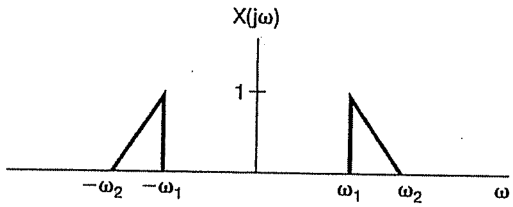
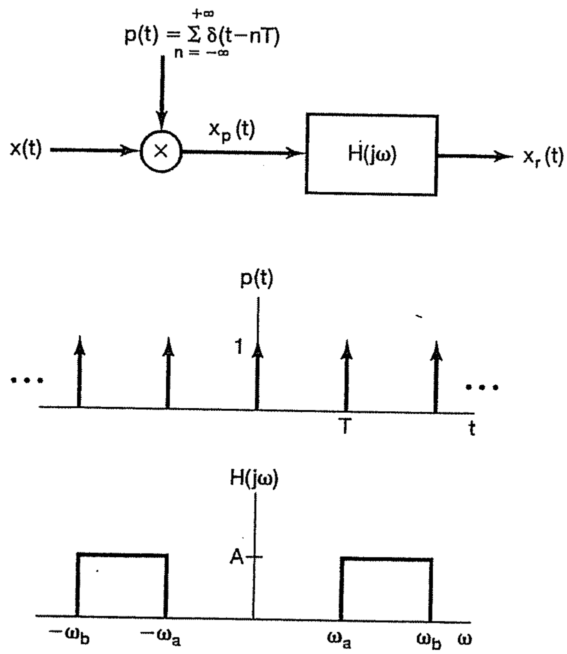
The sampling theorem, as we have derived it, states that a signal $x(t)$ must be sampled at a rate greater than its bandwidth (or equivalently, a rate greater than twice its highest frequency). This implies that if $x(t)$ has a spectrum as indicated in Figure P7.26(a) then $x(t)$ must be sampled at a rate greater than $2\omega_2$. However, since the signal has most of its energy concentrated in a narrow band, it would seem reasonable to expect that a sampling rate lower than twice the highest frequency could be used. A signal whose energy is concentrated in a frequency band is often referred to as a *bandpass signal*. There are a variety of techniques for sampling such signals, generally referred to as *bandpass-sampling* techniques.



(a)



(b)

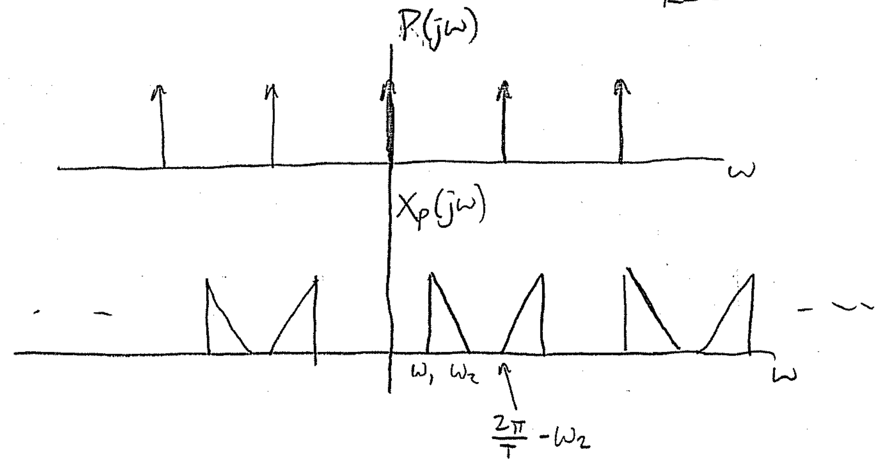
To examine the possibility of sampling a bandpass signal as a rate less than the total bandwidth, consider the system shown in Figure P7.26(b). Assuming that $\omega_1 > \omega_2 - \omega_1$, find the maximum value of T and the values of the constants A, ω_a, and ω_b such that $x_r(t) = x(t)$.

We start with the fact that

$$P(j\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega - k\frac{2\pi}{T}\right)$$

and since $x_p(t) = x(t)\,p(t)$,

$$X_p(j\omega) = \frac{1}{2\pi}\left\{X(j\omega) * P(j\omega)\right\} = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(j\left(\omega - k\frac{2\pi}{T}\right)\right)$$



As T increases, $\frac{2\pi}{T} - \omega_2$ decreases. Aliasing occurs when the triangles bump into each other. The overlapping will start when $\frac{2\pi}{T} - \omega_2 < \omega_2$, and it will continue as long as $\omega_1 < \frac{2\pi}{T} - \omega_2 + (\omega_2 - \omega_1)$

we can re-arrange this last expression as $2\omega_1 - \omega_2 < \frac{2\pi}{T} - \omega_2$, and combining with the first expression yields

$$2\omega_1 - \omega_2 < \frac{2\pi}{T} - \omega_2 < \omega_2 \quad \leftarrow \text{aliasing occurs under these conditions}$$

It is given that $0 < 2\omega_1 - \omega_2$, so if we pick T such that

$$0 < \frac{2\pi}{T} - \omega_2 < 2\omega_1 - \omega_2,$$ there will be <u>no</u> <u>aliasing</u>

For maximum T, choose minimum value of $\frac{2\pi}{T} - \omega_2$ (which is zero)

$T_{max} = \frac{2\pi}{\omega_2}$ $\quad \omega_b = \frac{2\pi}{T}$

$A = T$ $\quad \omega_a = \omega_1$