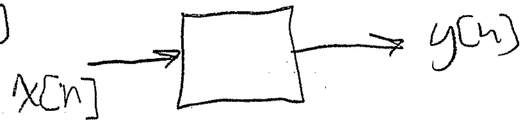
Consider the discrete-time system with excitation $x[n]$ and response $y[n]$



where $y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$, n_0 is a positive, finite integer

(a) Is the system linear?

let $x_1[n] = g[n] \longrightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} g[k]$

$x_2[n] = h[n] \longrightarrow y_2[n] = \sum_{k=n-n_0}^{n+n_0} h[k]$

$x_3[n] = \alpha g[n] + \beta h[n] \rightarrow y_3[n] = \sum_{k=n-n_0}^{n+n_0} \left(\alpha g[k] + \beta h[k]\right)$

$\boxed{\text{yes it is linear}}$ $\qquad = \alpha y_1[n] + \beta y_2[n]$

(b) Is this system time invariant?

Let $x_1[n] = g[n] \longrightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} g[k]$

Let $x_2[n] = g[n-n_1] \longrightarrow y_2[n] = \sum_{k=n-n_0}^{n+n_0} g[k-n_1]$

Let $m = k - n_1 \Rightarrow k = m + n_1$

$\boxed{\text{yes it is time invariant}}$ $\qquad y_2[n] = \sum_{m=n-n_1-n_0}^{n-n_1+n_0} g[m] = y_1[n-n_1]$

(c) If $|x[n]| < B$ for all n (bounded input)

then $y[n] = \sum_{k=n-n_0}^{n+n_0} x[n] \leq \sum_{k=n-n_0}^{n+n_0} |x[n]| \leq \sum_{k=n-n_0}^{n+n_0} B = (2n_0+1)B$

so a bounded input leads to a bounded output