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Approximate Geometric Methods in Application to the Modeling of Fiber Placed Composite Structures

Fiber placement is a modern method of constructing composite structures with complex curved surfaces. Modeling individual tows in a fiber placed part constitutes the challenge addressed by methods presented in this paper. One method is presented to approximate offset curves on a free form surface using the geometric constraints of the fiber placement process. A second method is presented to approximate a curve on a free form surface that can be used to generate a laminate family ply. We demonstrate that these approximation methods are sufficient for the accuracy of the fiber placement machine.
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Introduction

Developmental efforts in composites technology include a wide variety of primary and secondary structures in industries ranging from sporting goods to civil structures to high performance aircraft. In addition to potentially significant weight savings, composites offer the benefits of increased fatigue life, corrosion resistance, and life cycle cost savings. One such composite manufacturing technique is fiber placement. The basic component of a fiber placed part is the tow. The tow consists of a long and narrow, ~0.3175 cm, strip of resin impregnated fiber, ~0.025 cm in thickness. The minimum length of the tow is limited by the fiber placement machine-usually ~10 cm. The maximum length of a tow is limited only by the amount wound on the supply spool-typically in excess of 1500 meters. The tow itself consists of thousands of hair-like fibers aligned along the length of the tow and impregnated with resin. When cured with heat and pressure, the resin impregnated tow imparts strength and stiffness along its length. A fiber placement machine delivers from one up to approximately 30 adjacent tows in a single pass of the machine head. The tackiness of the tows allows them to be compacted onto a surface, which can contain concave or convex features, although concavity is limited by the head geometry. The adjacent tows placed by the fiber placement machine in a single pass are known as a course. Figure 1 shows a schematic of the fiber placement machine head placing a single course. Multiple courses placed alongside each other form a single layer, known as a ply, of the composite tow material. Multiple plies are placed atop each other to impart thickness to the shell-like structure.

Each tow in a course can be independently cut (dropped) or restarted (added), allowing precise control over the amount of material placed on the structure, and hence differing thickness regions on the part. In addition to drop and add capabilities, the feed rate of each tow is individually controlled, allowing a curved course with tows at the outside of the curve to be fed faster than those on the inside of the curve (see Fig. 2). The ability to feed tows at differential rates allows the fibers of the tow material to be steered in a relatively tight radius without buckling the tow material. Steered fibers can thus be aligned with the calculated loads of the entire structure, allowing continuous orientation of fibers along the load paths of the structure. Steered fiber architecture has the potential to offer significant weight savings by improving tailoring of local fiber orientation to the specific internal load path of the structure [1].

In a similar manner to Computer Numerical Control (CNC) machining, a fiber placement machine includes a programming system to plan the motions of the machine. Given a surface on which to place tows, a curve representing ideal fiber orientation, and specification of orientation control method; the offline programming system computes the path that the machine will take as it lays down each course. Paths of individual tows are not calculated. In addition, current offline programming systems convert free form surfaces to a discretized mesh on which the course paths are calculated. Meshing accuracy (the distance between a mesh element surface and the free form surface) has been found to be as loose as 13 mm [2], which translates to similar errors when calculating course paths. These errors result in unanticipated gaps or overlaps in tow material that must be detected *after* the part is manufactured.

The goal of the work presented in this paper is to develop algorithmic methods that simulate the position of individual tows in the fiber placed part. To eliminate errors associated with the offline programming systems' discretized representation of the design surface, our methods utilize NURBS representations of the design surface. The methods build upon an existing solid modeling kernel which implement NURBS surfaces and curves, as well as geometric calculations based on them. Our methods were developed using the Adaptive Modeling Language (AML) integrated development environment [3], enabling an object oriented imple-

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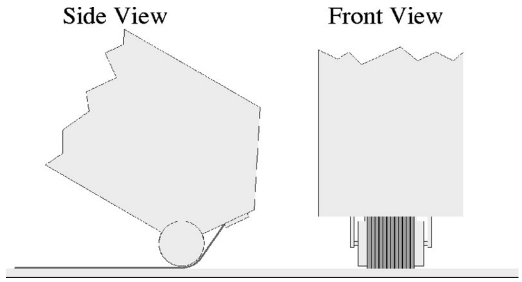

