EECS 360 Signal and System
Lab 7. Approximation of the Continuous Time Fourier Transform

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The continuous time Fourier Transform can be approximated by the following sum:

$$\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt = \lim_{\tau\to 0}\sum_{n=-\infty}^{\infty} x(m\tau)e^{-j2\pi f_n\tau}\tau \tag{1}$$

Given that you have a record of T seconds sampled every τ seconds resulting in a total of N samples (Note: $T = N * \tau$ of the signal $x(t)$ then the continuous time Fourier Transform can be approximated by:

$$\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt = \int_{0}^{T} x(t)e^{-j2\pi ft}\,dt \approx \sum_{m=-0}^{N-1} x(m\tau)e^{-j2\pi f_m\tau}\tau \tag{2}$$

Let

$$f_0 = \frac{1}{T} = \frac{1}{N\tau} \quad \text{so} \quad f = nf_0 = \frac{n}{N\tau} \tag{3}$$

and define:

$$X_n = \sum_{m=0}^{N-1} x(m\tau)e^{-j2\pi nf_0 m\tau}\tau = \tau\sum_{m=0}^{N-1} x(m\tau)e^{-j2\pi nm/N} \tag{4}$$

Now X_n can be viewed as an approximation for the continuous time Fourier Transform of $x(t)$ at $f = nf_0$. Note that X_n is a complex number.

(a). Given $x(t)$, T, τ, write a Matlab routine to find X_n.
(b). Test and validate your routine for signal $x(t) = e^{-t}$ and $x(t) = \sin(2\pi 10t)$ with $\tau = 0.01$ second and $T = 1.28$ seconds by analytically determining the FT for signal $x(t)$. Graph $|X_n|$ and the phase of X_n.
(c). Repeat part (b) for $T = 5.12$ seconds.
(d). Repeat part (b) for $\tau = 0.1$ second and $T = 12.8$ seconds.
(e). Comment on the accuracy of the approximation as T and τ change.

Hint:
Recall the matrices multiplication:

$$\mathbf{X} * \mathbf{Y} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \sum_{n=1}^{4} x[n] * y[n] = 4$$