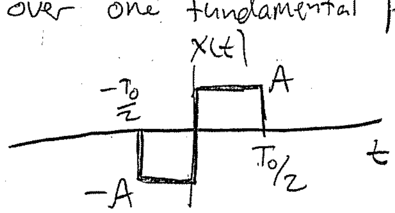
Problem 8.27 from Roberts

A periodic signal x(t) is described over one fundamental period by

$$x(t) = \begin{cases} -A, & -\frac{T_0}{2} < t < 0 \\ A, & 0 < t < \frac{T_0}{2} \end{cases}$$



Find X[k], then use the integration property to find the CTFS of the integral of x(t) and graph the resulting CTFS assuming the average value of the integration is zero.

Let $T_F = T_0$

$$X[k] = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-j2\pi k f_0 t} dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{0} (-A) e^{-j2\pi k f_0 t} dt + \frac{1}{T_0} \int_{0}^{T_0/2} (A) e^{-j2\pi k f_0 t} dt$$

$$= \frac{-A}{-j2\pi k f_0 T_0} e^{-j2\pi k f_0 t}\Big|_{t=-T_0/2}^{0} + \frac{A}{-j2\pi k f_0 T_0} e^{-j2\pi k f_0 t}\Big|_{t=0}^{T_0/2}$$

$$= \frac{A}{-j2\pi k}\Big[-1 + \underbrace{e^{j2\pi k f_0 T_0/2} + e^{-j2\pi k f_0 T_0/2}}_{2\cos(k\pi)} - 1\Big]$$

$$= jA \frac{\cos(k\pi) - 1}{k\pi}$$

Let $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$ using the integration property

$$y(t) \xleftrightarrow{FS} \frac{X[k]}{j2\pi k f_0} \quad k = 0, \text{ if } X[0] = 0$$ (we can see that the DC value of x(t) is 0)

Therefore $Y[k] = \frac{A T_0}{2} \frac{\cos(k\pi) - 1}{(k\pi)^2}, \quad k \neq 0$

and we are told to assume $Y[0] = 0$