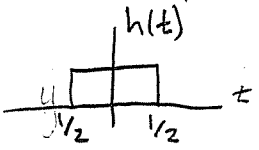
Suppose the signal $x(t) = u(t+0.5) - u(t-0.5)$ is convolved with the signal $h(t) = e^{j\omega_0 t}$

(a) Determine a value of ω_0 which ensures that $y(0) = 0$.

Solution Step #1: compute $y(t) = x(t) * h(t)$



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-1/2}^{1/2} e^{j\omega_0(t-\tau)}\, d\tau$$

$$= e^{j\omega_0 t} \int_{-1/2}^{1/2} e^{-j\omega_0 \tau}\, d\tau$$

$$= e^{j\omega_0 t} \cdot \frac{1}{-j\omega_0} e^{-j\omega_0 \tau} \Big|_{\tau=-1/2}^{1/2}$$

$$= e^{j\omega_0 t} \cdot \frac{2}{\omega_0} \cdot \sin(\omega_0/2)$$

Step #2: set $y(0) = 0$ and solve for ω_0

$$y(0) = e^{j\omega_0 \cdot 0} \cdot \frac{2}{\omega_0} \cdot \sin(\omega_0/2)$$ ← this is zero when $\omega_0 = 2\pi$

(b) Is the answer in part (a) unique?

No, $\omega_0 = 2\pi, 4\pi, 6\pi, \cdots$ are all valid solutions