EECS 360 Short Quiz #4
Signal and System Analysis
December 9, 2014

Name: KEY

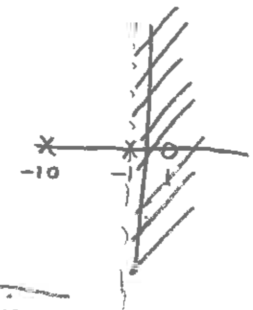
Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (35 %) Let $h(t)$ be a right-sided impulse response of a system and its Laplace tranform is given by

$$H(s) = \frac{10(-s+1)}{(s+10)(s+1)}.$$

   (a) Sketch the pole-zero plot of this system and indicate the region of convergence (ROC).
   (b) Is this system stable? Justify your answer.
   (c) Let $H_I(s)$ be the transfer function of the *inverse* system of $H(s)$, i.e., $H_I(s)H(s) = 1$. We require $H_I(s)$ to be *stable*, specify $H_I(s)$ and its region of convergence.
   (d) Now we require $H_I(s)$ to be *causal*, does your answer from part (c) change?
   (e) For this system, is it possible for the inverse system to be causal *and* stable? Justify your answer.

(a) $H(s)$ has poles at $s=-10$ and $s=-1$.
Because $h(t)$ is given to be right sided,
the ROC is to the right of $s=-1$ => ROC $\mathrm{Re}\{s\}>-1$



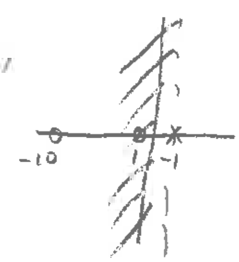
(b) It is stable because the ROC includes the $j\omega$-axis

(c) The algebraic form of $H_I(s)$ is simple

$$\boxed{H_I(s) = \frac{1}{H(s)} = \frac{(s+10)(s+1)}{10(-s+1)}},$$

because we require stability, and there is a pole at $s=1$ the ROC must be to the left of $s=1$, so that it includes the $j\omega$-axis

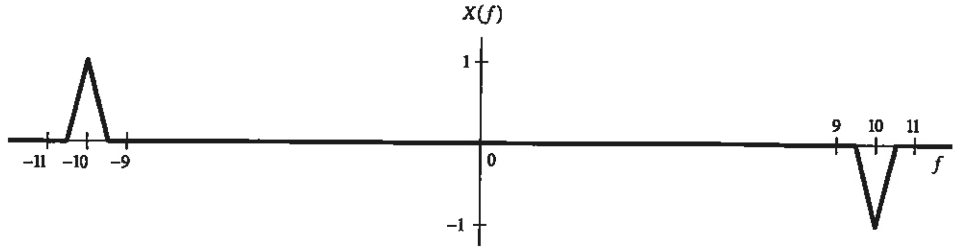
$\boxed{\text{ROC } \mathrm{Re}\{s\} < +1}$



(d) The answer in part (c) is left-sided (non-causal). If we have $\boxed{\text{ROC } \mathrm{Re}\{s\} > +1}$ this will give us a causal (but unstable!) system

(e) Because the poles and zeros are on opposite sides of the $j\omega$-axis, there is no way to have the ROC of both the original and inverse systems to be right-sided (causal) and include the $j\omega$-axis (stable).

2. (35 %) The continuous-time signal $x(t)$ with CTFT $X(f)$ as depicted below is impulse sampled.



(a) Provide a carefully labeled sketch of the CTFT of the impulse sampled signal [i.e., sketch $X_\delta(f)$] for the following sampling intervals:

i. $T_s = \frac{1}{28}$

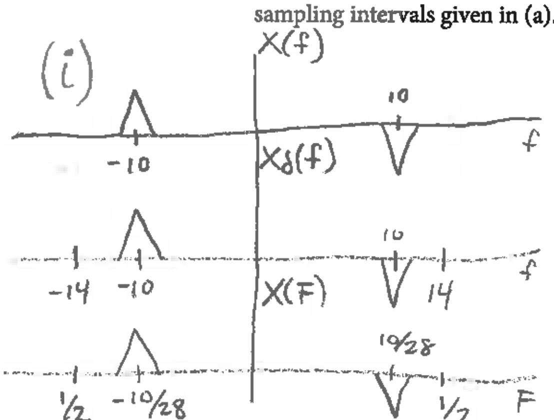
ii. $T_s = \frac{1}{14}$

iii. $T_s = \frac{1}{10}$

In each case, identify whether aliasing occurs.

(b) Let $x[n] = x(nT_s)$. Provide a carefully labeled sketch of the DTFT of $x[n]$ [i.e., sketch $X(F)$] for each of the sampling intervals given in (a).

