

Let  $x(t)$  be a periodic signal whose CTFS harmonic function is

$$X[k] = \begin{cases} 2, & k=0 \\ j\left(\frac{1}{2}\right)^{|k|}, & \text{otherwise} \end{cases}$$

Use CTFS properties to answer the following questions:

(a) Is  $x(t)$  real?

According to O+W Table 3.1, if  $x(t)$  is real then

$$X[k] = X^*[-k]$$

$$\underbrace{j\left(\frac{1}{2}\right)^{|k|}}_{X[k]} \stackrel{?}{=} \underbrace{-j\left(\frac{1}{2}\right)^{|-k|}}_{X^*[-k]} \quad \leftarrow \text{no, equality does not hold, } x(t) \text{ is not real}$$

(b) Is  $x(t)$  even?

According to p. 273 and O+W Table 3.1, the time reversal

property is

$$x(t) \xleftrightarrow{\text{FS}} X[k]$$

$$x(-t) \xleftrightarrow{\text{FS}} X[-k]$$

so if  $x(t)$  is even, then  $x(t) = x(-t) \Rightarrow X[k] = X[-k]$

$$j\left(\frac{1}{2}\right)^{|k|} \stackrel{?}{=} j\left(\frac{1}{2}\right)^{|-k|} \quad \leftarrow \text{equality does hold, } x(t) \text{ is even}$$

(c) Is  $\frac{d}{dt}x(t)$  even?

According to p 273 and O+W Table 1

$$\frac{d}{dt}x(t) \xleftrightarrow{\text{FS}} j2\pi k f_0 X[k] = \begin{cases} 0, & k=0 \\ -2\pi k f_0 \left(\frac{1}{2}\right)^{|k|}, & \text{otherwise} \end{cases}$$

$\frac{d}{dt}x(t)$  is not even

If this function satisfies

$$X[k] = X[-k] \text{ then } \frac{d}{dt}x(t) \text{ is even. } \underline{\underline{\text{It does not}}}$$