EECS 360 Short Quiz #2

Signal and System Analysis

October 4, 2016

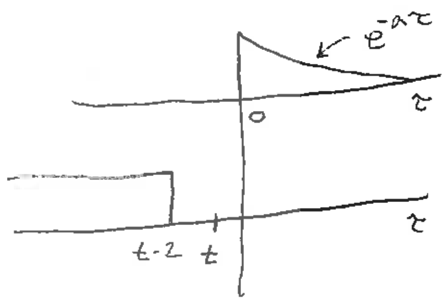
Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (35 %) Determine the output $y(t)$ of a system with impulse response $h(t) = \delta(t+1) + u(t-2)$ and input $x(t) = e^{-at}u(t)$.

Because of linearity, we can convolve these two separately

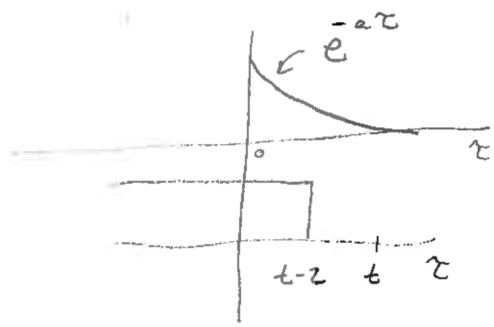
$e^{-at}u(t) * u(t-2)$



← these two "overlap" when $0 < t-2$

$\Rightarrow t > 2$

$x(t) * \delta(t+1) = x(t+1)$



$$\begin{cases} \int_0^{t-2} e^{-a\tau}\, d\tau = \frac{1}{a}\left[1 - e^{-a(t-2)}\right] & t > 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow y(t) = e^{-a(t+1)}u(t+1) + \frac{1}{a}\left[1 - e^{-a(t-2)}\right]u(t-2)$$

2. (40 %) Determine the stability and causality of the LTI systems with the following impulse responses:

(a) $h[n] = 2^{-n}u[n-1]$
(b) $h(t) = e^{3t}u(1-t)$
(c) $\cancel{h(t)}$ $h[n]$ $= 2^{n}u[n]$
(d) $h(t) = e^{-3|t|}$
(e) $h(t) = te^{-3t}u(t)$
(f) $h(t) = e^{-t}\cos(3t)u(t)$

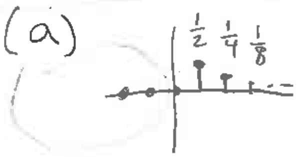
Test for causality $h(t) \overset{?}{=} 0$ for $t < 0$

$h[n] = 0$ for $n < 0$

Test for stability $\int_{-\infty}^{\infty} |h(t)|\,dt \overset{?}{<} \infty$

$\sum_{n=-\infty}^{\infty} |h[n]| \overset{?}{<} \infty$

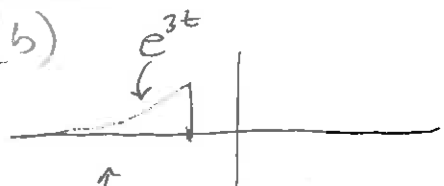
(a)



$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ $\quad 1 \overset{\checkmark}{<} \infty$

zero for $n < 0$ $\Rightarrow$ causal and stable

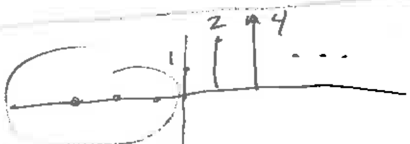
(b)



$\int_{-\infty}^{-1} e^{3t} = \frac{1}{3}[e^{-1} - 0] = \frac{1}{3e} \overset{\checkmark}{<} \infty$

non-zero for $t<0$ $\Rightarrow$ non-causal and stable

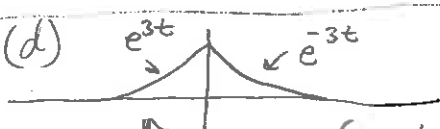
(c)



$\sum_{n=0}^{\infty} 2^n = \infty$ $\Rightarrow$ causal and unstable

zero for $n<0$

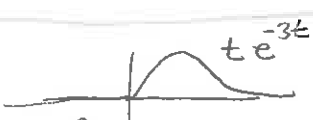
(d)



$\int_{-\infty}^{\infty} e^{-3|t|}\,dt = 2\int_0^{\infty} e^{-3t}\,dt = \frac{2}{3} \overset{\checkmark}{<} \infty$ $\Rightarrow$ noncausal and stable

non-zero for $t<0$

(e)



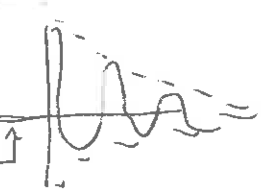
$u = t \quad dv = e^{-3t}$
$du = 1 \quad v = -\frac{1}{3}e^{-3t}$

$\int_0^{\infty} te^{-3t}\,dt = \underbrace{-\frac{t}{3}e^{-3t}\Big|_{t=0}^{\infty}}_{0} + \underbrace{\frac{1}{3}\int_0^{\infty} e^{-3t}\,dt}_{1/9 < \infty}$

zero for $t<0$

$\Rightarrow$ causal and stable

(f)



zero for $t<0$

$\int_0^{\infty} e^{-t}\cos(3t)\,dt < \int_0^{\infty} e^{-t}\,dt = 1 \overset{\checkmark}{<} \infty$

$\Rightarrow$ causal and stable

3. (25 %) Given the convolution sum

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m],$$

show that this can also be expressed as

$$y[n] = h[n] * x[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m].$$

$$\sum_{m=-\infty}^{m=\infty} x(m)h(n-m) \qquad \text{let } \ell = n-m \Rightarrow m = n-\ell$$

$$= \sum_{n-\ell=-\infty}^{n-\ell=\infty} x[n-\ell]h[\ell]$$

$$= \sum_{\ell=-\infty-n}^{\infty-n} x[n-\ell]h[\ell]$$

$$\boxed{= \sum_{\ell=-\infty}^{\infty} x[n-\ell]h[\ell]}$$