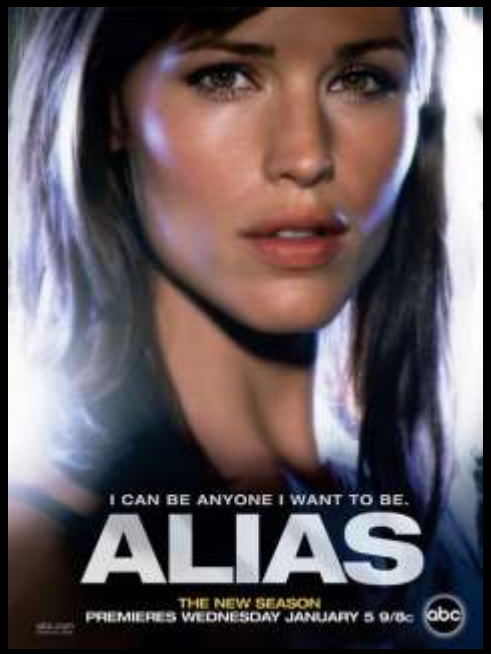
ALIASing

Lab # 11

Warning:
This is not your early-2000s Primetime Show!



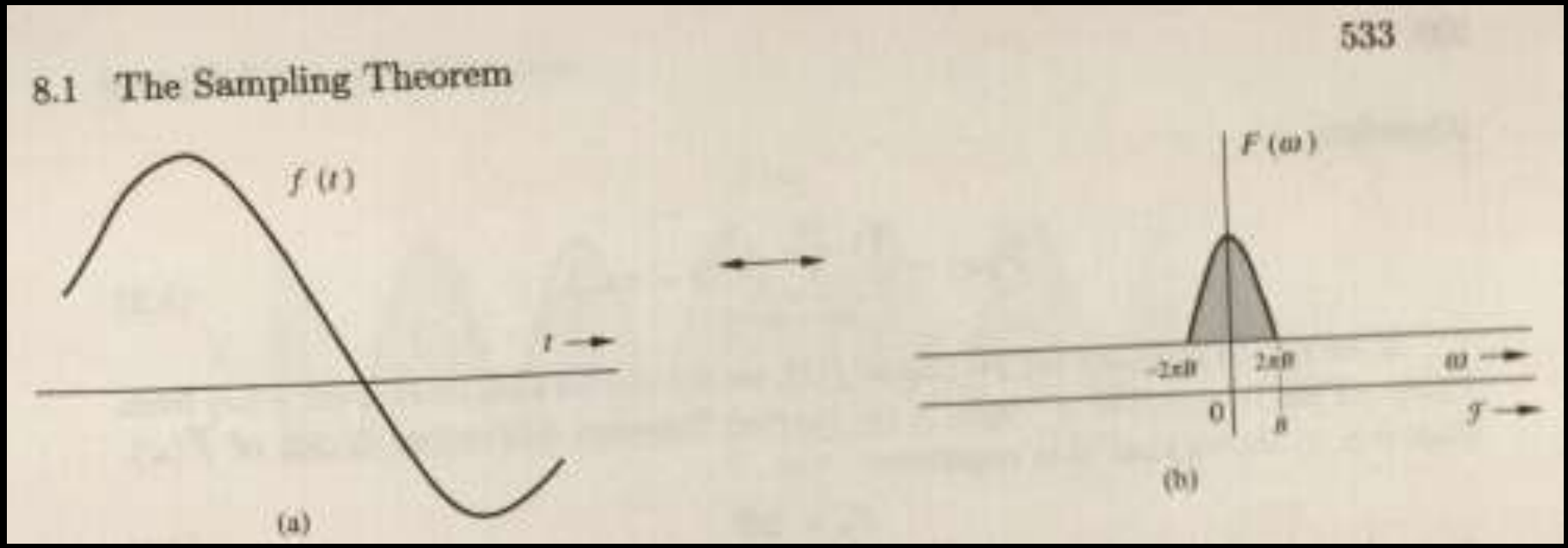
Example

↓

http://www.youtube.com/watch?v=jHS9JGkEOmA

Aliasing is not an esoteric Mathematical result derived from laborious amounts of Calculus. In fact it is commonly observed.