Fig. 1 Fiber placement head delivers multiple tows in a course

mentation of the Parasolid [4] solid modeling kernel, although any geometry engine supporting the required operations can be used (or developed). The methods described in this paper have been implemented in the Steered Composite Analysis and Design System (SCADS) [5,6], representing an integrated design for manufacturing/fiber steering system for the fiber placement process. Improved design tools such as SCADS are critical, because current capabilities of existing fiber placement hardware far exceed the capabilities of design engineering tools, particularly with respect to the ability to fabricate structures with controlled curvilinear fiber paths. The lack of robust analytical tools and design environments capable of modeling the complexity of steered fibers in the preliminary design phase are prohibiting optimal design solutions.

The first method presented in this paper uses the free form design surface, and a curve representing an initial fiber path, to calculate piecewise linear approximations of a series of offset curves on the surface. The method is used to represent the position of individual tows in a single course of the fiber placed part. By approximating curves on the actual design surface, our method avoids unanticipated gaps and overlaps associated with a discretized surface. It should be noted that the method is specifically motivated by, and limited to, a domain where the offset distance between curves (e.g. the width of the fiber placed tow) is no more than half the local radius of curvature of features on the surface (e.g. the fiber placement surface).

A second method addresses orientation of fibers to create a quasi isotropic material layup. As mentioned previously, a frequent design goal is to orient tows in a continuous manner along the primary axial load path of the structure. In addition, there will generally be in-plane loads in directions other than those of the primary load path. These local loads can always be resolved to principal components of axial, transverse and shear loads. If the local axial load direction is specified as $0°$, fibers can oriented at $90°$ to handle transverse loads, and $\pm 45°$ for shear loads. The fiber placement process allows the local definition of $0°$ to change continuously along a curvilinear load path. For purposes of this paper, a ply with fibers aligned along the primary load path is known as a $0°$ ply. A ply containing tows aligned at a fixed angle to the primary load path (usually $90°$ or $\pm 45°$) is known as a laminate family ply. The goal of the composite designer is to design a through-the-thickness sequence of $0°$ and laminate family plies that meets the strength requirements of the part. In a fiber placed part, where tows of the $0°$ ply are placed along a curved primary load path, a challenge is to model an appropriate definition of a laminate family ply.

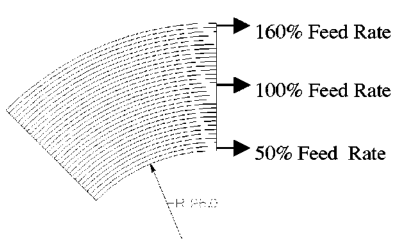

Fig. 2 Steered course illustrates difference in feed rate across tows in the course

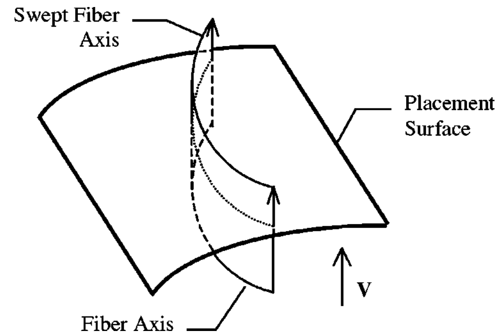

Fig. 3 The fiber axis is swept along *V* to create the initial reference curve

The tows in a $0°$ ply are modeled entirely with offset curves on the design surface. The second method presented in this paper is used to approximate a laminate family curve. The method uses each tow representation in the $0°$ ply to calculate a piecewise linear curve that preserves a fixed angle of intersection between the laminate family curve and each tow in the $0°$ ply. By preserving the angle of intersection between the laminate family curve and tows in the $0°$ ply, the fiber orientation of the laminate family ply can be kept constant.

