Assume that $x[n]$ is real and even, i.e. $x[n] = x[-n]$.
Further assume that z_0 is a zero of $X(z)$, i.e. $X(z_0) = 0$

(a) Show that $1/z_0$ is also a zero of $X(z)$

(b) Are there other zeros of $X(z)$ implied by the information given?

(a) According to the definition of the z-transform, we have

$$\sum_{n=-\infty}^{\infty} x[n] z^{-n} = X(z)$$

and we also have

Let $n=-m$

$$\sum_{n=-\infty}^{\infty} x[-n] z^{-n} = \sum_{m=-\infty}^{\infty} x[n] z^{n} = \boxed{X(z^{-1}) = X(z)}$$

If we expand $X(z)$ as a factored rational polynomial we have

$$X(z) = X(z^{-1}) = \frac{(z-z_0)(z-z_1)\cdots(z-z_m)}{(z-p_0)(z-p_1)\cdots(z-p_N)} = \frac{(z^{-1}-z_0)(z^{-1}-z_1)\cdots(z^{-1}-z_m)}{(z^{-1}-p_0)(z^{-1}-p_1)\cdots(z^{-1}-p_N)}$$

The only way this can be true is if the zeros (and poles!) of $X(z)$ come in pairs $z = z_0$ and $z = 1/z_0$

(b) Yes, since $x[n]$ is given to be real, its poles and zeros come in complex-conjugate pairs, so if you are given a zero $z = z_0$, you also know there are zeros at $z = z_0^*$, $z = 1/z_0$, and $z = 1/z_0^*$