For each of the following input-output relationships, determine whether the corresponding system is linear, time invariant, or both

(a) $y(t) = t^2 x(t-1)$

linear
nonlinear

$x_1(t) = g(t) \rightarrow y_1(t) = t^2 g(t-1)$, $x_2(t) = h(t) \rightarrow y_2(t) = t^2 h(t-1)$

$\alpha x_1(t) + \beta x_2(t) \rightarrow t^2(\alpha g(t-1) + \beta h(t-1)) = \alpha y_1(t) + \beta y_2(t)$

time-invariant
time-varying

$y(t) = t^2 x(t-1)$

the system needs to know what time it is to compute its output

(b) $y[n] = x^2[n-2]$

linear
nonlinear

$\alpha g[n] + \beta h[n] \rightarrow (\alpha g[n-2] + \beta h[n-2])^2$

this is a nonlinearity

time-invariant
time-varying

$x_1[n] = g[n] \rightarrow y_1[n] = g^2[n-2]$

$x_2[n] = g[n-n_0] \rightarrow y_2[n] = g^2[n-n_0-2] \boxed{= y_1[n-n_0]}$

(c) $y[n] = x[n+1] - x[n-1]$

linear
nonlinear

$\alpha g[n] + \beta h[n] \rightarrow \alpha\overbrace{(g[n+1] - g[n-1])}^{y_1[n]} + \beta\overbrace{(h[n+1] - h[n-1])}^{y_2[n]}$

time-invariant
time-varying

$x[n] = g[n-n_0] \rightarrow g[n-n_0+1] - g[n-n_0-1] = y[n-n_0]$

(d) $y(t) = Od\{x(t)\} = \frac{x(t) - x(-t)}{2}$

linear
nonlinear

$\alpha g(t) + \beta h(t) \rightarrow \alpha \overbrace{\frac{g(t) - g(-t)}{2}}^{od(g(t))} + \beta \overbrace{\frac{h(t) - h(-t)}{2}}^{od(h(t))}$

time-invariant
time-varying

$x_1(t) = g(t) \rightarrow y_1(t) = \frac{g(t) - g(-t)}{2}$

$x_2(t) = g(t-t_0) \rightarrow y_2(t) = \frac{g(t-t_0) - g(-t-t_0)}{2}$

$\boxed{\neq y_1(t-t_0)}$