Approximation of an offset curve on a surface

In this section, we describe our method of generating piecewise linear approximations to offset curves on an arbitrarily curved surface. The purpose of the method is to place points on a surface (the placement surface) that, when interpolated with a curve on the surface, will form a curve that is a constant offset from a reference curve on the surface. An offset curve on the surface is defined:

Given: a surface, $s(u,v)$; two curves on the surface, $r(u(t),v(t))$ and $r_d(u_d(t),v_d(t))$; and a point P on r. Let $D(P)$ be the shortest distance, measured along the surface, from P to r_d. If, $\forall P \in r$, $D(P)=d$, then r_d is an offset curve at distance, d, from r on the surface, s.

The method presented in this section produces an approximation of r_d given r and s. The method generates a piecewise linear approximation of r_d by using points on r to generate points on r_d. There are similarities between the offset curve as defined for fiber placement, and tool path curves for 5-axis CNC machining [7–16]. None of the methods developed for CNC machining attempt to preserve the property of constant offset distance as measured on the surface.

The inputs to the method are a placement surface, fiber axis, projection vector, curve generation tolerance and offset distance. All defining points of the offset curves are generated on the placement surface. The placement surface must be a surface whose partial derivatives are G^1 continuous. Furthermore, it must not intersect itself.

The method begins by creating an initial reference curve. The fiber axis is projected along the projection vector to the placement surface to create the initial reference curve. The fiber axis is a NURBS space curve that may or may not lie on the placement surface. Given the input, the method creates a swept surface with cross section defined by the fiber axis definition curve, and axis defined by the projection vector (Fig. 3). The curve generation tolerance is used to generate points on the reference curve. The points on the curve form a piecewise linear approximation, such

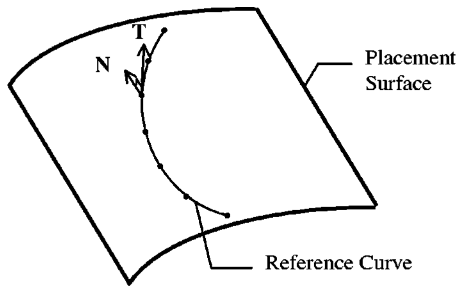

Fig. 4 Points are created along the reference curve, with normal and tangent vectors calculated at each point

that the maximum distance between the curve and any line segment defined by two adjacent points will be bounded by the tolerance.

An offset curve is approximated from a reference curve by interpolating points that are offset from points on the reference curve. The curve generation method creates a new curve by offsetting each defining point of the parent curve by a constant distance along the placement surface. This is approximated by calculating, at each defining point in the reference curve, the tangent vector to the curve (**T** in Fig. 4) and the normal vector to the surface at that point (**N** in Fig. 4). Using these vectors, an offset vector is calculated by taking the cross product of the normal vector and the tangent vector. The resulting vector, **O**, is tangent to the surface, and perpendicular to the curve. A new point is calculated by projecting the original point, P, along the offset vector by the offset distance, d. Finally, this point is projected along a surface normal to the nearest point on the surface using a Global Closest technique, resulting in P'. Figure 5 shows how P' is determined using P and **O**. The curve shows the placement surface cross section, while the vector shows the projection of P by distance d along the vector **O**. After all points on a reference curve have been offset, the offset points are interpolated with a curve which becomes the new reference curve for the next offset curve.

Resulting Error of Method to Generate Offset Curves on a Surface

In Fig. 5, it can be seen that the offset distance is used to calculate a point that typically does not lie on the surface. That point is then projected back to the surface. Using the method of Fig. 4 and Fig. 5, each point on the initial reference curve is offset n times, where n is the number of offset curves. Given that the distance between the points along the surface is not equal to the offset distance on non-planar surfaces, error can accumulate as multiple offset curves are generated. Two methods of approximating error are presented. The first method presented can be used to quickly approximate the error each time a point is offset during the curve generation process. The method can be used to apply a correction as each offset point is generated, although we will show that this is unnecessary within the application domain of fiber placement where permissible placement error is 100% of the offset distance over 50 offsets [2]. Further supporting this, material width variations of +/−3% may be expected [2], which cannot be factored into optimal tow path as these variances occur over the continuous path.

