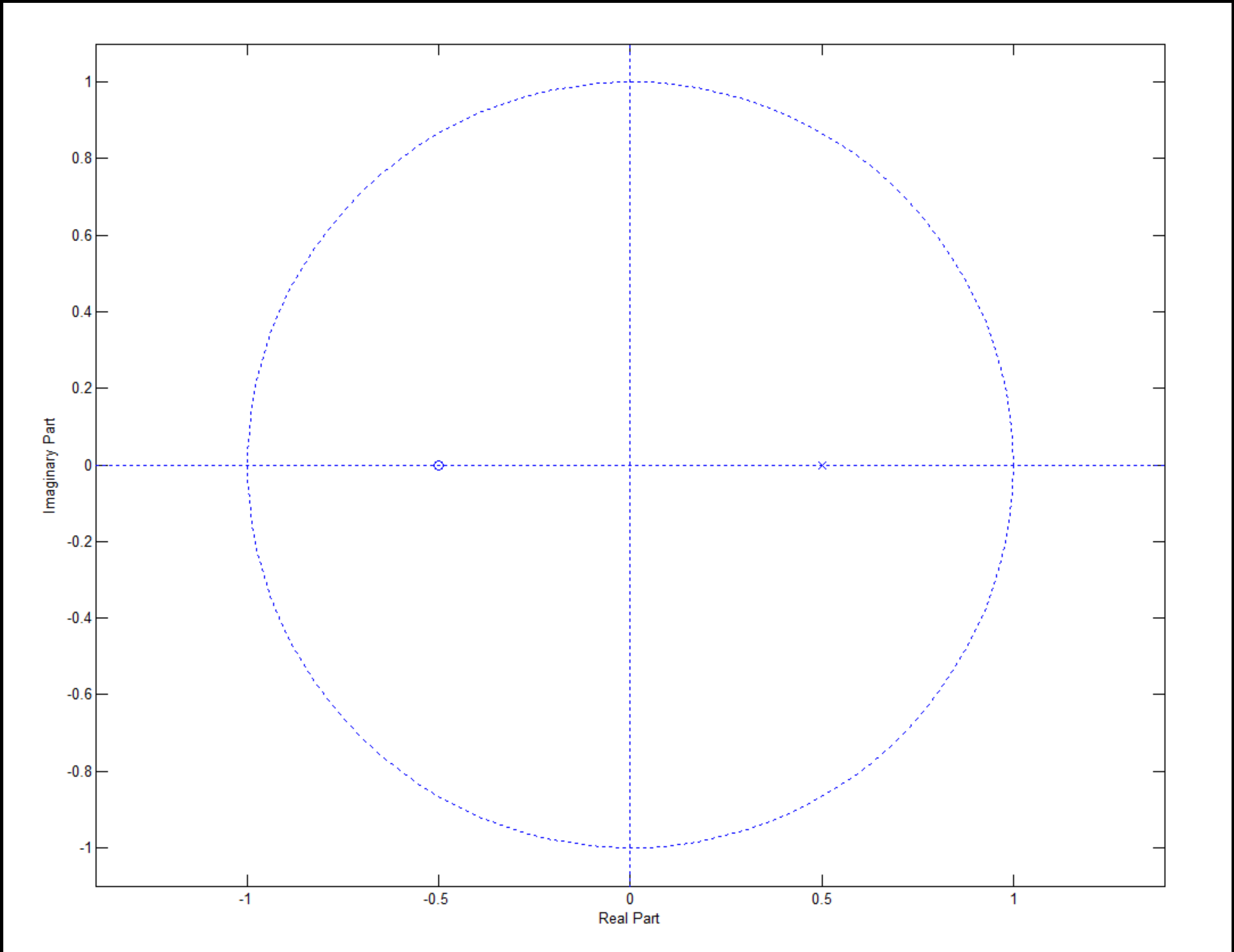
the sum is just the geometric series

$$\sum_{n=0}^{\infty} 0.5^{n}\, z^{-n} = \sum_{n=0}^{\infty} \left(\frac{0.5}{z}\right)^{n} = \frac{1}{1-\left(\frac{0.5}{z}\right)} = \frac{z}{z-0.5}, \qquad |z| > 0.5$$

$$\therefore X(z) = \frac{2z}{z-0.5} - 1 = \frac{z+0.5}{z-0.5}, \qquad |z| > 0.5$$

