Let $x[n]$ be a discrete-time signal and let

$$y_1[n] = x[2n] \quad \text{and} \quad y_2[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

$y_1[n]$ is a speeded up version of $x[n]$, and $y_2[n]$ is a slowed down version of $x[n]$

Consider the following statements

(1) If $x[n]$ is periodic, then $y_1[n]$ is periodic

(2) If $y_1[n]$ is periodic, then $x[n]$ is periodic

(3) If $x[n]$ is periodic, then $y_2[n]$ is periodic

(4) If $y_2[n]$ is periodic, then $x[n]$ is periodic

Determine whether each of these is true, if so determine the relationship between the fundamental periods of the two signals. If false, produce a counterexample to the statement

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(1) True $x[n] = x[n+N]$; $y_1[n] = y[n+N_0]$, i.e. $y[n]$ is periodic with period $N_0 = N/2$ if N is even, and $N_0 = N$ if N is odd

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(2) False, $y_1[n]$ has less information than $x[n]$, and we don't know what was in the samples of $x[n]$ that are not present in $y_1[n]$

Let $x[n] = g[n] + h[n]$, $g[n] = \begin{cases} 1 & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$ $h[n] = \begin{cases} 0, & n \text{ even} \\ n, & n \text{ odd} \end{cases}$

periodic (g[n]) aperiodic (h[n])

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(3) True $x[n] = x[n+N]$; $y_2[n] = y_2[n+N_0]$, where $N_0 = 2N$

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(4) True $y_2[n] = y_2[n+N]$; $x[n] = x[n+N_0]$, where $N_0 = N/2$