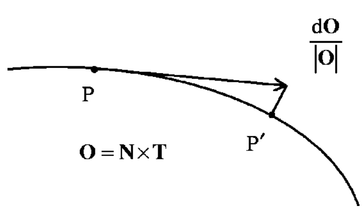

Fig. 5 Cross section of placement surface showing calculation of offset point

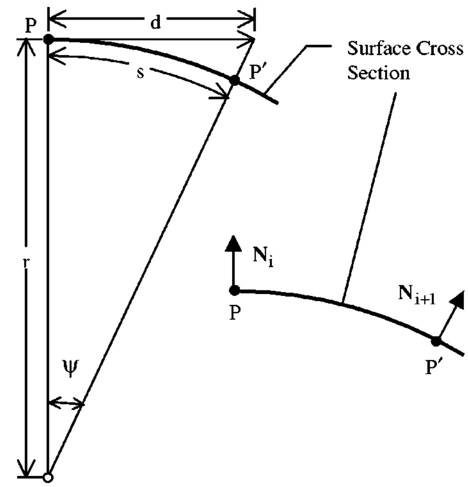

Fig. 6 Error associated with placing a point on a surface with circular arc cross section

To determine the error at each offset step, it is necessary to compare the offset distance to the length of a plane curve that results from slicing the placement surface perpendicular to the offset curve. Assuming that the offset distance is very small compared to the local surface curvature and extent, a circular approximation of the plane curve is appropriate. In the domain of fiber placement, surfaces are typically smooth (e.g., airplane wings) and contain five or less features that exhibit significant changes in curvature. The cross section of the tow is approximated by a rectangle whose width is equal to the offset distance, and a height of less than 1/10 the offset distance. The material properties of the tow do not allow the cross section to bend significantly, or it will split. This limitation allows a maximum curvature corresponding to a local surface cross section radius of 0.635 cm [2], or about twice the tow width. Surface features of this radius occur in five or less offsets over a part containing 10^3 to 10^4 offsets, with a more typical radius of 50 or more times the offset distance.

We define our offset error measure as the difference between the desired offset, d, and the actual distance, s, between a point and its offset as measured on the surface:

$$\text{Error} = (d - s) \tag{1}$$

A closed form equation for the error of Eq. (1) can be derived when the cross section of the placement surface, between the two points shown in Fig. 6, is a circular arc. It should be noted that the surface cross section shown in Fig. 6 is repeated so that normal vectors to the surface can be clearly seen.

$$l = r\psi$$

$$r = \frac{d}{\tan \Psi}$$

$$\Psi = \cos^{-1}\left(\frac{\mathbf{N}_i \cdot \mathbf{N}_{i+1}}{|\mathbf{N}_i||\mathbf{N}_{i+1}|}\right)$$

$$\text{Error} = d\left(1 - \frac{\Psi}{\tan \Psi}\right) \tag{2}$$

Thus, a closed form solution for the error is possible for a design surface whose cross section as described by corresponding points of a series of offset curves is a circular arc. This is rarely the case in our application, hence Eq. (2) is simply an approximated error measure. It is useful to consider two cases where curvature of the surface does change over the offset distance, and to determine whether these cases invalidate the use of the circular approximation in determining the error of the offset process. The first case involves the first half of the offset distance being flat, while the second half exhibits maximum curvature of twice the offset distance. In such a case, the actual error occurs over half the offset distance. The angle measured between normal vectors will be $\tan^{-1}(1/4)$. Given the offset distance, d, the calculated error is $0.02d$, while the actual error is $0.01d$. A second case involves an inflection in the plane curve with equal portions of the curve ahead of and behind the inflection. The bending properties of the tow allow an inflection, but the radius of curvature on either side of the inflection point must be at least twice the minimum cross section radius mentioned above. The normal vectors in this case will be equal, leading to zero approximated error. The actual error is $0.005d$. In a domain where 100% error is permissible over 50 offsets, these approximation errors are not significant. Additionally, it can be seen that the maximum true error for a single offset is $0.073d$ when the placement surface exhibits maximum curvature of radius $2d$, corresponding to $\psi=26.5°$. Where 100% error over 50 offsets is permissible, approximately 14 offsets of maximum curvature must occur out of 50. Thus, in the domain of fiber placement, where features of high or rapidly changing curvature occur with very small frequency, correction to the original offset method is not required.

