EECS 360 Lab 11

Laplace Transform

The Laplace transform of a signal $x(t)$,

$$x(t) \xrightarrow{L} X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

is a generalization of the continuous-time Fourier transform that is useful for studying CT signals and system. When $s = j\omega$, i.e. the Laplace transform reduces to the CTFT.

Most of the times, the Laplace transform can be represented as a ratio of polynomials in s:

$$X(s) = \frac{N(s)}{D(s)}$$

which is also known as rational transforms. Rational transforms can be completely determined by the roots of the polynomial $N(s)$ and $D(s)$, known as zeros and poles.

1. pole-zero diagram.
   A pole-zero diagram displays the "poles" and "zeros" of the rational transform by placing an 'x' at each pole location and an 'o' at each zero location in the complex *s*-plane.
   Poles and zeros can be found out by using *roots* function in matlab, i.e.

```
Transfer function:
           s - 1
H(s) = -------------
       s^2 + 3 s + 2

>>b = [1 -1];  % Numerator coefficients
>>a = [1 3 2]; % Demoninator coefficients
>>zs = roots(b)
zs =
     1
>>ps = roots(a)
ps =
    -2
    -1
>>pzmap(ps,zs) % generates pole-zero diagram
```

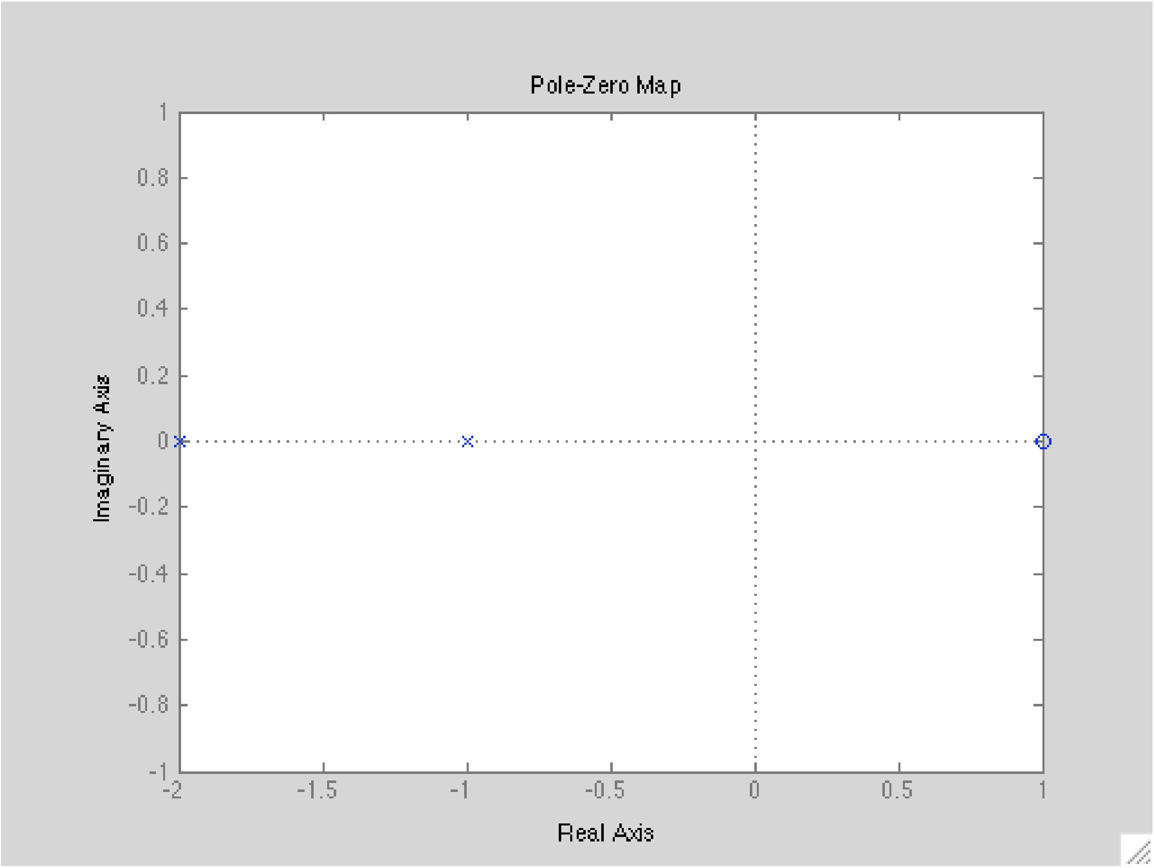

Fig.1 Pole-zero diagram

Using the method given above, find out the zeros and poles of the following system functions and plot them:

(1). $H(s) = \dfrac{s+5}{s^2 + 2s + 3}$

(2). $H(s) = \dfrac{2s^2 + 5s + 12}{s^2 + 2s + 10}$

(3). $H(s) = \dfrac{2s^2 + 5s + 12}{(s^2 + 2s + 10)(s + 2)}$

Graph the ROC of each system function manually on your figures. (You can also graph ROC superimposed on your pole-zero diagram if you would.)

2. Surface plots of Laplace transforms

In this exercise, you will visually explore the surfaces defined by rational Laplace transforms and the relationship between these surfaces and the CTFT.

Consider a transfer function:

$$H(s) = \frac{s^2 + 2s + 17}{s^2 + 4s + 104}$$

(1). Define the numerator and denominator polynomial coefficients as vector b and a respectively.

(2). Use the *freqs* function to evaluate the frequency response of a Laplace transform.

```
H = freqs(b,a,omega);
```

where $-20 \le \omega \le 20$ (omega) is the frequency vector in rad/s. (Hint: use *linspace* to generate a vector with 200 samples.)

(3). Graph the magnitude and phase of the frequency response.

(4). Complex number *s* in the Laplace transform is represented as:

$$s = \sigma + j\omega$$

A 3-D surface plot of the system transform function *H(s)* at the range of interests, i.e. $-20 \le \omega \le 20$ and, $-5 \le \sigma \le 5$ is extremely useful to illustrate the relationship between the frequency response $H(s)$ and the pole-zero locations.

the system response matrix *s* can be generated from ω and σ using *meshgrid* function:

```
[sigmagrid,omegagrid] = meshgrid(sigma,omega);
```

hence, $s = \sigma + j\omega$ is:

```
sgrid = sigmagrid+j*omegagrid;
```

use function *polyval* to evaluate the numerator and denominator polynomials at the specific range:

```
H1 = polyval(b,sgrid)./polyval(a,sgrid);
```

Finally, use *mesh()* function to generate the surface graph of the magnitude of *H(s)* in dB:

```
mesh(sigma,omega,10*log10(abs(H1)))
```

Where are the poles and zeros on the surface plot? What's the relationship between the surface plot and the plot in 2.(2)?