Ideal



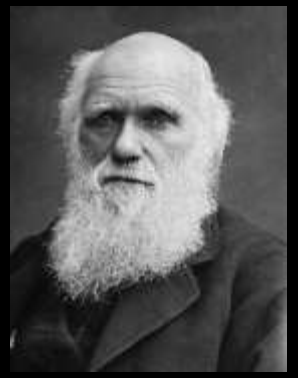
"Uncle" Nyquist

Nyquist Sampling Theorem

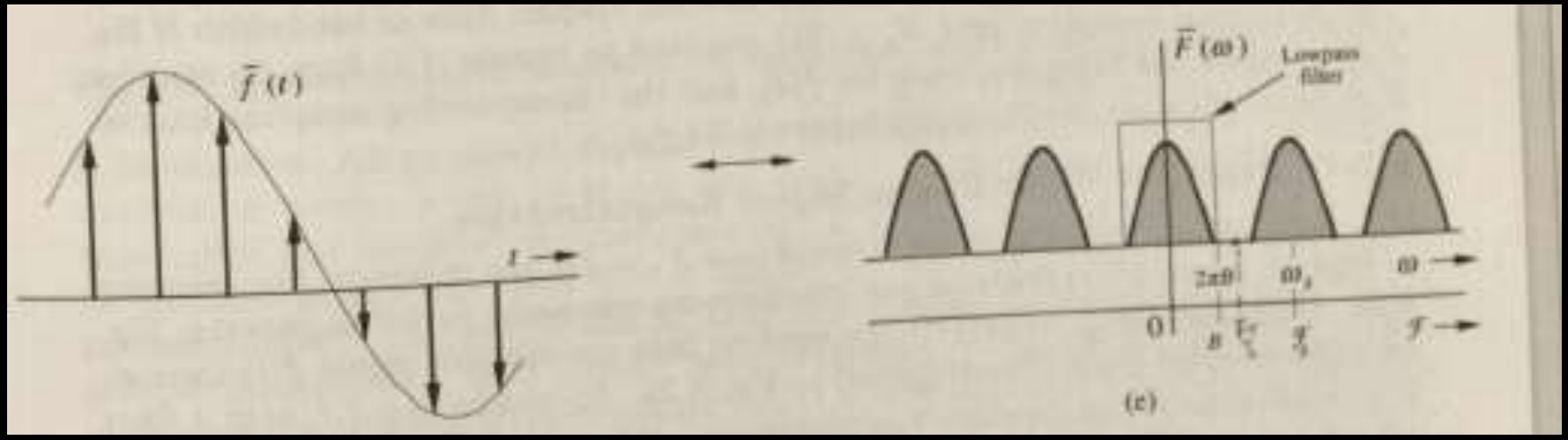
$$\therefore x(t)\delta_{T_s} \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{T_s}\delta_{f_s} * X(f)$$

Sampling signal every T_s in the time domain results in frequency domain "copies" every integer multiple of f_s. The "copies" are called aliases.

Reality- “The Origin of the Alias”



“Not an EE’s Uncle” Darwin



Nyquist's Theorem

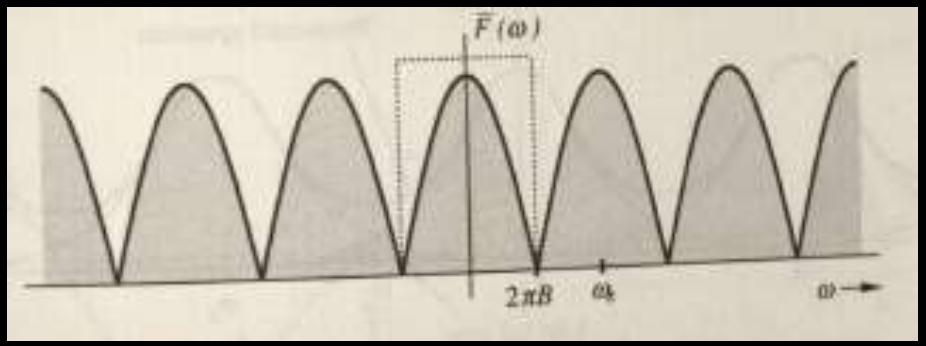
If the sampling rate, f_s, is not large enough (larger than twice the bandlimit, B) then the aliases will overlap: an effect known as Aliasing.

$$f_s > 2B$$

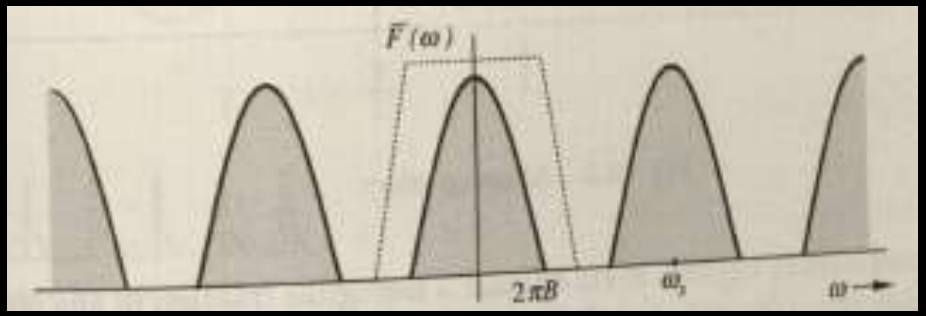
If and only if a signal is sampled at this frequency (or above) can the original signal be reconstructed in the time-domain.

The Nyquist Rate

Sampled at Nyquist Rate



Sampled above Nyquist Rate



Note how the aliases are spreading out as you increase F_s!