An alternative method of approximating error uses length calculations on a curve constructed on the surface, but we will show that this method does not offer an advantage over the method described above. As offset curves are successively placed on the surface, corresponding points on each curve will lie at approximately the offset distance from each other. These corresponding points can be used to construct a parametric curve in the placement surface. The parametric curve will intersect the offset curves at approximately 90°. The arc length along this parametric curve (with parameterization variable u) between points P and P' is then defined by [17]:

$$l=\int_{P}^{P'}\sqrt{\left(\frac{d(x(u))}{du}\right)^2+\left(\frac{d(y(u))}{du}\right)^2+\left(\frac{d(z(u))}{du}\right)^2}du \quad (3)$$

Typical implementations compute approximations to Eq. (3) (e.g. [18]) because a closed form solution is not generally available. The high cost of this method motivates an approximate method that uses the calculations initially used to generate the corresponding points on the offset curves. The results of quantitative studies provide evidence that the method using approximating circular cross sections is sufficient in the domain of fiber placement.

Both approximation methods (Eq. (2) and Eq. (3)) are used to estimate the cumulative error of placing offset curves on an arbitrarily curved surface, and the results compared. Figure 7 and Fig. 8 show two placement surfaces with offset curves placed by the SCADS application using the method of this paper. A cross section of each surface is shown below the isometric view of the surface.

Figure 7 shows an arbitrary surface with multiple inflection points, created to gauge the effect of those inflection points on the error approximation. The surface represents more severe curvature than is actually observed in fiber placed parts. The methods of Eq. (2) and Eq. (3) are used to estimate the placement error in locating the 124 points shown in Fig. 7. The nominal length measured along the surface between P_1 and P_n is calculated from $d(n-1)$ using non-specific length units. With an offset distance, d, of 0.150, the nominal length is 18.450. Two approximations of error are calculated-one using approximating circles, and one using a spline interpolating the points P_1 through P_n. As offset curves are generated, the surface normal vector is queried at each of the points shown in Fig. 7. No attempt is made to correct offset points

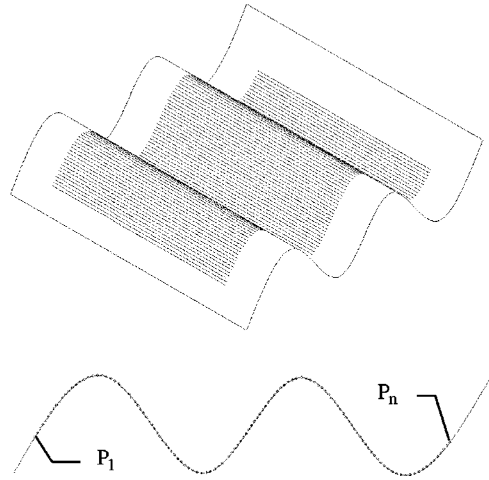

Fig. 7 Offset curves on surface with inflections

as they are generated. Using Eq. (2), the total error is estimated to be 0.0508 length units, or 33.9% of the offset distance, a single tow width. As a comparison, an interpolating spline is constructed that passes through points P_1 through P_n. The length of the interpolating spline is approximated using a query to the geometry engine. The length calculation is accurate to 10^{-8} for a length less than 1000 [3]. The length of the interpolating spline is approximated to be 18.400. The difference between the approximated length of the interpolating spline (18.400) and the nominal length (18.450) is the approximated error: 0.050, or 33.3% of the offset distance. We can see that the two approximation methods produce an error estimate that is of the same magnitude.