In the z-plane

$$X(z) = \frac{z + 0.5}{z - 0.5}$$



Poles and Zeros

Just like the Laplace transforms, z Transforms have poles and zeros also

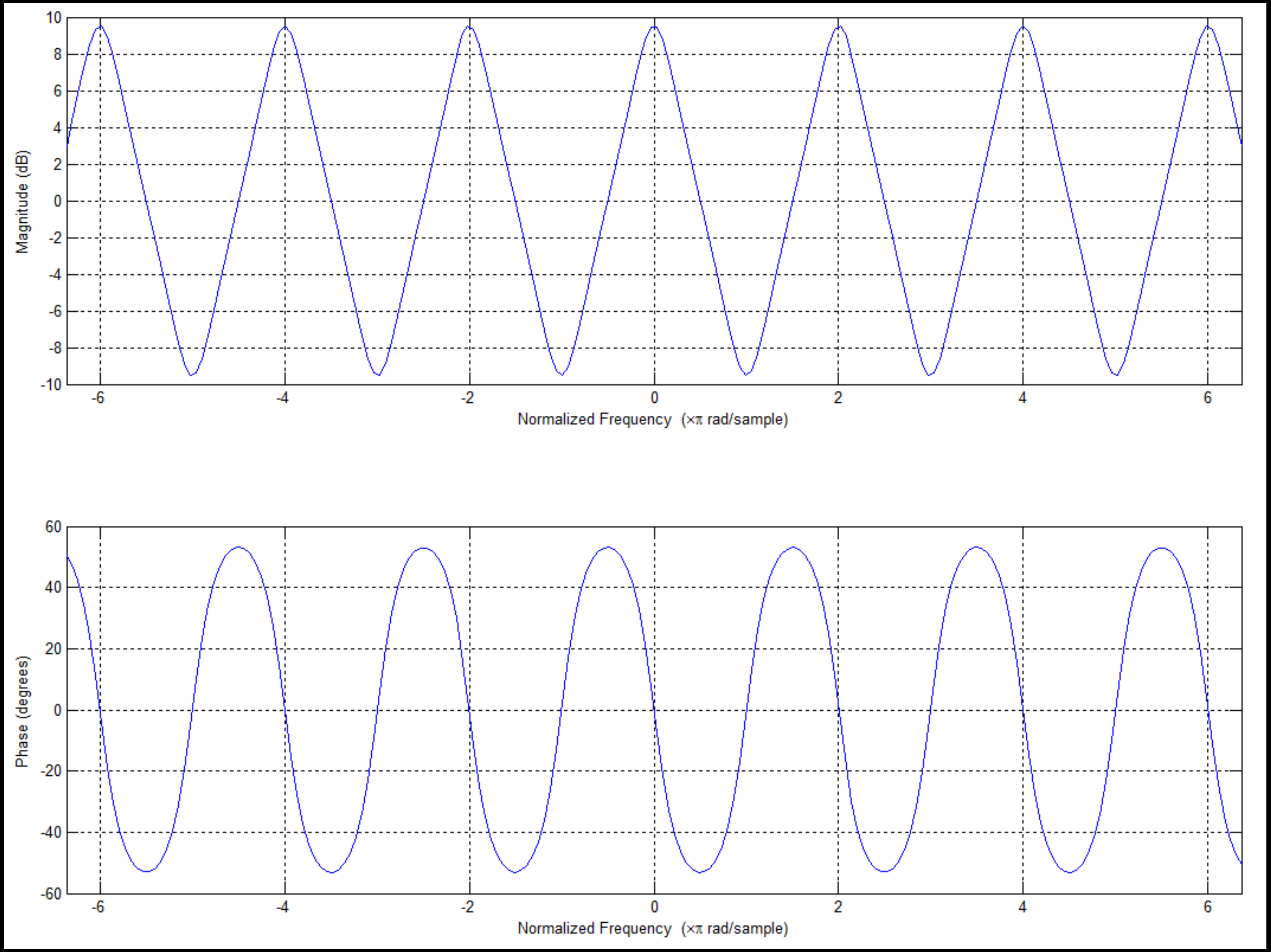
The function in matlab to map these on the complex plane is called zplane().

This is similar to pzmap().

Code

```
num1=[1 0.5];
den1=[1 -0.5];
zs1=roots(num1);
ps1=roots(den1);
zplane(zs1,ps1)
```

Frequency Response



Frequency Response

Additionally the function in matlab to show the frequency response of a discrete-time system is called freqz().

This is similar to freqs() for continuous-time systems.

Code

```
num1=[1 0.5];
den1=[1 -0.5];
omega=linspace(-20,20,200);
freqz(num1,den1,omega)
```

Multiplying polynomials

Question: How do I use MATLAB to multiply this expression?

$$(z^2 + 2z + 10)(z + 2)$$

Answer: Use conv()!

```
conv([1 2 10],[1 2])
ans =
      1      4      14      20
```

Note this function can be nested also!

Unilateral z-Transform

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

Region of Convergence for the *unilateral z-Transform* is always the open exterior of a circle, centered at the origin of the z-plane whose radius is the largest finite pole magnitude.

Contrast this with the Region of Convergence for the *unilateral Laplace Transform* which is always the region of the s-plane to the right of all the finite poles.