EECS 360 Short Quiz #1

Signal and System Analysis

September 11, 2014

Name: KEY

Open book, open notes, no calculator. Be neat, write legibly. For full credit you must show all work and justify each answer. You may write on the both sides of the paper, and use additional sheets of paper if needed.

1. (25 %) Find the even part, $x_e(t)$, and the odd part, $x_o(t)$, of the following signals:

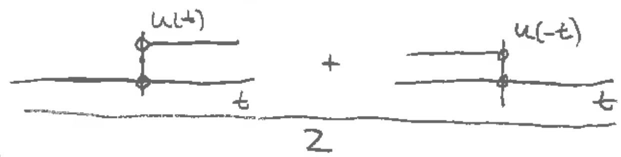
   (a) $x(t) = u(t)$.

   (b) $x(t) = 4t - 1$.

   (c) $x(t) = e^{-j8\pi t} + e^{j11\pi t}$.

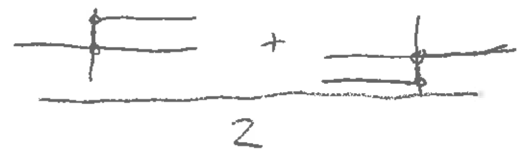
$$\boxed{x_e(t) = \frac{x(t)+x(-t)}{2} \qquad x_o(t) = \frac{x(t)-x(-t)}{2}}$$

(a) $x_e(t) = \frac{u(t)+u(-t)}{2} =$



$$\boxed{= \tfrac{1}{2} \leftarrow \text{for all } t}$$

$x_o(t) = \frac{u(-t)-u(-t)}{2} =$



$$\boxed{= \tfrac{1}{2}\,\mathrm{sgn}(t)}$$

(b) $x_e(t) = \frac{4t-1 + (4(-t)-1)}{2} = \frac{4t-1-4t-1}{2}$ $\boxed{= -1}$

$x_o(t) = \frac{4t-1 - (4(-t)-1)}{2} = \frac{4t-1+4t+1}{2}$ $\boxed{= 4t}$

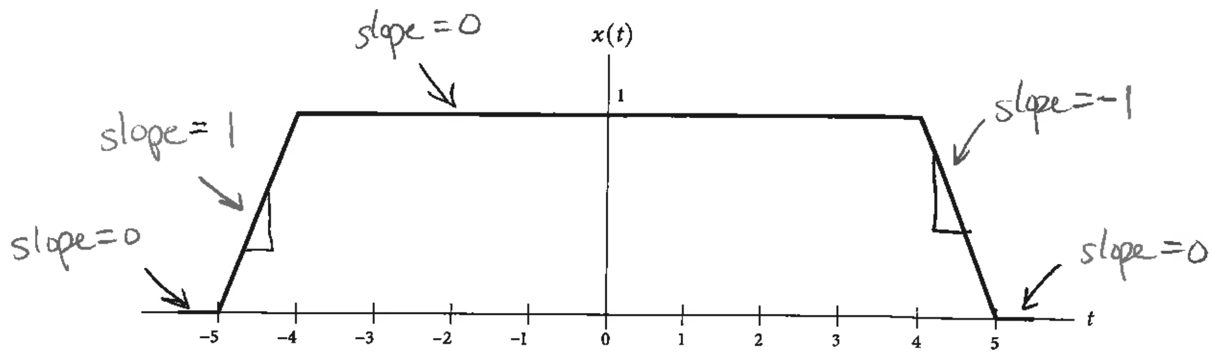
(c) If we are really clever, we would recall that sin(t) is odd and cos(t) is even, and we would break these into even/odd with Euler's identity. But if we are not clever...

$$x_e(t) = \frac{(e^{-j8\pi t} + e^{j11\pi t}) + (e^{j8\pi t} + e^{-j11\pi t})}{2} = \frac{(e^{j8\pi t} + e^{-j8\pi t})}{2} + \frac{(e^{j11\pi t} + e^{-j11\pi t})}{2} \boxed{= \cos(8\pi t) + \cos(11\pi t)}$$

$$x_o(t) = \frac{(e^{-j8\pi t} + e^{j11\pi t}) - (e^{j8\pi t} + e^{-j11\pi t})}{2} = -j\frac{(e^{j8\pi t} - e^{-j8\pi t})}{2j} + j\frac{(e^{j11\pi t} - e^{-j11\pi t})}{2} \boxed{= -j\sin(8\pi t) + j\sin(11\pi t)}$$

... then we get to see Euler's identity in action

2. (35 %) Consider the trapeziodal-shaped pulse $x(t)$ below.



(a) Sketch the output $y(t)$ of the system $y(t) = x(5t)$.

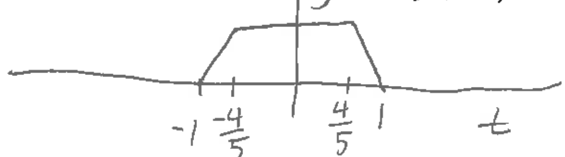
(b) Sketch the output $y(t)$ of the system $y(t) = x(10t - 5)$.

(c) Sketch the output $y(t)$ of the system $y(t) = \frac{d}{dt}x(t)$.