Another example of a placement surface with a non-circular cross section is the outer surface of an airplane wing (Fig. 8). For this example, a wing surface, based on the Raytheon Premier I small business jet [19], is constructed. One half of the wing (to the left of the fuselage) is constructed with a root chord length, C_r, 225.81 cm, tip chord length, C_t, 113.03 cm and a root to tip span of 731.01 cm. Because the airfoil shapes of the Raytheon Premier I remain proprietary, representative airfoil shapes are chosen from publicly available literature. The assumed root airfoil shape is of the NACA 23014 profile, while the tip airfoil is of the NACA 23012 profile [20]. The surface is created as a ruled surface between the root and the tip airfoil. Figure 8 shows a wireframe representation of the placement surface. Figure 8 also shows the region of the surface on which offset curves are generated. This region is represented as the shaded portion of Fig. 8. Using an initial curve that coincides with the leading edge of the wing, 790 offset curves are placed within the shaded region, corresponding

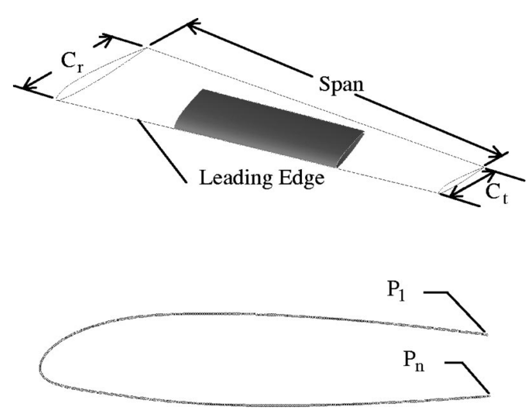

Fig. 8 Offset curves on representative wing surface

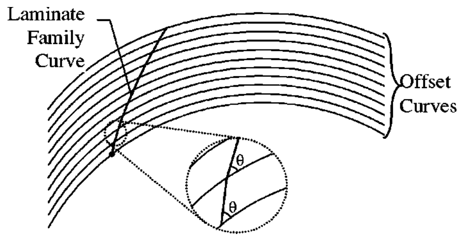

Fig. 9 A laminate family curve at angle θ to offset curves

to a single ply of a theoretical wing skin panel fabricated using fiber placement techniques. Figure 8 also shows a cross section (taken by aligning the normal of a cutting plane with the leading edge) of offset curve defining points corresponding to the point of each offset curve that is closest to the root of the wing surface.

The method of the previous example is used to estimate the placement error in locating the 790 points shown in the cross section of Fig. 8. The error is first approximated by constructing an interpolating spline that passes through points P_1 through P_n, approximating its length (implemented by the geometry engine), and comparing that length to the nominal length of $d(n-1)$. The length of the curve constructed through points P_1 through P_n is approximated as 300.591 cm. For the offset distance, d, of 0.381 cm, the nominal length is 300.609 cm. Therefore, the error approximated by constructing an interpolating spline is 0.018 cm, or 4.7% of the offset distance. As offset curves are generated, the surface normal vector is queried at each of the points from P_1 to P_n. Using Eq. (2), the total error is estimated to be 0.024 cm, or 6.2% of the offset distance. The difference between the two error approximations is 0.006 cm. Again, the two approximation methods predict error of the same order. It should be noted that the approximated placement error, after placing 790 points is just 4.7% of the offset distance, a single tow width. So, the error of our approximation method is significantly better than the previously stated permissible error of 100% of the offset distance over 50 offsets.