(i)

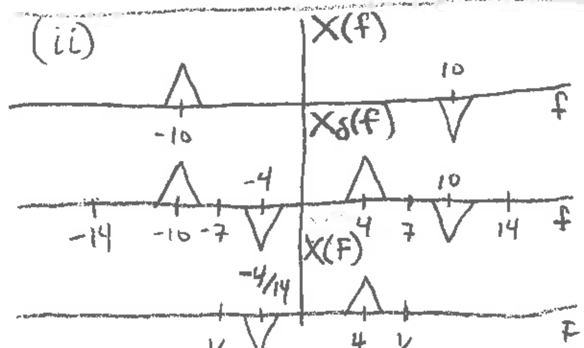


$T_s = \frac{1}{28}$, $f_s = 28$ $\frac{f_s}{2} = 14$

← periodic with period of $f_s = 28$, I have sketched one period, no aliasing

← always periodic with period 1, I have sketched one period, no aliasing

(ii)

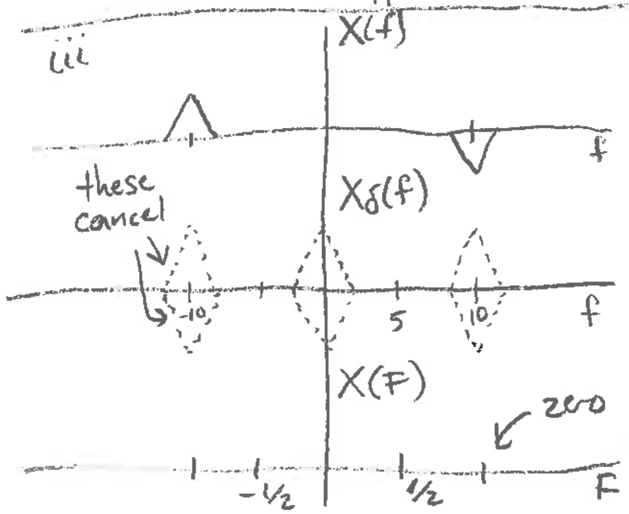


$T_s = \frac{1}{14}$ $f_s = \frac{1}{14}$ $\frac{f_s}{2} = 7$

← periodic with period of $f_s = 14$, I have sketched two periods, aliasing

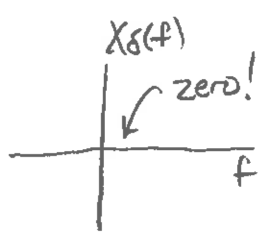
← always periodic with period 1, I have sketched one period, aliasing

iii



$T_s = \frac{1}{10}$ $f_s = 10$ $\frac{f_s}{2} = 5$

→



← periodic with period $f_s = 10$

aliasing

aliasing

3. (30 %) Consider the causal LTI system described by the difference equation

$$y[n] = x[n] - y[n-1] + \frac{3}{4}y[n-2].$$

(a) Determine the transfer function, $H(z)$, of this system.

(b) Sketch the corresponding pole-zero plot and indicate the region of convergence (ROC).

(c) Based on the pole-zero plot, is this system stable? Justify your answer.

(d) Determine the output of the system, $y[n]$, when the input is $x[n] = \delta[n] + \frac{3}{2}\delta[n-1]$.

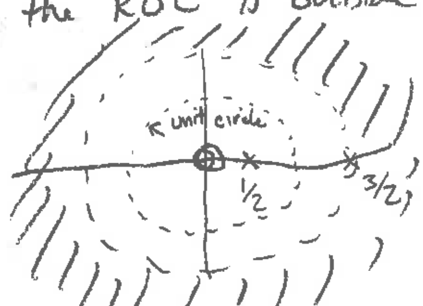
$$\boxed{X(z) = 1 + \tfrac{3}{2}z^{-1}}$$

(a) $y[n] + y[n-1] - \frac{3}{4}y[n-2] = x[n]$

$$\downarrow z$$

$$Y(z)\left[1 + z^{-1} - \tfrac{3}{4}z^{-2}\right] = X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)} = \boxed{\frac{1}{1+z^{-1}-\frac{3}{4}z^{-2}} = \frac{z^2}{z^2+z-\frac{3}{4}} = \frac{z^2}{(z-\frac{1}{2})(z+\frac{3}{2})}}$$

(b) There are poles at $z = 1/2$ and $z = -3/2$, and two zeros at $z = 0$. Because the system is given to be causal, the ROC is outside the outermost pole



(c) Because the ROC does not include the unit circle, the system is not stable.

(d) $$Y(z) = H(z)\cdot X(z) = \underbrace{\frac{1}{(1-\frac{1}{2}z^{-1})\cancel{(1+\frac{3}{2}z^{-1})}}}_{H(z)} \cdot \underbrace{\frac{\cancel{(1+\frac{3}{2}z^{-1})}}{1}}_{X(z)} = \underbrace{\boxed{\frac{1}{1-\frac{1}{2}z^{-1}}}}_{Y(z)}$$

$$\boxed{y[n] = \left(\tfrac{1}{2}\right)^n u(n)} \xleftrightarrow{z} Y(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$