(d) Determine the total energy of your answer, $y(t)$, in part (c).

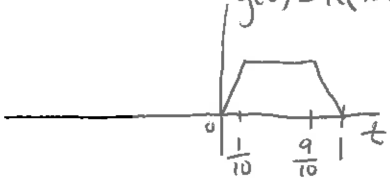
(a)

$y(t) = x(5t)$



(b)

$y(t) = x(10(t - \frac{1}{2}))$



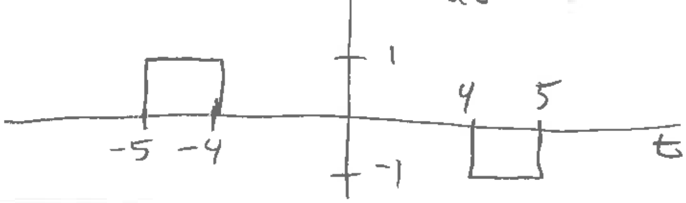
$y(t) = x(10t - 5) = x(10(t - \frac{1}{2}))$

compress by a factor of 10

delay by $\frac{1}{2}$

(c)

$y(t) = \frac{d}{dt} x(t)$



energy = 1 energy = 1

$$\int_{-\infty}^{\infty} \left(\frac{d}{dt} x(t)\right)^2 dt = 1 + 1 \boxed{= 2}$$

3. (40 %) The systems below have excitation $x(t)$ or $x[n]$ and response $y(t)$ or $y[n]$. For each system, determine whether it is (i) linear, (ii) time invariant, (iii) causal, and (iv) stable.

(a) $y[n] = |x[n]|$

(b) $y(t) = x(t-3) + 2\int_{-\infty}^{0.5t} x(\tau)\,d\tau$

(a) Linear ? $\boxed{\text{No}}$ $x_1[n] = g[n] \rightarrow y_1[n] = |g_1[n]|$ and $K y_1[n] = \underbrace{K|g_1[n]|}$

$x_2[n] = Kg[n] \rightarrow y_2[n] = \underbrace{|K||g_1[n]|}$

these two are not the same

Time Invariant $\boxed{\text{Yes}}$ $x_1[n] = g[n] \rightarrow y_1[n] = |x_1[n]| = g[n] \xrightarrow[n_0]{\text{delay}} y_1[n-n_0] = \underbrace{|g[n-n_0]|}$

$x_2[n] = g[n-n_0] \rightarrow y_2[n] = \underbrace{|g[n-n_0]|}$

these two are the same

Causal $\boxed{\text{Yes}}$ $y[n]$ depends only on $x[n]$ (the current input) and no future inputs, which makes it causal. It is in fact memoryless

Stable $\boxed{\text{Yes}}$ If $x[n]$ is bounded by $\pm B$, then $y[n]$ is bounded by 0 and B

(b) Linear $\boxed{\text{Yes}}$ Let the input be $\alpha x_1(t) + \beta x_2(t) \longrightarrow (\alpha x_1(t-3) + \beta x_2(t-3)) + 2\int_{-\infty}^{0.5t}(\alpha x_1(\tau) + \beta x_2(\tau))\,d\tau$

$$= \underbrace{\alpha\left(x_1(t-3) + 2\int_{-\infty}^{0.5t} x_1(\tau)\,d\tau\right)}_{\alpha y_1(t)} + \underbrace{\beta\left(x_2(t-3) + 2\int_{-\infty}^{0.5t} x_2(\tau)\,d\tau\right)}_{\beta y_2(t)}$$

Time invariant $\boxed{\text{No}}$ $x_1(t) = g(t) \longrightarrow y_1(t) = x_1(t-3) + 2\int_{-\infty}^{0.5t} x_1(\tau)\,d\tau$

$= g(t-3) + 2\int_{-\infty}^{0.5t} g(\tau)\,d\tau \xrightarrow[t_0]{\text{delay}}$ $\boxed{y_1(t-t_0) = g(t-t_0-3) + 2\int_{-\infty}^{0.5t-0.5t_0} g(\tau)\,d\tau}$

$x_2(t) = g(t-t_0) \longrightarrow y_2(t) = x_2(t-3) + 2\int_{-\infty}^{0.5t} x_2(\tau)\,d\tau$

$= g(t-3-t_0) + 2\int_{\tau=-\infty}^{\tau=0.5t} g(\tau - t_0)\,d\tau$ $\boxed{= g(t-t_0-3) + 2\int_{-\infty}^{0.5t-t_0} g(\tau)\,d\tau}$

not the same

change of variables

let $\lambda = \tau - t_0$

$\Rightarrow \tau = \lambda + t_0$

$d\tau = d\lambda$

Causal? $\boxed{\text{No}}$

In order to compute	we need
$y(-100)$	$x(t)$ from $-\infty \le t \le \underbrace{-50}$

relative to $y(-100)$, $x(-50)$ is in the future!

Stable? $\boxed{\text{No}}$

Let $x(t) = u(t)$ which is a <u>bounded</u> <u>input</u>

because of the limits of integration we will have

$$y(t) = u(t-3) + 2 \cdot \text{ramp}(0.5t)$$
$$= u(t-3) + \underbrace{\text{ramp}(t)}$$

and this is unbounded!