Approximation of a Laminate Family Curve

The purpose of the method presented in this section is to construct a laminate family curve based on a field of offset curves in an arbitrary surface. The method is a companion to the method used to place offset curves on a surface. A discussion of the application of the laminate family curve in the domain of fiber placement, and consequent choices for the designer, is followed by the method of approximation. A laminate family curve is defined:

Given: n offset curves, $\{C_1, \ldots, C_i, \ldots, C_n\}$; a curve, l, in the surface of the offset curves that does not intersect itself and intersects each C_i exactly once at point S_i; and a laminate family angle, θ. If, $\forall S_i$, the angle between the tangent to l at S_i and the tangent to C_i at S_i equals θ, then l is a laminate family curve.

The method presented here computes a piecewise linear approximation of a laminate family curve. Figure 9 shows a piecewise linear approximation to a laminate family curve, calculated using a set of offset curves. The curves are offset by a constant distance, the tow width. In keeping with the terminology of the introduction, the set of offset curves is called the 0° ply. The detail of Fig. 9 shows that the intersection of each linear segment of the piecewise linear laminate family curve with one of the offset curves occurs at the laminate family angle, θ.

Where the motivation of the offset curves of the 0° ply is to follow the primary load path in a structure, the laminate family curve is aligned to coincide with offset load directions defined at a constant angle to the primary load path. The curve is used to orient tows at a nominal angle of intersection to the 0° ply.

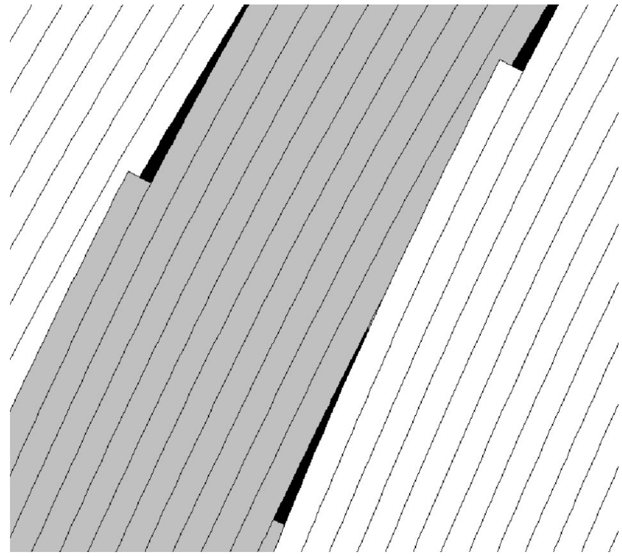

Fig. 10 Tows are dropped to control overlaps and gaps (shaded black) between neighboring courses

As with a 0° ply, the tows in a laminate family ply are delivered as a series of courses, with tows in a single course preserving constant offset to each other. Therefore, the laminate family curve described here is used as a reference from which offset curves for the course are constructed. It is obvious that, as tows are offset from the original laminate family curve, the prescribed angle of intersection cannot be preserved within a single course. For each subsequent course, a new laminate family curve can be generated to approximate the desired angle of intersection. As a consequence, tows of neighboring courses may begin to overlap each other, necessitating the discontinuation (dropping) of a tow to avoid undesired buildup of material. Figure 10 shows a close-up of three courses each generated with a laminate family curve. It can be seen that dropping of tows introduces gaps and overlaps of material, which in turn affects the strength of the part. The ratio of overlap to gap areas can be controlled by specifying the amount of overlap that occurs before a tow is dropped. Thus, a choice results for the designer:

1. Fill a ply with minimal material gaps and overlaps by preserving constant offsets with the potential result of large differences from nominal orientation angle as offsets are placed
2. Fill a ply with minimal error in orientation angle with the potential result of large numbers of material gaps and overlaps.

The former lends itself to a band offset ply of stated orientation, as previously described, and the latter lends itself to a laminate family ply.

The laminate family curve is constructed from a specified point on one of the offset curves in the 0° ply. To calculate the next point, P', two vectors are computed at point P; the tangent vector to the curve, **T**, and the normal vector to the placement surface, **N**. An orientation vector, **O**, is determined by rotating **T** about **N** by the laminate family angle, θ. The vectors **O** and **N** are used to orient a plane containing those vectors. P' is then found by intersecting the plane with the next offset curve. Figure 11 shows the vectors used to orient the plane, which in turn, is used to determine P'.