ALIASing

Sampled below Nyquist Rate

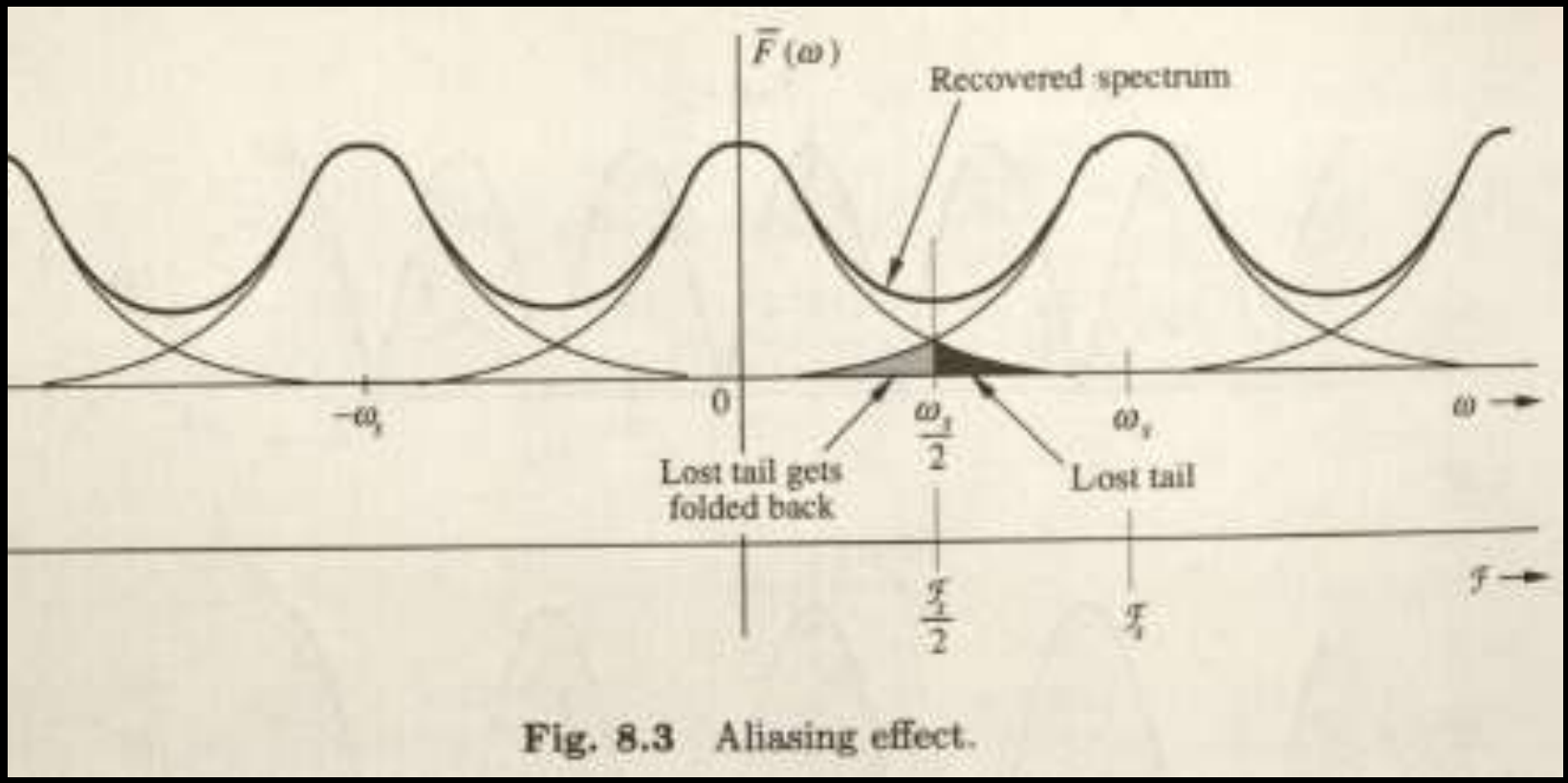


Fig. 8.3 Aliasing effect.

Example

$$x(t) = \cos(60t)$$

What is the bandlimit, B?

$$\cos(2\pi f_0 t) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2}[\delta(f + f_0) + \delta(f - f_0)]$$

So,

$$\cos(60t) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2}\left[\delta\left(f + \frac{60}{2\pi}\right) + \delta\left(f - \frac{60}{2\pi}\right)\right]$$

Thus the bandlimit is

$$B = \frac{60}{2\pi} \cong 9.5493\ Hz$$

Example

$$x(t) = \cos(60 * t)$$

What should the minimum Sampling Frequency (also called the Nyquist rate), F_s, be to prevent aliasing?

Using the Nyquist Sampling Theorem,

$$F_s > 2B \cong 19.0986 \text{ Hz}$$

$$\therefore F_s > 19.1 \text{ Hz}$$

Lab Report

You are given four different sampling rates (1000 Hz, 100 Hz, 20 Hz and 10 Hz).

It is obvious that the 10 Hz sampling rate is clearly less than the minimum frequency required to prevent aliasing (the Nyquist rate).

Recommended Reading

Read the following website for a better understanding of when one encounters Aliasing and the methods to avoid it:

http://www.ni.com/white-paper/3000/en/