Let $y[n] = \left(\dfrac{\sin(\frac{\pi}{4}n)}{\pi n}\right)^2 * \left(\dfrac{\sin(\omega_c n)}{\pi n}\right)$

where $|\omega_c| \leq \pi$. Determine a stricter constraint on ω_c which ensures that

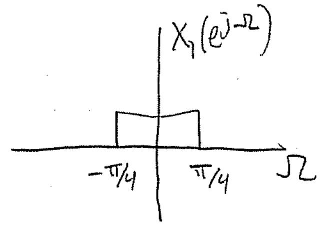
$$y[n] = \left(\frac{\sin(\frac{\pi}{4}n)}{\pi n}\right)^2$$

We first consider the signal

$$X_1[n] = \frac{\sin(\frac{\pi}{4}n)}{\pi n}.$$

Using DTFT transform pairs and properties, we see that the DTFT of $x_1[n]$ is

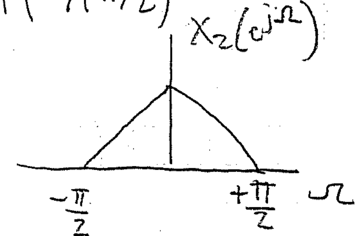
$$X_1(e^{j\Omega}) = \begin{cases} 1 & 0 \leq |\omega| \leq \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\omega| < \pi \end{cases}$$



$$= \mathrm{rect}(\omega/(\pi/2))$$

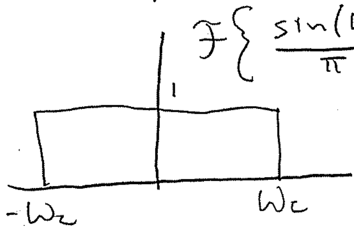
Since $X_2[n] = X_1[n]x_1[n] = \left(\dfrac{\sin(\frac{\pi}{4}n)}{\pi n}\right)^2$, then

$$X_2(e^{j\Omega}) = X_1(e^{j\Omega}) * X_1(e^{j\Omega}) = \mathrm{tri}(\omega/(\pi/2))$$



Since $X_2[n]$ is our desired output, the value of ω_c has to be just large enough to accomodate all of $X_2[n]$'s bandwidth

$\mathcal{F}\left\{\dfrac{\sin(\omega_c n)}{\pi n}\right\}$



Select ω_c anywhere in the range

$$\boxed{\frac{\pi}{2} \leq \omega_c \leq \pi}$$