It is obvious that the laminate family angle is only preserved between Curve$_i$ and segment PP', while error can occur in the intersection of segment PP' with Curve$_{i+1}$. As noted previously, the minimum radius of curvature of the placement surface is twice

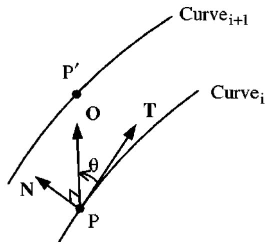

Fig. 11 P' is found by orienting a plane at P and intersecting with the next curve

the offset distance between Curve$_i$ and Curve$_{i+1}$. To prevent buckling of the tow material, the minimum radius of curvature of any offset curve is 63.5 cm, or about 180 times the offset distance [2]. As such, we can assume that the offset curves in the neighborhood of segment PP' are well approximated by circular arcs and a closed form solution exists for the angle of intersection between segment PP' and Curve$_{i+1}$. In the case of an interpolating spline, this error represents an upper bound on the error that occurs at point P'.

Assuming that segment PP' is coplanar with approximating arcs of Curve$_i$ and Curve$_{i+1}$, all can be expressed in planar polar coordinates with the origin located at the center of curvature of the approximating arcs. If the angle of intersection between Curve$_i$ and segment PP' is θ, the error is equal to the angular distance between P and P'. This error can be expressed:

$$\text{Error} = \theta - \cos^{-1}\left(\frac{R\cos\theta}{R+d}\right) \tag{4}$$

Where R is the radius of the approximating arc to Curve$_i$ and d is the offset distance to Curve$_{i+1}$. Recall that d is equal to a single tow width in the domain of fiber placement. The closed form solution for error shown in Eq. (4) can be used to determine whether the accuracy of the approximation is sufficient for the application domain of fiber placement. The allowable error is described first.

The United States Federal Aviation Administration (FAA) allows for an orientation error of 2.08 cm in one meter when fabricating parts to be used for mechanical testing [21]. This is equivalent to a 1.2° variation if linear, and a higher variation if the fiber curves over and then back across the nominal orientation. The error is allowed in specimens that are fabricated for the purpose of establishing material properties. This allowable represents a lower bound for the error that can be accepted in a manufactured part. Common practice allows for 2.0° variation in manufactured parts. It should be noted that the allowable error described by the FAA represents error in the part *after* manufacture and is presented here as a comparison for the approximation error.

In fiber placed parts, laminate family angle magnitudes less than 30° are generally not found, with angles commonly expressed between $-90°$ and $90°$. As mentioned previously, tows in a 0° ply can exhibit a minimum radius of curvature of 63.5 cm. Tow width (offset) values can vary from 0.250 to 0.0.635 cm. It can be seen that the combination of large tow width (d) and small radius of curvature (R) in the 0° ply, and small magnitude of laminate family angle (θ) result in the greatest error. A maximum error case in fiber placement thus occurs with $d=0.635$, $R=63.5$, and $\theta=30°$. The resulting error is about 1.0°, or about half that allowed in current common practice. Thus, the method of approximating a laminate family curve provides sufficient accuracy.

Conclusions

Fiber placement of composite structures promises reduced cost in the form of production waste savings, and increased structural strength-to-weight ratios that can be realized with tailored fiber orientations. To date, reduced production costs have been realized with fiber placement, but a lack of robust design and analysis tools prevents designers from fully exploring weight saving design solutions in the preliminary design phase. A design and analysis tool for fiber placed composites must be capable of modeling the part down to the level of the tow, in turn requiring a method to represent the position of individual tows on the design surface. In this paper, we have formally defined two types of curves required to represent individual tows in a fiber placed part; the offset curve on a surface, and the laminate family curve; and present approximate methods to compute both types. Accuracy of these methods is shown to be acceptable for production use.

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