Consider a signal $x(t)$ with CTFT $X(j\omega)$. Suppose we are given the following facts:

1. $x(t)$ is real and nonnegative
2. $\mathcal{F}^{-1}\{(1+j\omega)X(j\omega)\} = Ae^{-2t}u(t)$, where A is independent of t
3. $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi$

Determine a closed-form expression for $x(t)$

We start by taking the CTFT of both sides of fact 2 and solve for $X(j\omega)$

$$Ae^{-2t}u(t) \Longleftrightarrow \frac{A}{2+j\omega}, \quad \text{so}$$

$$X(j\omega) = \frac{A}{(1+j\omega)(2+j\omega)}$$

which we use partial fraction expansion to get

$$= A\left(\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right)$$

These are found in Table 10.1

$$x(t) = A\left(e^{-t} - e^{-2t}\right)u(t)$$

Using Parseval's relation we have

using fact 3

$$\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt \Rightarrow \int_{-\infty}^{\infty} |x(t)|^2 dt = 1$$

$$A^2 \int_0^{\infty} \left[e^{-t} - e^{-2t}\right]^2 dt = A^2 \int_0^{\infty} \left[e^{-2t} + e^{-4t} - e^{-3t}\right] dt = \frac{A^2}{12} = 1$$

$$\boxed{\Rightarrow A = \sqrt{12}}$$

We choose $A = \sqrt{12}$ instead of $A = -\sqrt{12}$ because of fact 1