Let's compute $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$

where $x(t) = e^{-at} u(t)$, $a > 0$

and $h(t) = u(t)$.

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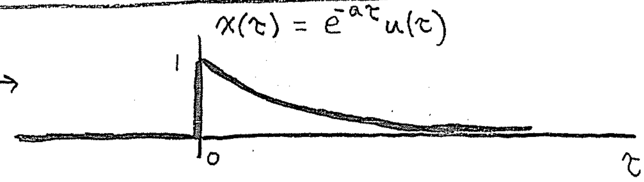
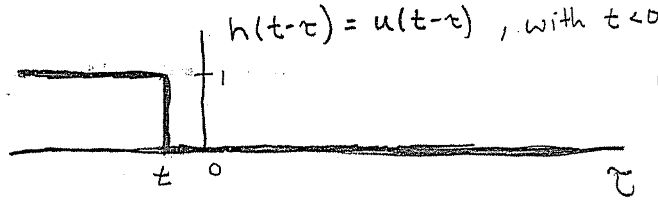
**Step 0)** Draw $x(\tau) \longrightarrow$




(1)

**Step 1)** Pick a value for $t \longrightarrow$ I'll do all values of $t < 0$

**Step 2)** Draw $h(t-\tau) \rightarrow$

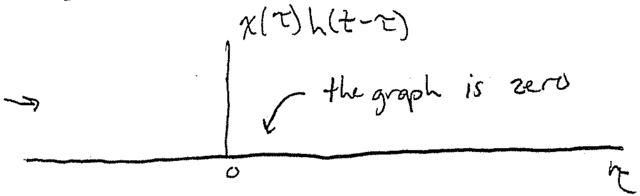



(2)

Recall that the unit step is "on" when its argument is positive. In the case of $h(t-\tau) = u(t-\tau)$, we want to find the range of values of $\tau$ that satisfy $t - \tau > 0 \Rightarrow \boxed{\tau < t}$

Therefore, the graph of $h(t-\tau)$ turns "on" at $t$, and stays on for all values of $\tau < t$.

**Step 3)** Draw $x(\tau)h(t-\tau) \rightarrow$
(i.e. multiply the two graphs above)




(3)

**Step 4)** Integrate $x(\tau)h(t-\tau) \rightarrow \int_{-\infty}^{\infty} (0)\, d\tau = 0$
(i.e. find the area under the curve)

so far we have $y(t) = \begin{cases} 0, & t < 0 \\ & \end{cases}$

**Step 5)** Go back to step 1

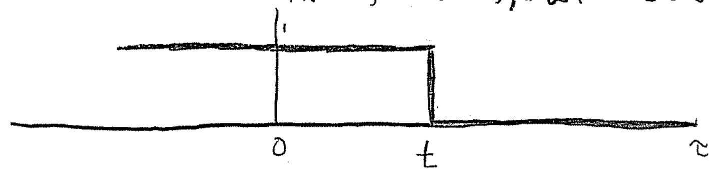
$\longrightarrow$ next page

→ continued from previous page

<u>Step 1)</u> Last time we were at step 1), we picked all values of $t<0$, Based on the way graphs (1) and (2) turned out, it is pretty easy to see that when $t$ "slides" anywhere over the range $t<0$, there will never be any interesting overlap between the two graphs, and the area under the overlap will be zero. Now we pick $t>0$ and go through the steps again

<u>Step 2)</u> Draw $h(t-\tau)$

$h(t-\tau) = u(t-\tau)$, with $t>0$

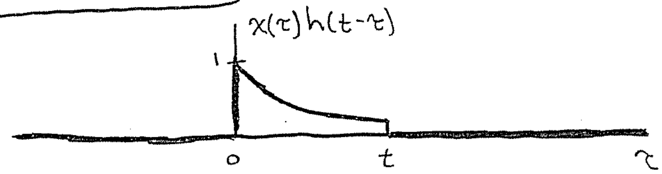



(4)

Once again $u(\underbrace{t-\tau})$ is "on" when $t-\tau>0 \Rightarrow$ when $\tau<t$

<u>Step 3)</u> Draw $x(\tau)h(t-\tau)$

(i.e. multply graphs (1) and (4))




(5)

<u>Step 4)</u> Integrate $x(\tau)h(t-\tau)$

(i.e. find the area under the curve

$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau, \text{ with } t>0$$

$$= \int_{-\infty}^{\infty}\Big(e^{-a\tau}\underbrace{u(\tau)}\Big)\Big(\underbrace{u(t-\tau)}\Big)d\tau$$

Based on graph (5), we easily see that this affects the upper limit of integration

Based on graph (5), we easily see that this affects the lower limit of integration

$$= \int_{0}^{t} e^{-a\tau}\,d\tau$$

$$= -\frac{1}{a}\left[e^{-a\tau}\right]_{\tau=0}^{t} = \frac{1}{a}\left(1-e^{-at}\right)$$

so far we have $y(t) = \begin{cases} 0, & t<0 \\ \frac{1}{a}(1-e^{-at}) & t>0 \end{cases}$ (6)

<u>Step 5)</u> there are no values of $t$ left!

we can simplify Eq. (6) as $\boxed{y(t) = \frac{1}{a}(1-e^{-at})\,u(t)}$